

KRISHNA PUBLIC SCHOOL

XII Physics Advanced Study
Material

1. Electrostatics
2. Current electricity
3. Effects of electric current
4. Electromagnetic Induction and Alternating Current
5. Electromagnetic Waves and Wave Optics
6. Atomic physics
7. Dual Nature of Radiation and Matter and Relativity
8. Nuclear Physics
9. Semiconductor Devices and their Applications
10. Communication Systems

1. Electrostatics

Electrostatics is the branch of Physics, which deals with static electric charges or charges at rest. In this chapter, we shall study the basic phenomena about static electric charges. The charges in an electrostatic field are analogous to masses in a gravitational field. These charges have forces acting on them and hence possess potential energy. The ideas are widely used in many branches of electricity and in the theory of atom.

1.1 Electrostatics – frictional electricity

In 600 B.C., Thales, a Greek Philosopher observed that, when a piece of amber is rubbed with fur, it acquires the property of attracting light objects like bits of paper. In the 17th century, William Gilbert discovered that, glass, ebonite etc, also exhibit this property, when rubbed with suitable materials.

The substances which acquire charges on rubbing are said to be 'electrified' or charged. These terms are derived from the Greek word elektron, meaning amber. The electricity produced by friction is called frictional electricity. If the charges in a body do not move, then, the frictional electricity is also known as Static Electricity.

1.1.1 Two kinds of charges

(i) If a glass rod is rubbed with a silk cloth, it acquires positive charge while the silk cloth acquires an equal amount of negative charge.

(ii) If an ebonite rod is rubbed with fur, it becomes negatively charged, while the fur acquires equal amount of positive charge. This classification of positive and negative charges were termed by American scientist, Benjamin Franklin.

Thus, charging a rod by rubbing does not create electricity, but simply transfers or redistributes the charges in a material.

1.1.2 Like charges repel and unlike charges attract each other – experimental verification.

A charged glass rod is suspended by a silk thread, such that it swings horizontally. Now another charged glass rod is brought near the end of the suspended glass rod. It is found that the ends of the two rods repel each other (Fig 1.1). However, if a charged ebonite rod is brought near the end of the suspended rod, the two rods attract each other (Fig 1.2). The above experiment shows that like charges repel and unlike charges attract each other.

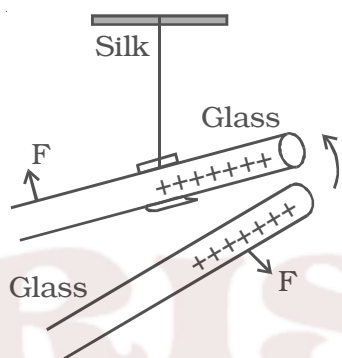


Fig. 1.1 Two charged rods of same sign

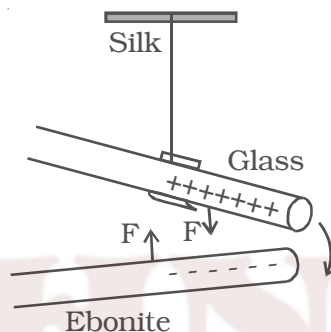


Fig 1.2 Two charged rods of opposite sign

The property of attraction and repulsion between charged bodies have many applications such as electrostatic paint spraying, powder coating, fly-ash collection in chimneys, ink-jet printing and photostat copying (Xerox) etc.

1.1.3 Conductors and Insulators

According to the electrostatic behaviour, materials are divided into two categories : conductors and insulators (dielectrics). Bodies which allow the charges to pass through are called conductors. e.g. metals, human body, Earth etc. Bodies which do not allow the charges to pass through are called insulators. e.g. glass, mica, ebonite, plastic etc.

1.1.4 Basic properties of electric charge

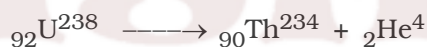
(i) Quantisation of electric charge

The fundamental unit of electric charge (e) is the charge carried by the electron and its unit is coulomb. e has the magnitude 1.6×10^{-19} C.

In nature, the electric charge of any system is always an integral multiple of the least amount of charge. It means that the quantity can take only one of the discrete set of values. The charge, $q = ne$ where n is an integer.

(ii) Conservation of electric charge

Electric charges can neither be created nor destroyed. According to the law of conservation of electric charge, the total charge in an isolated system always remains constant. But the charges can be transferred from one part of the system to another, such that the total charge always remains conserved. For example, Uranium (${}_{92}\text{U}^{238}$) can decay by emitting an alpha particle (${}_{2}\text{He}^4$ nucleus) and transforming to thorium (${}_{90}\text{Th}^{234}$).



Total charge before decay = $+92e$, total charge after decay = $90e + 2e$. Hence, the total charge is conserved. i.e. it remains constant.

(iii) Additive nature of charge

The total electric charge of a system is equal to the algebraic sum of electric charges located in the system. For example, if two charged bodies of charges $+2q$, $-5q$ are brought in contact, the total charge of the system is $-3q$.

1.1.5 Coulomb's law

The force between two charged bodies was studied by Coulomb in 1785.

Coulomb's law states that the force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between

them. The direction of forces is along the line joining the two point charges.

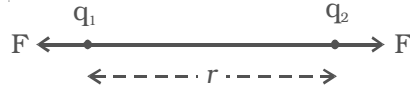


Fig 1.3a Coulomb forces

Let q_1 and q_2 be two point charges placed in air or vacuum at a distance r apart (Fig. 1.3a). Then, according to Coulomb's law,

$$F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad F = k \frac{q_1 q_2}{r^2}$$

where k is a constant of proportionality. In air or vacuum, $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the permittivity of free space (i.e., vacuum) and the value of ϵ_0 is $8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \dots(1)$$

$$\text{and} \quad \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

In the above equation, if $q_1 = q_2 = 1\text{C}$ and $r = 1\text{m}$ then,

$$F = (9 \times 10^9) \frac{1 \times 1}{1^2} = 9 \times 10^9 \text{ N}$$

One Coulomb is defined as the quantity of charge, which when placed at a distance of 1 metre in air or vacuum from an equal and similar charge, experiences a repulsive force of $9 \times 10^9 \text{ N}$.

If the charges are situated in a medium of permittivity ϵ , then the magnitude of the force between them will be,

$$F_m = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \quad \dots(2)$$

Dividing equation (1) by (2)

$$\frac{F}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$$

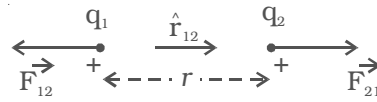
The ratio $\frac{\epsilon}{\epsilon_0} = \epsilon_r$, is called the relative permittivity or dielectric constant of the medium. The value of ϵ_r for air or vacuum is 1.

$$\therefore \epsilon = \epsilon_0 \epsilon_r$$

Since $F_m = \frac{F}{\epsilon_r}$, the force between two point charges depends on the nature of the medium in which the two charges are situated.

Coulomb's law - vector form

If \vec{F}_{21} is the force exerted on charge q_2 by charge q_1 (Fig.1.3b),



$$\vec{F}_{21} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

where \hat{r}_{12} is the unit vector from q_1 to q_2 .

If \vec{F}_{12} is the force exerted on q_1 due to q_2 ,

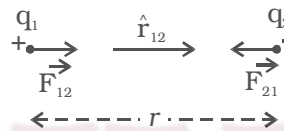


Fig 1.3b Coulomb's law in vector form

$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

where \hat{r}_{21} is the unit vector from q_2 to q_1 .

[Both \hat{r}_{21} and \hat{r}_{12} have the same magnitude, and are oppositely directed]

$$\therefore \vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} (-\hat{r}_{12})$$

$$\text{or } \vec{F}_{12} = -k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$\text{or } \vec{F}_{12} = -\vec{F}_{21}$$

So, the forces exerted by charges on each other are equal in magnitude and opposite in direction.

1.1.6 Principle of Superposition

The principle of superposition is to calculate the electric force experienced by a charge q_1 due to other charges q_2, q_3, \dots, q_n .

The total force on a given charge is the vector sum of the forces exerted on it due to all other charges.

The force on q_1 due to q_2

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

Similarly, force on q_1 due to q_3

$$\vec{F}_{13} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31}$$

The total force F_1 on the charge q_1 by all other charges is,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots + \vec{F}_{1n}$$

Therefore,

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31} + \dots + \frac{q_1 q_n}{r_{n1}^2} \hat{r}_{n1} \right]$$

1.2 Electric Field

Electric field due to a charge is the space around the test charge in which it experiences a force. The presence of an electric field around a charge cannot be detected unless another charge is brought towards it.

When a test charge q_o is placed near a charge q , which is the source of electric field, an electrostatic force F will act on the test charge.

Electric Field Intensity (E)

Electric field at a point is measured in terms of electric field intensity. Electric field intensity at a point, in an electric field is defined as the force experienced by a unit positive charge kept at that point.

It is a vector quantity. $|\vec{E}| = \frac{|\vec{F}|}{q_0}$. The unit of electric field intensity is N C^{-1} .

The electric field intensity is also referred as electric field strength or simply electric field. So, the force exerted by an electric field on a charge is $F = q_0 E$.

1.2.1 Electric field due to a point charge

Let q be the point charge placed at O in air (Fig.1.4). A test charge q_0 is placed at P at a distance r from q . According to Coulomb's law, the force acting on q_0 due to q is



Fig 1.4 Electric field due to a point charge

$$F = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2}$$

The electric field at a point P is, by definition, the force per unit test charge.

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

The direction of E is along the line joining O and P, pointing away from q , if q is positive and towards q , if q is negative.

In vector notation $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$, where \hat{r} is a unit vector pointing away from q .

1.2.2 Electric field due to system of charges

If there are a number of stationary charges, the net electric field (intensity) at a point is the vector sum of the individual electric fields due to each charge.

$$\begin{aligned} \vec{E} &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1^2} \hat{r}_1 + \frac{q_2}{r_2^2} \hat{r}_2 + \frac{q_3}{r_3^2} \hat{r}_3 + \dots \right] \end{aligned}$$

1.2.3 Electric lines of force

The concept of field lines was introduced by Michael Faraday as an aid in visualizing electric and magnetic fields.

Electric line of force is an imaginary straight or curved path along which a unit positive charge tends to move in an electric field.

The electric field due to simple arrangements of point charges are shown in Fig 1.5.

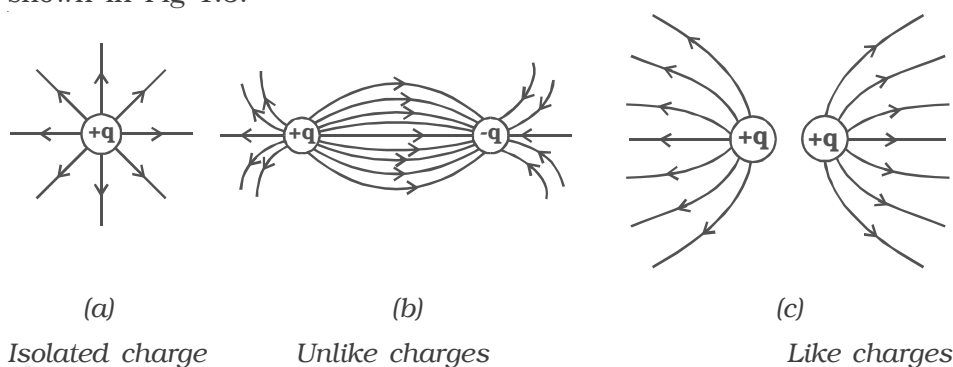


Fig1.5 Lines of Forces

Properties of lines of forces:

- (i) Lines of force start from positive charge and terminate at negative charge.
- (ii) Lines of force never intersect.
- (iii) The tangent to a line of force at any point gives the direction of the electric field (E) at that point.
- (iv) The number of lines per unit area, through a plane at right angles to the lines, is proportional to the magnitude of E . This means that, where the lines of force are close together, E is large and where they are far apart, E is small.
- (v) Each unit positive charge gives rise to $\frac{1}{\epsilon_0}$ lines of force in free space. Hence number of lines of force originating from a point charge q is $N = \frac{q}{\epsilon_0}$ in free space.

1.2.4 Electric dipole and electric dipole moment

Two equal and opposite charges separated by a very small distance constitute an electric dipole.

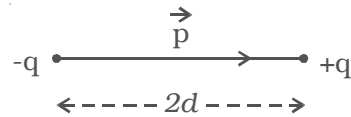


Fig 1.6 Electric dipole

Water, ammonia, carbon-dioxide and chloroform molecules are some examples of permanent electric dipoles. These molecules behave like electric dipole, because the centres of positive and negative charge do not coincide and are separated by a small distance.

Two point charges +q and -q are kept at a distance 2d apart (Fig.1.6). The magnitude of the dipole moment is given by the product of the magnitude of the one of the charges and the distance between them.

$$\therefore \text{Electric dipole moment, } p = q2d \text{ or } 2qd.$$

It is a vector quantity and acts from -q to +q. The unit of dipole moment is C m.

1.2.5 Electric field due to an electric dipole at a point on its axial line.

AB is an electric dipole of two point charges -q and +q separated by a small distance 2d (Fig 1.7). P is a point along the axial line of the dipole at a distance r from the midpoint O of the electric dipole.

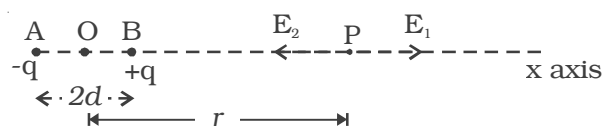


Fig 1.7 Electric field at a point on the axial line

The electric field at the point P due to +q placed at B is,

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-d)^2} \text{ (along BP)}$$

The electric field at the point P due to $-q$ placed at A is,

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r+d)^2} \text{ (along PA)}$$

E_1 and E_2 act in opposite directions.

Therefore, the magnitude of resultant electric field (E) acts in the direction of the vector with a greater magnitude. The resultant electric field at P is,

$$E = E_1 + (-E_2)$$

$$E = \left[\frac{1}{4\pi\epsilon_0} \frac{q}{(r-d)^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r+d)^2} \right] \text{ along BP.}$$

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2} \right] \text{ along BP}$$

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{4rd}{(r^2-d^2)^2} \right] \text{ along BP.}$$

If the point P is far away from the dipole, then $d \ll r$

$$\therefore E = \frac{q}{4\pi\epsilon_0} \frac{4rd}{r^4} = \frac{q}{4\pi\epsilon_0} \frac{4d}{r^3}$$

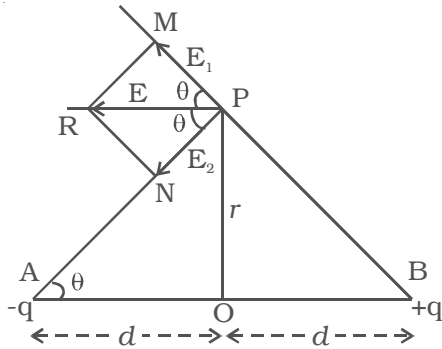
$$E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} \text{ along BP.}$$

[\because Electric dipole moment $p = q \times 2d$]

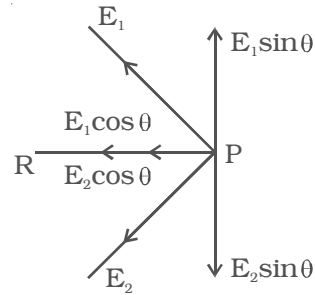
E acts in the direction of dipole moment.

1.2.6 Electric field due to an electric dipole at a point on the equatorial line.

Consider an electric dipole AB. Let $2d$ be the dipole distance and p be the dipole moment. P is a point on the equatorial line at a distance r from the midpoint O of the dipole (Fig 1.8a).



(a) Electric field at a point on equatorial line



(b) The components of the electric field

Fig 1.8

Electric field at a point P due to the charge $+q$ of the dipole,

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{BP^2} \text{ along BP.}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + d^2)} \text{ along BP } (\because BP^2 = OP^2 + OB^2)$$

Electric field (E_2) at a point P due to the charge $-q$ of the dipole

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{AP^2} \text{ along PA}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + d^2)} \text{ along PA}$$

The magnitudes of E_1 and E_2 are equal. Resolving E_1 and E_2 into their horizontal and vertical components (Fig 1.8b), the vertical components $E_1 \sin \theta$ and $E_2 \sin \theta$ are equal and opposite, therefore they cancel each other.

The horizontal components $E_1 \cos \theta$ and $E_2 \cos \theta$ will get added along PR.

Resultant electric field at the point P due to the dipole is

$$\begin{aligned} E &= E_1 \cos \theta + E_2 \cos \theta \text{ (along PR)} \\ &= 2 E_1 \cos \theta \quad (\because E_1 = E_2) \end{aligned}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + d^2)} \times 2 \cos \theta$$

$$\text{But } \cos \theta = \frac{d}{\sqrt{r^2 + d^2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + d^2)} \times \frac{2d}{(r^2 + d^2)^{1/2}} = \frac{1}{4\pi\epsilon_0} \frac{q2d}{(r^2 + d^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + d^2)^{3/2}} \quad (\because p = q2d)$$

For a dipole, d is very small when compared to r

$$\therefore E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

The direction of E is along PR, parallel to the axis of the dipole and directed opposite to the direction of dipole moment.

1.2.7 Electric dipole in a uniform electric field

Consider a dipole AB of dipole moment p placed at an angle θ in an uniform electric field E (Fig.1.9). The charge $+q$ experiences a force qE in the direction of the field. The charge $-q$ experiences an equal force in the opposite direction. Thus the net force on the dipole is zero. The two equal and unlike

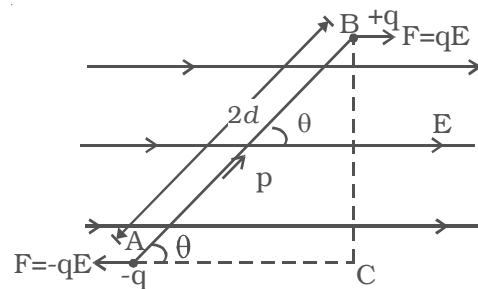


Fig 1.9 Dipole in a uniform field

parallel forces are not passing through the same point, resulting in a torque on the dipole, which tends to set the dipole in the direction of the electric field.

The magnitude of torque is,

$$\begin{aligned}\tau &= \text{One of the forces} \times \text{perpendicular distance between the forces} \\ &= F \times 2d \sin \theta \\ &= qE \times 2d \sin \theta = pE \sin \theta \quad (\because q \times 2d = P)\end{aligned}$$

In vector notation, $\vec{\tau} = \vec{p} \times \vec{E}$

Note : If the dipole is placed in a non-uniform electric field at an angle θ , in addition to a torque, it also experiences a force.

1.2.8 Electric potential energy of an electric dipole in an electric field.

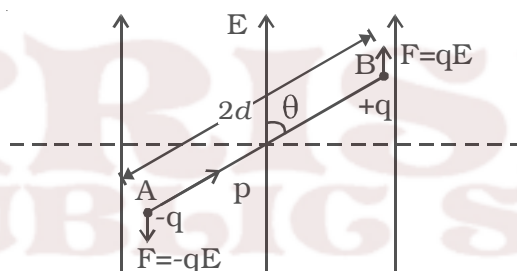


Fig 1.10 Electric potential energy of dipole

Electric potential energy of an electric dipole in an electrostatic field is the work done in rotating the dipole to the desired position in the field.

When an electric dipole of dipole moment p is at an angle θ with the electric field E , the torque on the dipole is

$$\tau = pE \sin \theta$$

Work done in rotating the dipole through $d\theta$,

$$\begin{aligned}dw &= \tau \cdot d\theta \\ &= pE \sin \theta \cdot d\theta\end{aligned}$$

The total work done in rotating the dipole through an angle θ is

$$\begin{aligned}W &= \int dw \\ W &= pE \int \sin \theta \cdot d\theta = -pE \cos \theta\end{aligned}$$

This work done is the potential energy (U) of the dipole.

$$\therefore U = -pE \cos \theta$$

When the dipole is aligned parallel to the field, $\theta = 0^\circ$

$$\therefore U = -pE$$

This shows that the dipole has a minimum potential energy when it is aligned with the field. A dipole in the electric field experiences a torque ($\vec{\tau} = \vec{p} \times \vec{E}$) which tends to align the dipole in the field direction, dissipating its potential energy in the form of heat to the surroundings.

Microwave oven

It is used to cook the food in a short time. When the oven is operated, the microwaves are generated, which in turn produce a non-uniform oscillating electric field. The water molecules in the food which are the electric dipoles are excited by an oscillating torque. Hence few bonds in the water molecules are broken, and heat energy is produced. This is used to cook food.

1.3 Electric potential

Let a charge $+q$ be placed at a point O (Fig 1.11). A and B are two points, in the electric field. When a unit positive charge is moved from A to B against the electric force, work is done. This work is the potential difference between these two points. i.e., $dV = W_{A \rightarrow B}$.

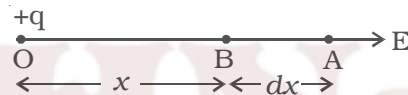


Fig1.11 Electric potential

The potential difference between two points in an electric field is defined as the amount of work done in moving a unit positive charge from one point to the other against the electric force.

The unit of potential difference is volt. The potential difference between two points is 1 volt if 1 joule of work is done in moving 1 Coulomb of charge from one point to another against the electric force.

The electric potential in an electric field at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electric forces.

Relation between electric field and potential

Let the small distance between A and B be dx . Work done in moving a unit positive charge from A to B is $dV = E \cdot dx$.

The work has to be done against the force of repulsion in moving a unit positive charge towards the charge +q. Hence,

$$dV = -E \cdot dx$$

$$E = \frac{-dV}{dx}$$

The change of potential with distance is known as potential gradient, hence the electric field is equal to the negative gradient of potential.

The negative sign indicates that the potential decreases in the direction of electric field. The unit of electric intensity can also be expressed as Vm^{-1} .

1.3.1 Electric potential at a point due to a point charge

Let +q be an isolated point charge situated in air at O. P is a point at a distance r from +q. Consider two points A and B at distances x and x + dx from the point O (Fig.1.12).

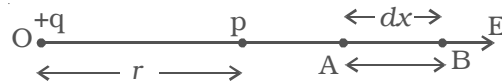


Fig 1.12 Electric potential due to a point charge

The potential difference between A and B is,

$$dV = -E dx$$

The force experienced by a unit positive charge placed at A is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}$$

$$\therefore dV = - \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \cdot dx$$

The negative sign indicates that the work is done against the electric force.

The electric potential at the point P due to the charge +q is the total work done in moving a unit positive charge from infinity to that point.

$$V = - \int_{\infty}^r \frac{q}{4\pi\epsilon_0 x^2} \cdot dx = \frac{q}{4\pi\epsilon_0 r}$$

1.3.2 Electric potential at a point due to an electric dipole

Two charges $-q$ at A and $+q$ at B separated by a small distance $2d$ constitute an electric dipole and its dipole moment is p (Fig 1.13).

Let P be the point at a distance r from the midpoint of the dipole O and θ be the angle between PO and the axis of the dipole OB. Let r_1 and r_2 be the distances of the point P from $+q$ and $-q$ charges respectively.

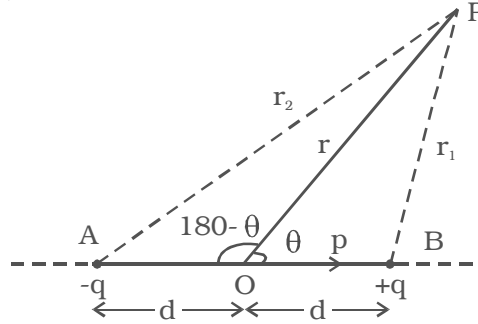


Fig 1.13 Potential due to a dipole

$$\text{Potential at P due to charge (+q)} = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1}$$

$$\text{Potential at P due to charge (-q)} = \frac{1}{4\pi\epsilon_0} \left(-\frac{q}{r_2} \right)$$

$$\text{Total potential at P due to dipole is, } V = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_2}$$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad \dots(1)$$

Applying cosine law,

$$r_1^2 = r^2 + d^2 - 2rd \cos \theta$$

$$r_1^2 = r^2 \left(1 - 2d \frac{\cos \theta}{r} + \frac{d^2}{r^2} \right)$$

Since d is very much smaller than r , $\frac{d^2}{r^2}$ can be neglected.

$$\therefore r_1 = r \left(1 - \frac{2d}{r} \cos \theta \right)^{\frac{1}{2}}$$

$$\text{or } \frac{1}{r_1} = \frac{1}{r} \left(1 - \frac{2d}{r} \cos \theta \right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\therefore \frac{1}{r_1} = \frac{1}{r} \left(1 + \frac{d}{r} \cos \theta \right) \quad \dots(2)$$

Similarly,

$$r_2^2 = r^2 + d^2 - 2rd \cos (180 - \theta)$$

$$\text{or } r_2^2 = r^2 + d^2 + 2rd \cos \theta.$$

$$r_2 = r \left(1 + \frac{2d}{r} \cos \theta \right)^{1/2} \quad (\because \frac{d^2}{r^2} \text{ is negligible})$$

$$\text{or } \frac{1}{r_2} = \frac{1}{r} \left(1 + \frac{2d}{r} \cos \theta \right)^{-1/2}$$

Using the Binomial theorem and neglecting higher powers,

$$\frac{1}{r_2} = \frac{1}{r} \left(1 - \frac{d}{r} \cos \theta \right) \quad \dots(3)$$

Substituting equation (2) and (3) in equation (1) and simplifying

$$\begin{aligned} V &= \frac{q}{4\pi\epsilon_0} \frac{1}{r} \left(1 + \frac{d}{r} \cos \theta - 1 + \frac{d}{r} \cos \theta \right) \\ \therefore V &= \frac{q \cdot 2d \cos \theta}{4\pi\epsilon_0 \cdot r^2} = \frac{1}{4\pi\epsilon_0} \frac{p \cdot \cos \theta}{r^2} \quad \dots(4) \end{aligned}$$

Special cases :

1. When the point P lies on the axial line of the dipole on the side of +q, then $\theta = 0$

$$\therefore V = \frac{p}{4\pi\epsilon_0 r^2}$$

2. When the point P lies on the axial line of the dipole on the side of -q, then $\theta = 180$

$$\therefore V = -\frac{p}{4\pi\epsilon_0 r^2}$$

3. When the point P lies on the equatorial line of the dipole, then, $\theta = 90^\circ$,

$$\therefore V = 0$$

1.3.3 Electric potential energy

The electric potential energy of two point charges is equal to the work done to assemble the charges or work done in bringing each charge or work done in bringing a charge from infinite distance.

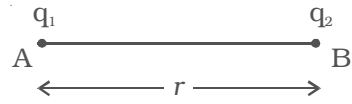


Fig 1.14a Electric potential energy

Let us consider a point charge q_1 , placed at A (Fig 1.14a).

The potential at a point B at a distance r from the charge q_1 is

$$V = \frac{q_1}{4\pi\epsilon_0 r}$$

Another point charge q_2 is brought from infinity to the point B. Now the work done on the charge q_2 is stored as electrostatic potential energy (U) in the system of charges q_1 and q_2 .

$$\therefore \text{work done, } w = Vq_2$$

$$\text{Potential energy (U)} = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

Keeping q_2 at B, if the charge q_1 is imagined to be brought from infinity to the point A, the same amount of work is done.

Also, if both the charges q_1 and q_2 are brought from infinity, to points A and B respectively, separated by a distance r , then potential energy of the system is the same as the previous cases.

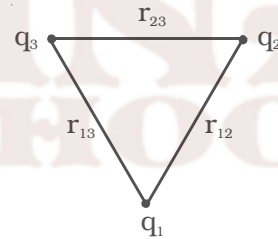


Fig 1.14b Potential energy of system of charges

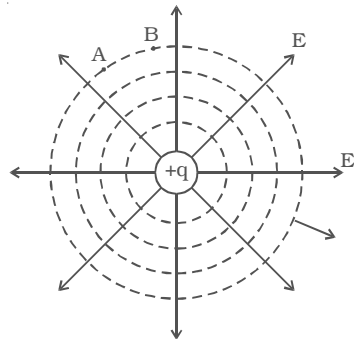
For a system containing more than two charges (Fig 1.14b), the potential energy (U) is given by

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

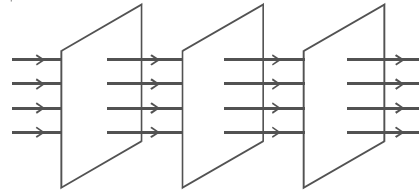
1.3.4 Equipotential Surface

If all the points of a surface are at the same electric potential, then the surface is called an equipotential surface.

(i) In case of an isolated point charge, all points equidistant from the charge are at same potential. Thus, equipotential surfaces in this



(a) Equipotential surface (spherical)



(b) For a uniform field (plane)

Fig 1.15

case will be a series of concentric spheres with the point charge as their centre (Fig 1.15a). The potential, will however be different for different spheres.

If the charge is to be moved between any two points on an equipotential surface through any path, the work done is zero. This is because the potential difference between two points A and B is defined as $V_B - V_A = \frac{W_{AB}}{q}$. If $V_A = V_B$ then $W_{AB} = 0$. Hence the electric field lines must be normal to an equipotential surface.

(ii) In case of uniform field, equipotential surfaces are the parallel planes with their surfaces perpendicular to the lines of force as shown in Fig 1.15b.

1.4 Gauss's law and its applications

Electric flux

Consider a closed surface S in a non-uniform electric field (Fig 1.16). Consider a very small area \vec{ds} on this surface. The direction of \vec{ds} is drawn normal to the surface outward. The electric field over \vec{ds} is supposed to be a constant \vec{E} . \vec{E} and \vec{ds} make an angle θ with each other.

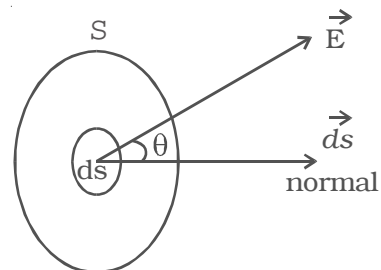


Fig1.16 Electric flux

The electric flux is defined as the total number of electric lines of force, crossing through the given area. The electric flux $d\phi$ through the

area ds is,

$$d\phi = \vec{E} \cdot \vec{ds} = E ds \cos \theta$$

The total flux through the closed surface S is obtained by integrating the above equation over the surface.

$$\phi = \oint d\phi = \oint \vec{E} \cdot \vec{ds}$$

The circle on the integral indicates that, the integration is to be taken over the closed surface. The electric flux is a scalar quantity.

Its unit is $\text{N m}^2 \text{C}^{-1}$

1.4.1 Gauss's law

The law relates the flux through any closed surface and the net charge enclosed within the surface. The law states that the total flux of the electric field E over any closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

$$\phi = \frac{q}{\epsilon_0}$$

This closed imaginary surface is called Gaussian surface. Gauss's law tells us that the flux of E through a closed surface S depends only on the value of net charge inside the surface and not on the location of the charges. Charges outside the surface will not contribute to flux.

1.4.2 Applications of Gauss's Law

i) Field due to an infinite long straight charged wire

Consider an uniformly charged wire of infinite length having a constant linear charge density λ (charge per unit length). Let P be a point at a distance r from the wire (Fig. 1.17) and E be the electric field at the point P . A cylinder of length l , radius r , closed at each end by plane caps normal to the axis is chosen as Gaussian surface. Consider a very small area ds on the Gaussian surface.

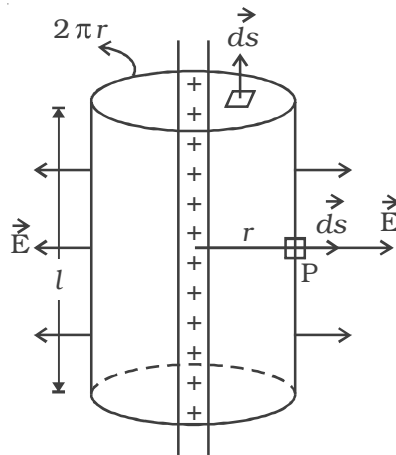


Fig 1.17 Infinitely long straight charged wire

By symmetry, the magnitude of the electric field will be the same at all points on the curved surface of the cylinder and directed radially outward. \vec{E} and \vec{ds} are along the same direction.

The electric flux (ϕ) through curved surface = $\oint \vec{E} \cdot \vec{ds} \cos \theta$

$$\phi = \oint \vec{E} \cdot \vec{ds} \quad [\because \theta = 0; \cos \theta = 1]$$

$$= E (2\pi r l)$$

(\because The surface area of the curved part is $2\pi r l$)

Since \vec{E} and \vec{ds} are right angles to each other, the electric flux through the plane caps = 0

\therefore Total flux through the Gaussian surface, $\phi = E \cdot (2\pi r l)$

The net charge enclosed by Gaussian surface is, $q = \lambda l$

\therefore By Gauss's law,

$$E (2\pi r l) = \frac{\lambda l}{\epsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi\epsilon_0 r}$$

The direction of electric field E is radially outward, if line charge is positive and inward, if the line charge is negative.

1.4.3 Electric field due to an infinite charged plane sheet

Consider an infinite plane sheet of charge with surface charge density σ . Let P be a point at a distance r from the sheet (Fig. 1.18) and E be the electric field at P . Consider a Gaussian surface in the form of cylinder of cross-sectional area A and length $2r$ perpendicular to the sheet of charge.

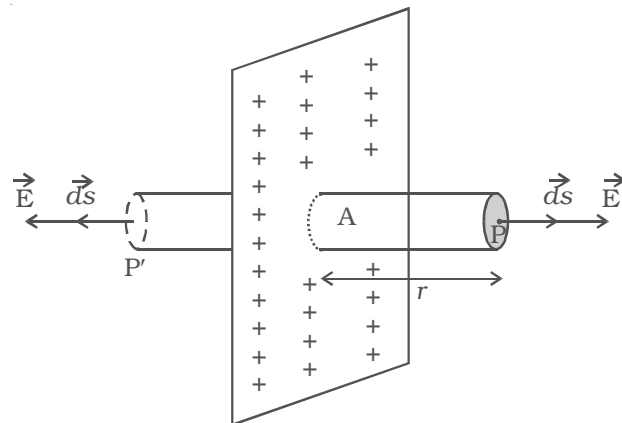


Fig 1.18 Infinite plane sheet

By symmetry, the electric field is at right angles to the end caps and away from the plane. Its magnitude is the same at P and at the other cap at P'.

Therefore, the total flux through the closed surface is given by

$$\begin{aligned} \phi &= \left[\oint E \cdot ds \right]_P + \left[\oint E \cdot ds \right]_{P'} \quad (\because \theta = 0, \cos \theta = 1) \\ &= EA + EA = 2EA \end{aligned}$$

If σ is the charge per unit area in the plane sheet, then the net positive charge q within the Gaussian surface is, $q = \sigma A$

Using Gauss's law,

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$\therefore E = \frac{\sigma}{2\epsilon_0}$$

1.4.4 Electric field due to two parallel charged sheets

Consider two plane parallel infinite sheets with equal and opposite charge densities $+\sigma$ and $-\sigma$ as shown in Fig 1.19. The magnitude of electric field on either side of a plane sheet of charge is $E = \sigma/2\epsilon_0$ and acts perpendicular to the sheet, directed outward (if the charge is positive) or inward (if the charge is negative).

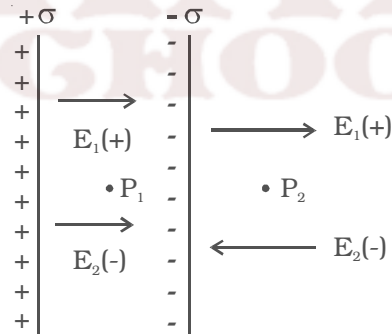


Fig 1.19 Field due to two parallel sheets

(i) When the point P_1 is in between the sheets, the field due to two sheets will be equal in magnitude and in the same direction. The resultant field at P_1 is,

$$E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \quad (\text{towards the right})$$

(ii) At a point P_2 outside the sheets, the electric field will be equal in magnitude and opposite in direction. The resultant field at P_2 is,

$$E = E_1 - E_2 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0.$$

1.4.5 Electric field due to uniformly charged spherical shell

Case (i) At a point outside the shell.

Consider a charged shell of radius R (Fig 1.20a). Let P be a point outside the shell, at a distance r from the centre O . Let us construct a Gaussian surface with r as radius. The electric field E is normal to the surface.

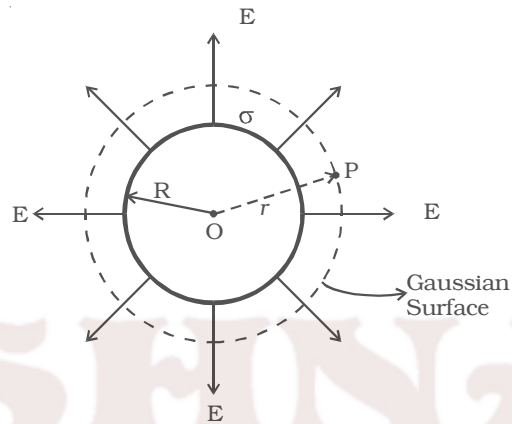


Fig1.20a. Field at a point outside the shell

The flux crossing the Gaussian sphere normally in an outward direction is,

$$\phi = \int_s \vec{E} \cdot \vec{ds} = \int_s E ds = E (4\pi r^2)$$

(since angle between E and ds is zero)

By Gauss's law, $E \cdot (4\pi r^2) = \frac{q}{\epsilon_0}$

or $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

It can be seen from the equation that, the electric field at a point outside the shell will be the same as if the total charge on the shell is concentrated at its centre.

Case (ii) At a point on the surface.

The electric field E for the points on the surface of charged spherical shell is,

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} (\because r = R)$$

Case (iii) At a point inside the shell.

Consider a point P' inside the shell at a distance r' from the centre of the shell. Let us construct a Gaussian surface with radius r'.

The total flux crossing the Gaussian sphere normally in an outward direction is

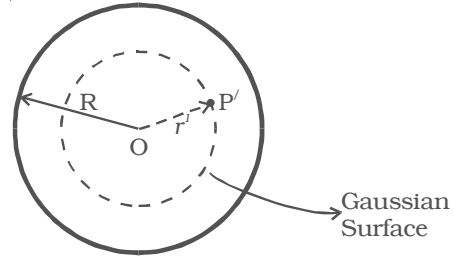


Fig 1.20b Field at a point inside the shell

$$\phi = \int_s \vec{E} \cdot \vec{ds} = \int_s E ds = E \times (4\pi r'^2)$$

since there is no charge enclosed by the gaussian surface, according to Gauss's Law

$$E \times 4\pi r'^2 = \frac{q}{\epsilon_0} = 0 \quad \therefore E = 0$$

(i.e) the field due to a uniformly charged thin shell is zero at all points inside the shell.

1.4.6 Electrostatic shielding

It is the process of isolating a certain region of space from external field. It is based on the fact that electric field inside a conductor is zero.

During a thunder accompanied by lightning, it is safer to sit inside a bus than in open ground or under a tree. The metal body of the bus provides electrostatic shielding, where the electric field is zero. During lightning the electric discharge passes through the body of the bus.

1.5 Electrostatic induction

It is possible to obtain charges without any contact with another charge. They are known as induced charges and the phenomenon of producing induced charges is known as electrostatic induction. It is used in electrostatic machines like Van de Graaff generator and capacitors.

Fig 1.21 shows the steps involved in charging a metal sphere by induction.

(a) There is an uncharged metallic sphere on an insulating stand.

(b) When a negatively charged plastic rod is brought close to the sphere, the free electrons move away due to repulsion and start piling up at the farther end. The near end becomes positively charged due to deficit of electrons. This process of charge distribution stops when the net force on the free electron inside the metal is zero (this process happens very fast).

(c) When the sphere is grounded, the negative charge flows to the ground. The positive charge at the near end remains held due to attractive forces.

(d) When the sphere is removed from the ground, the positive charge continues to be held at the near end.

(e) When the plastic rod is removed, the positive charge spreads uniformly over the sphere.

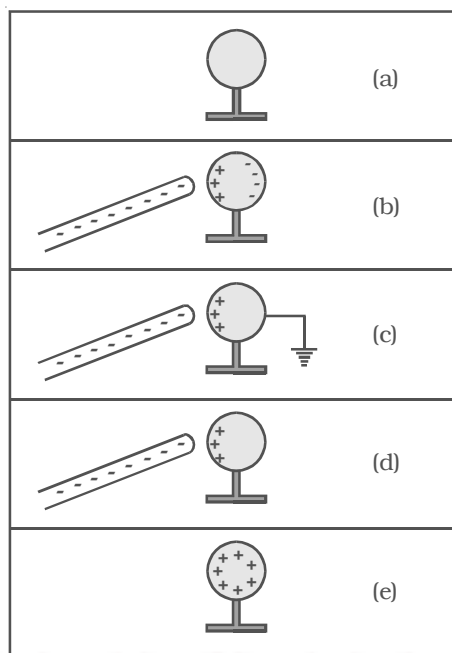


Fig 1.21 Electrostatic Induction

1.5.1 Capacitance of a conductor

When a charge q is given to an isolated conductor, its potential will change. The change in potential depends on the size and shape of the conductor. The potential of a conductor changes by V , due to the charge q given to the conductor.

$$q \propto V \text{ or } q = CV$$

i.e. $C = q/V$

Here C is called as capacitance of the conductor.

The capacitance of a conductor is defined as the ratio of the charge given to the conductor to the potential developed in the conductor.

The unit of capacitance is farad. A conductor has a capacitance of one farad, if a charge of 1 coulomb given to it, rises its potential by 1 volt.

The practical units of capacitance are μF and pF .

Principle of a capacitor

Consider an insulated conductor (Plate A) with a positive charge 'q' having potential V (Fig 1.22a). The capacitance of A is $C = q/V$. When another insulated metal plate B is brought near A, negative charges are induced on the side of B near A. An equal amount of positive charge is induced on the other side of B (Fig 1.22b). The negative charge in B decreases the potential of A. The positive charge in B increases the potential of A. But the negative charge on B is nearer to A than the positive charge on B. So the net effect is that, the potential of A decreases. Thus the capacitance of A is increased.

If the plate B is earthed, positive charges get neutralized (Fig 1.22c). Then the potential of A decreases further. Thus the capacitance of A is considerably increased.

The capacitance depends on the geometry of the conductors and nature of the medium. A capacitor is a device for storing electric charges.

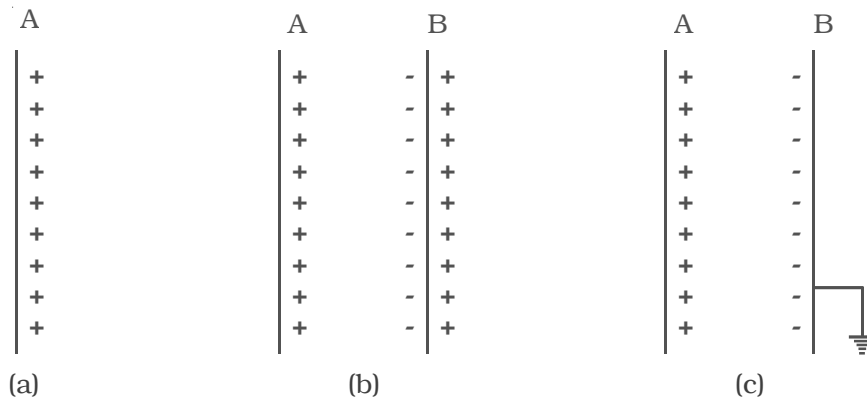


Fig 1.22 Principle of capacitor

1.5.2 Capacitance of a parallel plate capacitor

The parallel plate capacitor consists of two parallel metal plates X and Y each of area A, separated by a distance d , having a surface charge density σ (fig. 1.23). The medium between the plates is air. A charge $+q$ is given to the plate X. It induces a charge $-q$ on the upper surface of earthed plate Y. When the plates are very close to each other, the field is confined to the region between them. The electric lines of force starting from plate X and ending at the plate Y are parallel to each other and perpendicular to the plates.

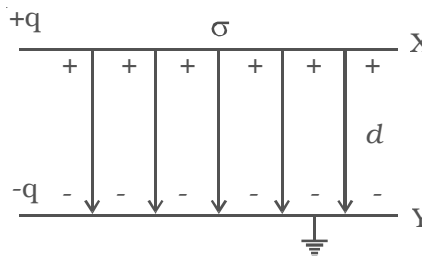


Fig 1.23 Parallel plate capacitor

By the application of Gauss's law, electric field at a point between the two plates is,

$$E = \frac{\sigma}{\epsilon_0}$$

Potential difference between the plates X and Y is

$$V = \int_d^0 -E \, dr = \int_d^0 -\frac{\sigma}{\epsilon_0} \, dr = \frac{\sigma d}{\epsilon_0}$$

The capacitance (C) of the parallel plate capacitor

$$C = \frac{q}{V} = \frac{\sigma A}{\sigma d / \epsilon_0} = \frac{\epsilon_0 A}{d} \quad \left[\text{since, } \sigma = \frac{q}{A} \right]$$

$$\therefore C = \frac{\epsilon_0 A}{d}$$

The capacitance is directly proportional to the area (A) of the plates and inversely proportional to their distance of separation (d).

1.5.3 Dielectrics and polarisation

Dielectrics

A dielectric is an insulating material in which all the electrons are tightly bound to the nucleus of the atom. There are no free electrons to carry current. Ebonite, mica and oil are few examples of dielectrics. The electrons are not free to move under the influence of an external field.

Polarisation

A nonpolar molecule is one in which the centre of gravity of the positive charges (protons) coincide with the centre

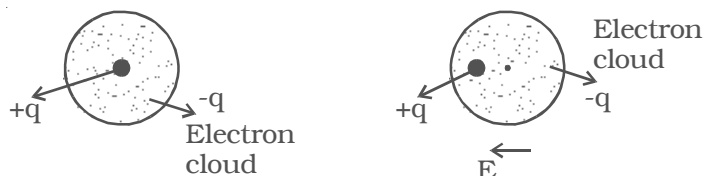
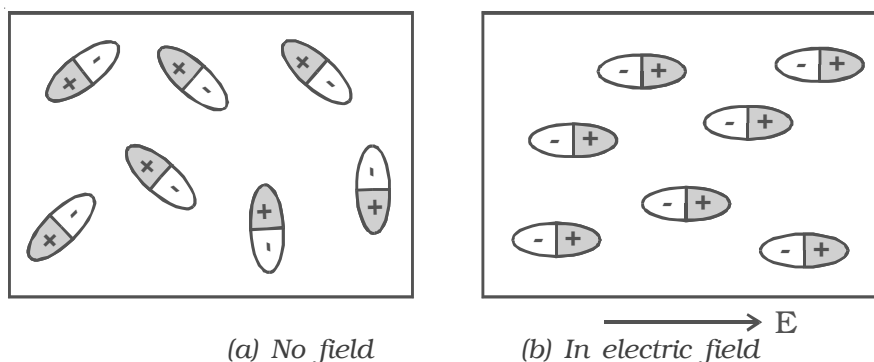


Fig 1.24 Induced dipole

of gravity of the negative charges (electrons). Example: O_2 , N_2 , H_2 . The nonpolar molecules do not have a permanent dipole moment.

If a non polar dielectric is placed in an electric field, the centre of charges get displaced. The molecules are then said to be polarised and are called induced dipoles. They acquire induced dipole moment p in the direction of electric field (Fig 1.24).

A polar molecule is one in which the centre of gravity of the positive charges is separated from the centre of gravity of the negative charges by a finite distance. Examples : N_2O , H_2O , HCl , NH_3 . They have a permanent dipole moment. In the absence of an external field, the dipole moments of polar molecules orient themselves in random directions. Hence no net dipole moment is observed in the dielectric. When an electric field is applied, the dipoles orient themselves in the direction of electric field. Hence a net dipole moment is produced (Fig 1.25).



(a) No field

(b) In electric field

Fig1.25 Polar molecules

The alignment of the dipole moments of the permanent or induced dipoles in the direction of applied electric field is called polarisation or electric polarisation.

The magnitude of the induced dipole moment p is directly proportional to the external electric field E .

$\therefore p \propto E$ or $p = \alpha E$, where α is the constant of proportionality and is called molecular polarisability.

1.5.4 Polarisation of dielectric material

Consider a parallel plate capacitor with $+q$ and $-q$ charges. Let E_0 be the electric field between the plates in air. If a dielectric slab is introduced in the space between them, the dielectric slab gets polarised. Suppose $+q_i$ and $-q_i$ be the induced surface charges on the face of dielectric opposite to the plates of capacitor (Fig 1.26). These induced charges produce their own field E_i which opposes the electric field E_0 . So, the resultant field, $E < E_0$. But the direction of E is in the direction of E_0 .

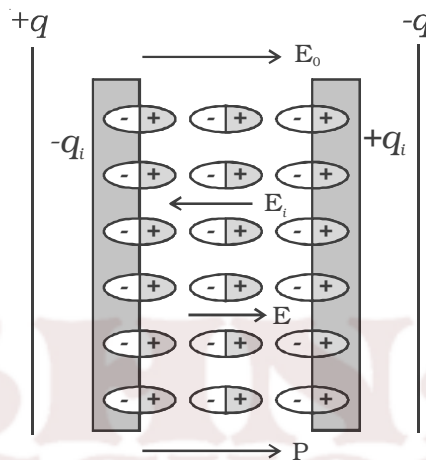


Fig1.26 Polarisation of dielectric material

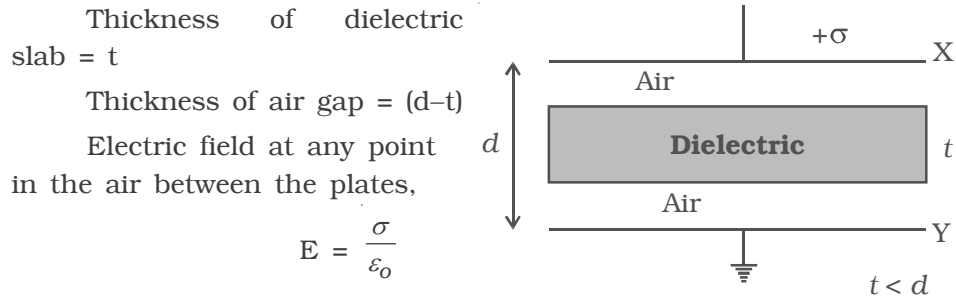
$$\therefore E = E_0 + (-E_i)$$

($\because E_i$ is opposite to the direction of E_0)

1.5.5 Capacitance of a parallel plate capacitor with a dielectric medium.

Consider a parallel plate capacitor having two conducting plates X and Y each of area A , separated by a distance d apart. X is given a positive charge so that the surface charge density on it is σ and Y is earthed.

Let a dielectric slab of thick-ness t and relative permittivity ϵ_r be introduced between the plates (Fig.1.27).



Electric field at any point, in Fig 1.27 Dielectric in capacitor

the dielectric slab $E' = \frac{\sigma}{\epsilon_r \epsilon_0}$

The total potential difference between the plates, is the work done in crossing unit positive charge from one plate to another in the field E over a distance $(d-t)$ and in the field E' over a distance t , then

$$V = E(d-t) + E't$$

$$= \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma t}{\epsilon_0 \epsilon_r}$$

$$= \frac{\sigma}{\epsilon_0} \left[(d-t) + \frac{t}{\epsilon_r} \right]$$

The charge on the plate X, $q = \sigma A$

Hence the capacitance of the capacitor is,

$$C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0} \left[(d-t) + \frac{t}{\epsilon_r} \right]} = \frac{\epsilon_0 A}{(d-t) + \frac{t}{\epsilon_r}}$$

Effect of dielectric

In capacitors, the region between the two plates is filled with dielectric like mica or oil.

The capacitance of the air filled capacitor, $C = \frac{\epsilon_0 A}{d}$

The capacitance of the dielectric filled capacitor, $C' = \frac{\epsilon_r \epsilon_0 A}{d}$

$$\therefore \frac{C'}{C} = \epsilon_r \text{ or } C' = \epsilon_r C$$

since, $\epsilon_r > 1$ for any dielectric medium other than air, the capacitance increases, when dielectric is placed.

1.5.6 Applications of capacitors.

(i) They are used in the ignition system of automobile engines to eliminate sparking.

(ii) They are used to reduce voltage fluctuations in power supplies and to increase the efficiency of power transmission.

(iii) Capacitors are used to generate electromagnetic oscillations and in tuning the radio circuits.

1.5.7 Capacitors in series and parallel

(i) Capacitors in series

Consider three capacitors of capacitance C_1 , C_2 and C_3 connected in series (Fig 1.28). Let V be the potential difference applied across the series combination. Each capacitor carries the same amount of charge q . Let V_1 , V_2 , V_3 be the potential difference across the capacitors C_1 , C_2 , C_3 respectively. Thus $V = V_1 + V_2 + V_3$

The potential difference across each capacitor is,

$$V_1 = \frac{q}{C_1}; V_2 = \frac{q}{C_2}; V_3 = \frac{q}{C_3}$$

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]$$

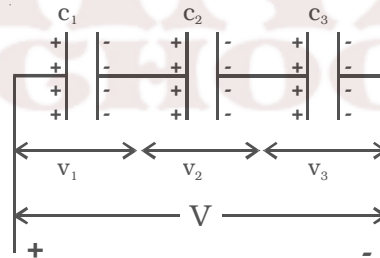


Fig 1.28 Capacitors in series

If C_S be the effective capacitance of the series combination, it should acquire a charge q when a voltage V is applied across it.

$$\text{i.e. } V = \frac{q}{C_S}$$

$$\frac{q}{C_S} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\therefore \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

when a number of capacitors are connected in series, the reciprocal of the effective capacitance is equal to the sum of reciprocal of the capacitance of the individual capacitors.

(ii) Capacitors in parallel

Consider three capacitors of capacitances C_1 , C_2 and C_3 connected in parallel (Fig.1.29). Let this parallel combination be connected to a potential difference V . The potential difference across each capacitor is the same. The charges on the three capacitors are,

$$q_1 = C_1V, \quad q_2 = C_2 V, \quad q_3 = C_3V.$$

The total charge in the system of capacitors is

$$q = q_1 + q_2 + q_3$$

$$q = C_1V + C_2V + C_3V$$

But $q = C_p \cdot V$ where C_p is the effective capacitance of the system

$$\therefore C_p V = V (C_1 + C_2 + C_3)$$

$$\therefore C_p = C_1 + C_2 + C_3$$

Hence the effective capacitance of the capacitors connected in parallel is the sum of the capacitances of the individual capacitors.

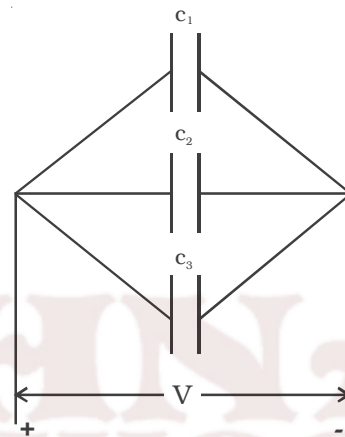


Fig 1.29 Capacitors in parallel

1.5.8 Energy stored in a capacitor

The capacitor is a charge storage device. Work has to be done to store the charges in a capacitor. This work done is stored as electrostatic potential energy in the capacitor.

Let q be the charge and V be the potential difference between the plates of the capacitor. If dq is the additional charge given to the plate, then work done is, $dw = Vdq$

$$dw = \frac{q}{C} dq \quad \left(\because V = \frac{q}{C} \right)$$

Total work done to charge a capacitor is

$$w = \int dw = \int_0^q \frac{q}{C} dq = \frac{1}{2} \frac{q^2}{C}$$

This work done is stored as electrostatic potential energy (U) in the capacitor.

$$U = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} CV^2 \quad (\because q = CV)$$

This energy is recovered if the capacitor is allowed to discharge.

1.5.9 Distribution of charges on a conductor and action of points

Let us consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a conducting wire (Fig 1.30). Let r_1 be greater than r_2 . A charge given to the system is distributed as q_1 and q_2 on the surface of the spheres A and B. Let σ_1, σ_2 be the charge densities on the sphere A and B.

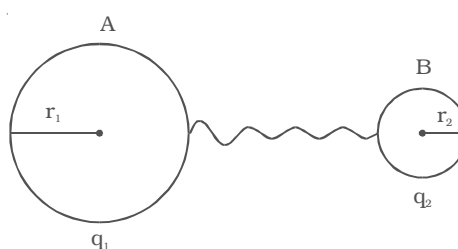


Fig 1.30 Distribution of charges

The potential at A,

$$V_1 = \frac{q_1}{4\pi\epsilon_0 r_1}$$

The potential at B, $V_2 = \frac{q_2}{4\pi\epsilon_0 r_2}$

Since they are connected, their potentials are equal

$$\frac{q_1}{4\pi\epsilon_0 r_1} = \frac{q_2}{4\pi\epsilon_0 r_2} \quad \left[\begin{array}{l} \because q_1 = 4\pi r_1^2 \sigma_1 \\ \text{and} \\ q_2 = 4\pi r_2^2 \sigma_2 \end{array} \right]$$

$$\sigma_1 r_1 = \sigma_2 r_2$$

i.e., σr is a constant. From the above equation it is seen that, smaller the radius, larger is the charge density.

In case of conductor, shaped as in Fig.1.31 the distribution is not uniform. The

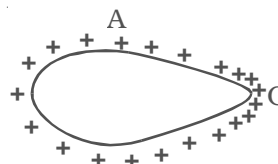


Fig 1.31 Action of point

charges accumulate to a maximum at the pointed end where the curvature is maximum or the radius is minimum. It is found experimentally that a charged conductor with sharp points on its surface, loses its charge rapidly.

The reason is that the air molecules which come in contact with the sharp points become ionized. The positive ions are repelled and the negative ions are attracted by the sharp points and the charge in them is therefore reduced.

Thus, the leakage of electric charges from the sharp points on the charged conductor is known as action of points or corona discharge. This principle is made use of in the electrostatic machines for collecting charges and in lightning arresters (conductors).

1.6 Lightning conductor

This is a simple device used to protect tall buildings from the lightning.

It consists of a long thick copper rod passing through the building to ground. The lower end of the rod is connected to a copper plate buried deeply into the ground. A metal plate with number of spikes is connected to the top end of the copper rod and kept at the top of the building.

When a negatively charged cloud passes over the building, positive charge will be induced on the pointed conductor. The positively charged sharp points will ionize the air in the vicinity. This will partly neutralize the negative charge of the cloud, thereby lowering the potential of the cloud. The negative charges that are attracted to the conductor travels down to the earth. Thereby preventing the lightning stroke from the damage of the building.

Van de Graaff Generator

In 1929, Robert J. Van de Graaff designed an electrostatic machine which produces large electrostatic potential difference of the order of 10^7 V.

The working of Van de Graaff generator is based on the principle of electrostatic induction and action of points.

A hollow metallic sphere A is mounted on insulating pillars as

shown in the Fig.1.32. A pulley B is mounted at the centre of the sphere and another pulley C is mounted near the bottom. A belt made of silk moves over the pulleys. The pulley C is driven continuously by an electric motor. Two comb-shaped conductors D and E having number of needles, are mounted near the pulleys. The comb D is maintained at a positive potential of the order of 10^4 volt by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere.

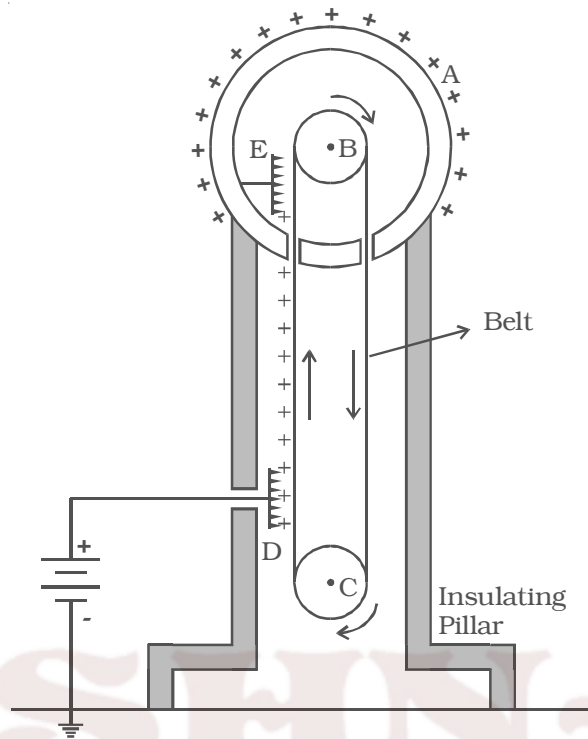


Fig 1.32 Van de Graaff Generator

Because of the high electric field near the comb D, the air gets ionised due to action of points, the negative charges in air move towards the needles and positive charges are repelled on towards the belt. These positive charges stick to the belt, moves up and reaches near the comb E.

As a result of electrostatic induction, the comb E acquires negative charge and the sphere acquires positive charge. The acquired positive charge is distributed on the outer surface of the sphere. The high electric field at the comb E ionises the air. Hence, negative charges are repelled to the belt, neutralises the positive charge on the belt before the belt passes over the pulley. Hence the descending belt will be left uncharged.

Thus the machine, continuously transfers the positive charge to the sphere. As a result, the potential of the sphere keeps increasing till it attains a limiting value (maximum). After this stage no more charge

can be placed on the sphere, it starts leaking to the surrounding due to ionisation of the air.

The leakage of charge from the sphere can be reduced by enclosing it in a gas filled steel chamber at a very high pressure.

The high voltage produced in this generator can be used to accelerate positive ions (protons, deuterons) for the purpose of nuclear disintegration.

Solved Problems

- 1.1 Three small identical balls have charges $-3 \times 10^{-12}\text{C}$, $8 \times 10^{-12}\text{C}$ and $4 \times 10^{-12}\text{C}$ respectively. They are brought in contact and then separated. Calculate (i) charge on each ball (ii) number of electrons in excess or deficit on each ball after contact.

Data : $q_1 = -3 \times 10^{-12}\text{C}$, $q_2 = 8 \times 10^{-12}\text{C}$, $q_3 = 4 \times 10^{-12}\text{C}$

Solution : (i) The charge on each ball

$$q = \frac{q_1 + q_2 + q_3}{3} = \left(\frac{-3 + 8 + 4}{3} \right) \times 10^{-12}$$
$$= 3 \times 10^{-12}\text{C}$$

(ii) Since the charge is positive, there is a shortage of electrons on each ball.

$$n = \frac{q}{e} = \frac{3 \times 10^{-12}}{1.6 \times 10^{-19}} = 1.875 \times 10^7$$

\therefore number of electrons = 1.875×10^7 .

- 1.2 Two insulated charged spheres of charges $6.5 \times 10^{-7}\text{C}$ each are separated by a distance of 0.5m. Calculate the electrostatic force between them. Also calculate the force (i) when the charges are doubled and the distance of separation is halved. (ii) when the charges are placed in a dielectric medium water ($\epsilon_r = 80$)

Data : $q_1 = q_2 = 6.5 \times 10^{-7}\text{C}$, $r = 0.5\text{m}$

Solution : $F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$

$$= \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2}$$

$$= 1.52 \times 10^{-2} \text{ N.}$$

(i) If the charge is doubled and separation between them is halved then,

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{2q_1 2q_2}{(r/2)^2}$$

$$F_1 = 16 \text{ times of } F.$$

$$= 16 \times 1.52 \times 10^{-2}$$

$$F_1 = 0.24 \text{ N}$$

(ii) When placed in water of $\epsilon_r = 80$

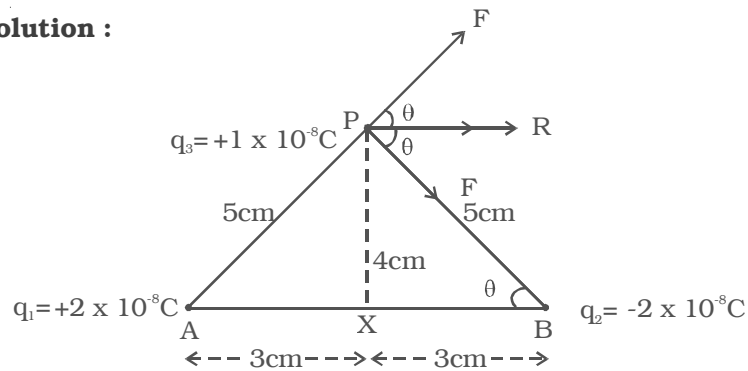
$$F_2 = \frac{F}{\epsilon_r} = \frac{1.52 \times 10^{-2}}{80}$$

$$F_2 = 1.9 \times 10^{-4} \text{ N}$$

1.3. Two small equal and unlike charges $2 \times 10^{-8} \text{ C}$ are placed at A and B at a distance of 6 cm. Calculate the force on the charge $1 \times 10^{-8} \text{ C}$ placed at P, where P is 4 cm on the perpendicular bisector of AB.

Data : $q_1 = +2 \times 10^{-8} \text{ C}$, $q_2 = -2 \times 10^{-8} \text{ C}$
 $q_3 = 1 \times 10^{-8} \text{ C}$ at P
 XP = 4 cm or 0.04 m, AB = 6 cm or 0.06 m

Solution :



From ΔAPX , $AP = \sqrt{4^2 + 3^2} = 5 \text{ cm}$ or $5 \times 10^{-2} \text{ m}$.

A repels the charge at P with a force F (along AP)

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-8} \times 1 \times 10^{-8}}{(5 \times 10^{-2})^2}$$

$$= 7.2 \times 10^{-4} \text{ N along AP.}$$

B attracts the charge at P with same F (along PB), because BP = AP = 5 cm.

To find R, we resolve the force into two components

$$\begin{aligned} R &= F \cos \theta + F \cos \theta = 2F \cos \theta \\ &= 2 \times 7.2 \times 10^{-4} \times \frac{3}{5} \quad \left[\because \cos \theta = \frac{BX}{PB} = \frac{3}{5} \right] \\ \therefore R &= 8.64 \times 10^{-4} \text{ N} \end{aligned}$$

- 1.4 Compare the magnitude of the electrostatic and gravitational force between an electron and a proton at a distance r apart in hydrogen atom. (Given : $m_e = 9.11 \times 10^{-31} \text{ kg}$; $m_p = 1.67 \times 10^{-27} \text{ kg}$; $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$; $e = 1.6 \times 10^{-19} \text{ C}$)

Solution :

The gravitational attraction between electron and proton is

$$F_g = G \frac{m_e m_p}{r^2}$$

Let r be the average distance between electron and proton in hydrogen atom.

The electrostatic force between the two charges.

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

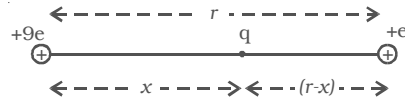
$$\begin{aligned} \therefore \frac{F_e}{F_g} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{G m_e m_p} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{G m_e m_p} \\ &= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 1.67 \times 10^{-27}} \\ \frac{F_e}{F_g} &= 2.27 \times 10^{39} \end{aligned}$$

This shows that the electrostatic force is 2.27×10^{39} times stronger than gravitational force.

- 1.5 Two point charges $+9e$ and $+e$ are kept at a distance of 16 cm from each other. At what point between these charges, should a third charge q to be placed so that it remains in equilibrium?

Data : $r = 16 \text{ cm}$ or 0.16 m ; $q_1 = 9e$ and $q_2 = e$

Solution : Let a third charge q be kept at a distance x from $+9e$ and $(r - x)$ from $+e$



$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{9e \times q}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{q e}{(r-x)^2}$$

$$\therefore \frac{x^2}{(r-x)^2} = 9$$

$$\frac{x}{r-x} = 3$$

or $x = 3r - 3x$

$$\therefore 4x = 3r = 3 \times 16 = 48 \text{ cm}$$

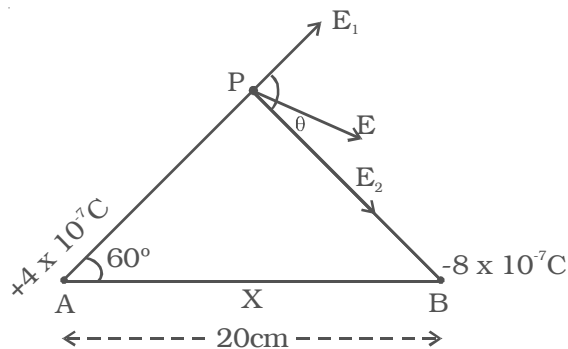
$$\therefore x = \frac{48}{4} = 12 \text{ cm or } 0.12 \text{ m}$$

\therefore The third charge should be placed at a distance of 0.12 m from charge $9e$.

- 1.6 Two charges $4 \times 10^{-7} \text{ C}$ and $-8 \times 10^{-7} \text{ C}$ are placed at the two corners A and B of an equilateral triangle ABP of side 20 cm. Find the resultant intensity at P.

Data : $q_1 = 4 \times 10^{-7} \text{ C}$; $q_2 = -8 \times 10^{-7} \text{ C}$; $r = 20 \text{ cm} = 0.2 \text{ m}$

Solution :



Electric field E_1 along AP

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} = \frac{9 \times 10^9 \times 4 \times 10^{-7}}{(0.2)^2} = 9 \times 10^4 \text{ N C}^{-1}$$

Electric field E_2 along PB.

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-7}}{0.04} = 18 \times 10^4 \text{ N C}^{-1}$$

$$\begin{aligned} \therefore E &= \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos 120^\circ} \\ &= 9 \times 10^4 \sqrt{2^2 + 1^2 + 2 \times 2 \times 1 \left(-\frac{1}{2}\right)} \\ &= 9\sqrt{3} \times 10^4 = 15.6 \times 10^4 \text{ N C}^{-1} \end{aligned}$$

- 1.7 Calculate (i) the potential at a point due a charge of $4 \times 10^{-7} \text{ C}$ located at 0.09 m away (ii) work done in bringing a charge of $2 \times 10^{-9} \text{ C}$ from infinity to the point.

Data : $q_1 = 4 \times 10^{-7} \text{ C}$, $q_2 = 2 \times 10^{-9} \text{ C}$, $r = 0.09 \text{ m}$

Solution :

(i) The potential due to the charge q_1 at a point is

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r} \\ &= \frac{9 \times 10^9 \times 4 \times 10^{-7}}{0.09} = 4 \times 10^4 \text{ V} \end{aligned}$$

(ii) Work done in bringing a charge q_2 from infinity to the point is

$$\begin{aligned} W &= q_2 V = 2 \times 10^{-9} \times 4 \times 10^4 \\ W &= 8 \times 10^{-5} \text{ J} \end{aligned}$$

- 1.8 A sample of HCl gas is placed in an electric field of $2.5 \times 10^4 \text{ N C}^{-1}$. The dipole moment of each HCl molecule is $3.4 \times 10^{-30} \text{ C m}$. Find the maximum torque that can act on a molecule.

Data : $E = 2.5 \times 10^4 \text{ N C}^{-1}$, $p = 3.4 \times 10^{-30} \text{ C m}$.

Solution : Torque acting on the molecule

$$\begin{aligned} \tau &= pE \sin \theta \quad \text{for maximum torque, } \theta = 90^\circ \\ &= 3.4 \times 10^{-30} \times 2.5 \times 10^4 \end{aligned}$$

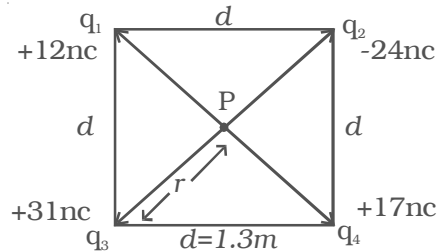
Maximum Torque acting on the molecule is $= 8.5 \times 10^{-26} \text{ N m}$.

- 1.9 Calculate the electric potential at a point P, located at the centre of the square of point charges shown in the figure.

Data : $q_1 = +12 \text{ nC}$;

$q_2 = -24 \text{ nC}$; $q_3 = +31 \text{ nC}$;

$q_4 = +17 \text{ nC}$; $d = 1.3 \text{ m}$



Solution :

Potential at a point P is

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r} \right]$$

The distance $r = \frac{d}{\sqrt{2}} = \frac{1.3}{\sqrt{2}} = 0.919 \text{ m}$

Total charge = $q_1 + q_2 + q_3 + q_4$
 $= (12 - 24 + 31 + 17) \times 10^{-9}$

$q = 36 \times 10^{-9}$

$\therefore V = \frac{9 \times 10^9 \times 36 \times 10^{-9}}{0.919}$

$V = 352.6 \text{ V}$

- 1.10 Three charges $-2 \times 10^{-9} \text{C}$, $+3 \times 10^{-9} \text{C}$, $-4 \times 10^{-9} \text{C}$ are placed at the vertices of an equilateral triangle ABC of side 20 cm. Calculate the work done in shifting the charges A, B and C to A_1 , B_1 and C_1 respectively which are the mid points of the sides of the triangle.

Data :

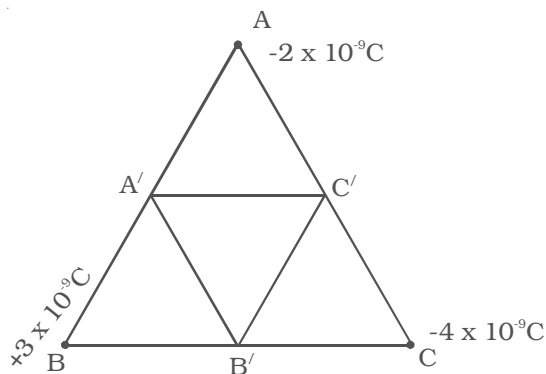
$q_1 = -2 \times 10^{-9} \text{C}$;

$q_2 = +3 \times 10^{-9} \text{C}$;

$q_3 = -4 \times 10^{-9} \text{C}$;

$AB = BC = CA = 20 \text{ cm}$

$= 0.20 \text{ m}$



Solution :

The potential energy of the system of charges,

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r} + \frac{q_2 q_3}{r} + \frac{q_3 q_1}{r} \right]$$

Work done in displacing the charges from A, B and C to A₁, B₁ and C₁ respectively

$$W = U_f - U_i$$

U_i and U_f are the initial and final potential energy of the system.

$$U_i = \frac{9 \times 10^9}{0.20} [-6 \times 10^{-18} - 12 \times 10^{-18} + 8 \times 10^{-18}]$$

$$= -4.5 \times 10^{-7} \text{ J}$$

$$U_f = \frac{9 \times 10^9}{0.10} [-6 \times 10^{-18} - 12 \times 10^{-18} + 8 \times 10^{-18}]$$

$$= -9 \times 10^{-7} \text{ J}$$

$$\therefore \text{work done} = -9 \times 10^{-7} - (-4.5 \times 10^{-7})$$

$$W = -4.5 \times 10^{-7} \text{ J}$$

- 1.11 An infinite line charge produces a field of $9 \times 10^4 \text{ N C}^{-1}$ at a distance of 2 cm. Calculate the linear charge density.

Data : $E = 9 \times 10^4 \text{ N C}^{-1}$, $r = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

$$\text{Solution : } E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\lambda = E \times 2\pi\epsilon_0 r$$

$$= 9 \times 10^4 \times \frac{1}{18 \times 10^9} \times 2 \times 10^{-2} \left(\because 2\pi\epsilon_0 = \frac{1}{18 \times 10^9} \right)$$

$$\lambda = 10^{-7} \text{ C m}^{-1}$$

- 1.12 A point charge causes an electric flux of $-6 \times 10^3 \text{ Nm}^2 \text{ C}^{-1}$ to pass through a spherical Gaussian surface of 10 cm radius centred on the charge. (i) If the radius of the Gaussian surface is doubled, how much flux will pass through the surface? (ii) What is the value of charge?

Data : $\phi = -6 \times 10^3 \text{ N m}^2 \text{ C}^{-1}$; $r = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

Solution :

- (i) If the radius of the Gaussian surface is doubled, the electric flux through the new surface will be the same, as it depends only on the net charge enclosed within and it is independent of the radius.

$$\therefore \phi = -6 \times 10^3 \text{ N m}^2 \text{ C}^{-1}$$

- (ii) $\therefore \phi = \frac{q}{\epsilon_0}$ or $q = -(8.85 \times 10^{-12} \times 6 \times 10^3)$
 $q = -5.31 \times 10^{-8} \text{ C}$

- 1.13 A parallel plate capacitor has plates of area 200 cm^2 and separation between the plates 1 mm . Calculate (i) the potential difference between the plates if 1 nC charge is given to the capacitor (ii) with the same charge (1 nC) if the plate separation is increased to 2 mm , what is the new potential difference and (iii) electric field between the plates.

Data: $d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$; $A = 200 \text{ cm}^2$ or $200 \times 10^{-4} \text{ m}^2$;
 $q = 1 \text{ nC} = 1 \times 10^{-9} \text{ C}$;

Solution : The capacitance of the capacitor

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 200 \times 10^{-4}}{1 \times 10^{-3}}$$
$$C = 0.177 \times 10^{-9} \text{ F} = 0.177 \text{ nF}$$

- (i) The potential difference between the plates

$$V = \frac{q}{C} = \frac{1 \times 10^{-9}}{0.177 \times 10^{-9}} = 5.65 \text{ V}$$

- (ii) If the plate separation is increased from 1 mm to 2 mm , the capacitance is decreased by 2, the potential difference increases by the factor 2

$$\therefore \text{New potential difference is } 5.65 \times 2 \\ = 11.3 \text{ V}$$

- (iii) Electric field is,

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{A \cdot \epsilon_0} = \frac{1 \times 10^{-9}}{8.85 \times 10^{-12} \times 200 \times 10^{-4}}$$
$$= 5650 \text{ N C}^{-1}$$

- 1.14 A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance, if the distance between the plates be reduced to half and the space between them is filled with a substance of dielectric constant 6.

Data : $C_0 = 8 \text{ pF}$, $\epsilon_r = 6$, distance d becomes, $d/2$ with dielectric

Solution : $C_0 = \frac{A\epsilon_0}{d} = 8 \text{ pF}$

when the distance is reduced to half and dielectric medium fills the gap, the new capacitance will be

$$C = \frac{\epsilon_r A \epsilon_0}{d/2} = \frac{2\epsilon_r A \epsilon_0}{d}$$

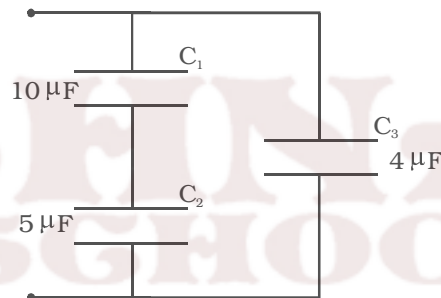
$$= 2\epsilon_r C_0$$

$$C = 2 \times 6 \times 8 = 96 \text{ pF}$$

- 1.15 Calculate the effective capacitance of the combination shown in figure.

Data : $C_1 = 10 \mu\text{F}$; $C_2 = 5 \mu\text{F}$; $C_3 = 4 \mu\text{F}$

Solution : (i) C_1 and C_2 are connected in series, the effective capacitance of the capacitor of the series combination is



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1}{10} + \frac{1}{5}$$

$$\therefore C_s = \frac{10 \times 5}{10 + 5} = \frac{10}{3} \mu\text{F}$$

(ii) This C_s is connected to C_3 in parallel.

The effective capacitance of the capacitor of the parallel combination is

$$C_p = C_s + C_3$$

$$= \left(\frac{10}{3} + 4 \right) = \frac{22}{3} \mu\text{F}$$

$$C_p = 7.33 \mu\text{F}$$

- 1.16 The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm . The capacitor is charged by connecting it to a 400 V supply. How much electrostatic energy is stored by the capacitor?

Data : $A = 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2$; $d = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$;

$$V = 400 \text{ V}$$

Solution : Capacitance of a parallel plate capacitor

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}}$$

$$= 3.186 \times 10^{-11} \text{ F}$$

Energy of the capacitor = $\left(\frac{1}{2} \right) CV^2$

$$= \frac{1}{2} \times 3.186 \times 10^{-11} \times (400)^2$$

$$\text{Energy} = 2.55 \times 10^{-6} \text{ J}$$

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Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

- 1.1 A glass rod rubbed with silk acquires a charge of $+8 \times 10^{-12}\text{C}$. The number of electrons it has gained or lost
- (a) 5×10^{-7} (gained) (b) 5×10^7 (lost)
(c) 2×10^{-8} (lost) (d) -8×10^{-12} (lost)
- 1.2 The electrostatic force between two point charges kept at a distance d apart, in a medium $\epsilon_r = 6$, is 0.3 N. The force between them at the same separation in vacuum is
- (a) 20 N (b) 0.5 N
(c) 1.8 N (d) 2 N
- 1.3 Electric field intensity is 400 V m^{-1} at a distance of 2 m from a point charge. It will be 100 V m^{-1} at a distance?
- (a) 50 cm (b) 4 cm
(c) 4 m (d) 1.5 m
- 1.4 Two point charges $+4q$ and $+q$ are placed 30 cm apart. At what point on the line joining them the electric field is zero?
- (a) 15 cm from the charge q (b) 7.5 cm from the charge q
(c) 20 cm from the charge $4q$ (d) 5 cm from the charge q
- 1.5 A dipole is placed in a uniform electric field with its axis parallel to the field. It experiences
- (a) only a net force
(b) only a torque
(c) both a net force and torque
(d) neither a net force nor a torque
- 1.6 If a point lies at a distance x from the midpoint of the dipole, the electric potential at this point is proportional to
- (a) $\frac{1}{x^2}$ (b) $\frac{1}{x^3}$
(c) $\frac{1}{x^4}$ (d) $\frac{1}{x^{3/2}}$

1.7 Four charges $+q$, $+q$, $-q$ and $-q$ respectively are placed at the corners A, B, C and D of a square of side a . The electric potential at the centre O of the square is

(a) $\frac{1}{4\pi\epsilon_0} \frac{q}{a}$ (b) $\frac{1}{4\pi\epsilon_0} \frac{2q}{a}$

(c) $\frac{1}{4\pi\epsilon_0} \frac{4q}{a}$ (d) zero

1.8 Electric potential energy (U) of two point charges is

(a) $\frac{q_1 q_2}{4\pi\epsilon_0 r^2}$ (b) $\frac{q_1 q_2}{4\pi\epsilon_0 r}$

(c) $pE \cos \theta$ (d) $pE \sin \theta$

1.9 The work done in moving $500 \mu\text{C}$ charge between two points on equipotential surface is

(a) zero (b) finite positive

(c) finite negative (d) infinite

1.10 Which of the following quantities is scalar?

(a) dipole moment (b) electric force

(c) electric field (d) electric potential

1.11 The unit of permittivity is

(a) $\text{C}^2 \text{N}^{-1} \text{m}^{-2}$ (b) $\text{N m}^2 \text{C}^{-2}$

(c) H m^{-1} (d) $\text{N C}^{-2} \text{m}^{-2}$

1.12 The number of electric lines of force originating from a charge of 1 C is

(a) 1.129×10^{11} (b) 1.6×10^{-19}

(c) 6.25×10^{18} (d) 8.85×10^{12}

1.13 The electric field outside the plates of two oppositely charged plane sheets of charge density σ is

(a) $\frac{+\sigma}{2\epsilon_0}$ (b) $\frac{-\sigma}{2\epsilon_0}$

(c) $\frac{\sigma}{\epsilon_0}$ (d) zero

- 1.14 The capacitance of a parallel plate capacitor increases from $5 \mu\text{f}$ to $60 \mu\text{f}$ when a dielectric is filled between the plates. The dielectric constant of the dielectric is
- (a) 65 (b) 55
(c) 12 (d) 10
- 1.15 A hollow metal ball carrying an electric charge produces no electric field at points
- (a) outside the sphere (b) on its surface
(c) inside the sphere (d) at a distance more than twice
- 1.16 State Coulomb's law in electrostatics and represent it in vector form.
- 1.17 What is permittivity and relative permittivity? How are they related?
- 1.18 Explain the principle of superposition.
- 1.19 Define electric field at a point. Give its unit and obtain an expression for the electric field at a point due to a point charge.
- 1.20 Write the properties of lines of forces.
- 1.21 What is an electric dipole? Define electric dipole moment?
- 1.22 Derive an expression for the torque acting on the electric dipole when placed in a uniform field.
- 1.23 What does an electric dipole experience when kept in a uniform electric field and non-uniform electric field?
- 1.24 Derive an expression for electric field due to an electric dipole (a) at a point on its axial line (b) at a point along the equatorial line.
- 1.25 Define electric potential at a point. Is it a scalar or a vector quantity? Obtain an expression for electric potential due to a point charge.
- 1.26 Distinguish between electric potential and potential difference.
- 1.27 What is an equipotential surface?
- 1.28 What is electrostatic potential energy of a system of two point charges? Deduce an expression for it.
- 1.29 Derive an expression for electric potential due to an electric dipole.
- 1.30 Define electric flux. Give its unit.

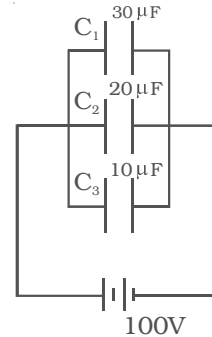
- 1.31 State Gauss's law. Applying this, calculate electric field due to
(i) an infinitely long straight charge with uniform charge density
(ii) an infinite plane sheet of charge of q .
- 1.32 What is a capacitor? Define its capacitance.
- 1.33 Explain the principle of capacitor. Deduce an expression for the capacitance of the parallel plate capacitor.
- 1.34 What is dielectric ? Explain the effect of introducing a dielectric slab between the plates of parallel plate capacitor.
- 1.35 A parallel plate capacitor is connected to a battery. If the dielectric slab of thickness equal to half the plate separation is inserted between the plates what happens to (i) capacitance of the capacitor (ii) electric field between the plates (iii) potential difference between the plates.
- 1.36 Deduce an expression for the equivalent capacitance of capacitors connected in series and parallel.
- 1.37 Prove that the energy stored in a parallel plate capacitor is $\frac{q^2}{2C}$.
- 1.38 What is meant by dielectric polarisation?
- 1.39 State the principle and explain the construction and working of Van de Graaff generator.
- 1.40 Why is it safer to be inside a car than standing under a tree during lightning?

Problems :

- 1.41 The sum of two point charges is $6 \mu\text{C}$. They attract each other with a force of 0.9 N , when kept 40 cm apart in vacuum. Calculate the charges.
- 1.42 Two small charged spheres repel each other with a force of $2 \times 10^{-3}\text{ N}$. The charge on one sphere is twice that on the other. When one of the charges is moved 10 cm away from the other, the force is $5 \times 10^{-4}\text{ N}$. Calculate the charges and the initial distance between them.
- 1.43 Four charges $+q$, $+2q$, $+q$ and $-q$ are placed at the corners of a square. Calculate the electric field at the intersection of the diagonals of the square of side 10 cm if $q = 5/3 \times 10^{-9}\text{ C}$.

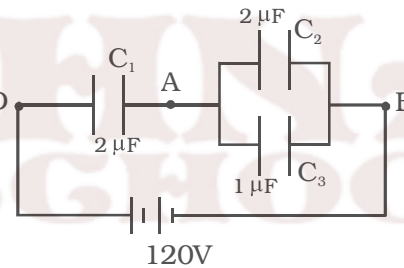
- 1.44 Two charges $10 \times 10^{-9} \text{ C}$ and $20 \times 10^{-9} \text{ C}$ are placed at a distance of 0.3 m apart. Find the potential and intensity at a point mid-way between them.
- 1.45 An electric dipole of charges $2 \times 10^{-10} \text{ C}$ and $-2 \times 10^{-10} \text{ C}$ separated by a distance 5 mm, is placed at an angle of 60° to a uniform field of 10 Vm^{-1} . Find the (i) magnitude and direction of the force acting on each charge. (ii) Torque exerted by the field
- 1.46 An electric dipole of charges $2 \times 10^{-6} \text{ C}$, $-2 \times 10^{-6} \text{ C}$ are separated by a distance 1 cm. Calculate the electric field due to dipole at a point on its. (i) axial line 1 m from its centre (ii) equatorial line 1 m from its centre.
- 1.47 Two charges $+q$ and $-3q$ are separated by a distance of 1 m. At what point in between the charges on its axis is the potential zero?
- 1.48 Three charges $+1 \mu\text{C}$, $+3 \mu\text{C}$ and $-5 \mu\text{C}$ are kept at the vertices of an equilateral triangle of sides 60 cm. Find the electrostatic potential energy of the system of charges.
- 1.49 Two positive charges of $12 \mu\text{C}$ and $8 \mu\text{C}$ respectively are 10 cm apart. Find the work done in bringing them 4 cm closer, so that, they are 6 cm apart.
- 1.50 Find the electric flux through each face of a hollow cube of side 10 cm, if a charge of $8.85 \mu\text{C}$ is placed at the centre.
- 1.51 A spherical conductor of radius 0.12 m has a charge of $1.6 \times 10^{-7} \text{ C}$ distributed uniformly on its surface. What is the electric field (i) inside the sphere (ii) on the sphere (iii) at a point 0.18 m from the centre of the sphere?
- 1.52 The area of each plate of a parallel plate capacitor is $4 \times 10^{-2} \text{ sq m}$. If the thickness of the dielectric medium between the plates is 10^{-3} m and the relative permittivity of the dielectric is 7. Find the capacitance of the capacitor.
- 1.53 Two capacitors of unknown capacitances are connected in series and parallel. If the net capacitances in the two combinations are $6 \mu\text{F}$ and $25 \mu\text{F}$ respectively, find their capacitances.
- 1.54 Two capacitances $0.5 \mu\text{F}$ and $0.75 \mu\text{F}$ are connected in parallel and the combination to a 110 V battery. Calculate the charge from the source and charge on each capacitor.

- 1.55 Three capacitors are connected in parallel to a 100 V battery as shown in figure. What is the total energy stored in the combination of capacitor?



- 1.56 A parallel plate capacitor is maintained at some potential difference. A 3 mm thick slab is introduced between the plates. To maintain the plates at the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.
- 1.57 A dielectric of dielectric constant 3 fills three fourth of the space between the plates of a parallel plate capacitor. What percentage of the energy is stored in the dielectric?

- 1.58 Find the charges on the capacitor shown in figure and the potential difference across them.



- 1.59 Three capacitors each of capacitance 9 pF are connected in series (i) What is the total capacitance of the combination? (ii) What is the potential difference across each capacitor, if the combination is connected to 120 V supply?

Answers

- 1.1** (b) **1.2** (c) **1.3** (c) **1.4** (c) **1.5** (d)
- 1.6** (a) **1.7** (d) **1.8** (b) **1.9** (a) **1.10** (d)
- 1.11** (a) **1.12** (a) **1.13** (d) **1.14** (c) **1.15** (c)
- 1.35** (i) increases (ii) remains the same (iii) remains the same
- 1.41** $q_1 = 8 \times 10^{-6} \text{C}$, $q_2 = -2 \times 10^{-6} \text{C}$
- 1.42** $q_1 = 33.33 \times 10^{-9} \text{C}$, $q_2 = 66.66 \times 10^{-9} \text{C}$, $x = 0.1 \text{ m}$
- 1.43** $0.9 \times 10^4 \text{Vm}^{-1}$
- 1.44** $V = 1800 \text{ V}$, $E = 4000 \text{ Vm}^{-1}$
- 1.45** $2 \times 10^{-9} \text{N}$, along the field, $\tau = 0.866 \times 10^{-11} \text{ Nm}$
- 1.46** 360 N/C , 180 N C^{-1}
- 1.47** $x = 0.25 \text{ m}$ from +q
- 1.48** -0.255 J
- 1.49** 5.70 J
- 1.50** $1.67 \times 10^5 \text{ Nm}^2 \text{C}^{-1}$
- 1.51** zero, 10^5 N C^{-1} , $4.44 \times 10^4 \text{ N C}^{-1}$
- 1.52** $2.478 \times 10^{-9} \text{F}$
- 1.53** $C_1 = 15 \mu\text{F}$, $C_2 = 10 \mu\text{F}$
- 1.54** $q = 137.5 \mu\text{C}$, $q_1 = 55 \mu\text{C}$, $q_2 = 82.5 \mu\text{C}$
- 1.55** 0.3 J
- 1.56** $\epsilon_r = 5$
- 1.57** 50%
- 1.58** $q_1 = 144 \times 10^{-6} \text{C}$, $q_2 = 96 \times 10^{-6} \text{C}$, $q_3 = 48 \times 10^{-6} \text{C}$
 $V_1 = 72 \text{ V}$, $V_2 = 48 \text{ V}$
- 1.59** 3 pF , each one is 40 V

2. Current Electricity

The branch of Physics which deals with the study of motion of electric charges is called current electricity. In an uncharged metallic conductor at rest, some (not all) electrons are continually moving randomly through the conductor because they are very loosely attached to the nuclei. The thermodynamic internal energy of the material is sufficient to liberate the outer electrons from individual atoms, enabling the electrons to travel through the material. But the net flow of charge at any point is zero. Hence, there is zero current. These are termed as free electrons. The external energy necessary to drive the free electrons in a definite direction is called electromotive force (emf). The emf is not a force, but it is the work done in moving a unit charge from one end to the other. The flow of free electrons in a conductor constitutes electric current.

2.1 Electric current

The current is defined as the rate of flow of charges across any cross sectional area of a conductor. If a net charge q passes through any cross section of a conductor in time t , then the current $I = q / t$, where q is in coulomb and t is in second. The current I is expressed in ampere. If the rate of flow of charge is not uniform, the current varies with time and the instantaneous value of current i is given by,

$$i = \frac{dq}{dt}$$

Current is a scalar quantity. The direction of conventional current is taken as the direction of flow of positive charges or opposite to the direction of flow of electrons.

2.1.1 Drift velocity and mobility

Consider a conductor XY connected to a battery (Fig 2.1). A steady electric field E is established in the conductor in the direction X to Y. In the absence of an electric field, the free electrons in the conductor move randomly in all possible directions.

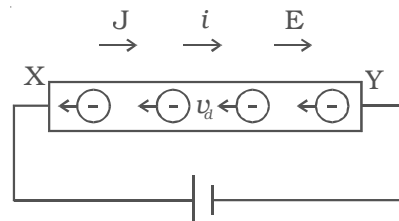


Fig 2.1 Current carrying conductor

They do not produce current. But, as soon as an electric field is applied, the free electrons at the end Y experience a force $F = eE$ in a direction opposite to the electric field. The electrons are accelerated and in the process they collide with each other and with the positive ions in the conductor.

Thus due to collisions, a backward force acts on the electrons and they are slowly drifted with a constant average drift velocity v_d in a direction opposite to electric field.

Drift velocity is defined as the velocity with which free electrons get drifted towards the positive terminal, when an electric field is applied.

If τ is the average time between two successive collisions and the acceleration experienced by the electron be a , then the drift velocity is given by,

$$v_d = a\tau$$

The force experienced by the electron of mass m is

$$F = ma$$

Hence $a = \frac{eE}{m}$

$$\therefore v_d = \frac{eE}{m}\tau = \mu E$$

where $\mu = \frac{e\tau}{m}$ is the mobility and is defined as the drift velocity acquired per unit electric field. It takes the unit $m^2V^{-1}s^{-1}$. The drift velocity of electrons is proportional to the electric field intensity. It is very small and is of the order of 0.1 cm s^{-1} .

2.1.2 Current density

Current density at a point is defined as the quantity of charge passing per unit time through unit area, taken perpendicular to the direction of flow of charge at that point.

The current density \mathbf{J} for a current I flowing across a conductor having an area of cross section A is

$$\mathbf{J} = \frac{(q/t)}{A} = \frac{I}{A}$$

Current density is a vector quantity. It is expressed in $A \text{ m}^{-2}$

* In this text book, the infinitesimally small current and instantaneous currents are represented by the notation i and all other currents are represented by the notation I .

2.1.3 Relation between current and drift velocity

Consider a conductor XY of length L and area of cross section A (Fig 2.1). An electric field E is applied between its ends. Let n be the number of free electrons per unit volume. The free electrons move towards the left with a constant drift velocity v_d .

The number of conduction electrons in the conductor = nAL

The charge of an electron = e

The total charge passing through the conductor $q = (nAL) e$

The time in which the charges pass through the conductor, $t = \frac{L}{v_d}$

The current flowing through the conductor, $I = \frac{q}{t} = \frac{(nAL)e}{(L/v_d)}$

$$I = nAev_d \quad \dots(1)$$

The current flowing through a conductor is directly proportional to the drift velocity.

From equation (1), $\frac{I}{A} = nev_d$

$$\mathbf{J} = nev_d \quad \left[\because \mathbf{J} = \frac{I}{A}, \text{current density} \right]$$

2.1.4 Ohm's law

George Simon Ohm established the relationship between potential difference and current, which is known as Ohm's law. The current flowing through a conductor is,

$$I = nAev_d$$

$$\text{But } v_d = \frac{eE}{m} \cdot \tau$$

$$\therefore I = nAe \frac{eE}{m} \tau$$

$$I = \frac{nAe^2}{mL} \tau V \quad \left[\because E = \frac{V}{L} \right]$$

where V is the potential difference. The quantity $\frac{mL}{nAe^2\tau}$ is a constant for a given conductor, called electrical resistance (R).

$$\therefore I \propto V$$

The law states that, at a constant temperature, the steady current flowing through a conductor is directly proportional to the potential difference between the two ends of the conductor.

$$(i.e) \quad I \propto V \quad \text{or} \quad I = \frac{1}{R} V$$

$$\therefore \quad V = IR \quad \text{or} \quad R = \frac{V}{I}$$

Resistance of a conductor is defined as the ratio of potential difference across the conductor to the current flowing through it. The unit of resistance is ohm (Ω)

The reciprocal of resistance is conductance. Its unit is mho (Ω^{-1}).

Since, potential difference V is proportional to the current I , the graph (Fig 2.2) between V and I is a straight line for a conductor. Ohm's law holds good only when a steady current flows through a conductor.

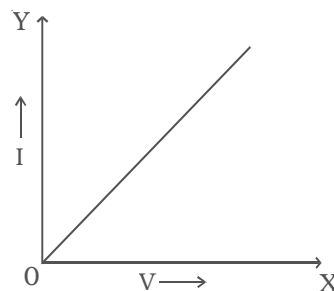


Fig 2.2 V-I graph of an ohmic conductor.

2.1.5 Electrical Resistivity and Conductivity

The resistance of a conductor R is directly proportional to the length of the conductor l and is inversely proportional to its area of cross section A .

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \frac{\rho l}{A}$$

ρ is called specific resistance or electrical resistivity of the material of the conductor.

$$\text{If } l = 1 \text{ m, } A = 1 \text{ m}^2, \text{ then } \rho = R$$

The electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section. The unit of ρ is ohm-m ($\Omega \text{ m}$). It is a constant for a particular material.

The reciprocal of electrical resistivity, is called electrical conductivity, $\sigma = \frac{1}{\rho}$

$$\text{The unit of conductivity is mho m}^{-1} (\Omega^{-1} \text{ m}^{-1})$$

2.1.6 Classification of materials in terms of resistivity

The resistivity of a material is the characteristic of that particular material. The materials can be broadly classified into conductors and insulators. The metals and alloys which have low resistivity of the order of $10^{-6} - 10^{-8} \Omega \text{ m}$ are good conductors of electricity. They carry current without appreciable loss of energy. Example : silver, aluminium, copper, iron, tungsten, nichrome, manganin, constantan. The resistivity of metals increase with increase in temperature. Insulators are substances which have very high resistivity of the order of $10^8 - 10^{14} \Omega \text{ m}$. They offer very high resistance to the flow of current and are termed non-conductors. Example : glass, mica, amber, quartz, wood, teflon, bakelite. In between these two classes of materials lie the semiconductors (Table 2.1). They are partially conducting. The resistivity of semiconductor is $10^{-2} - 10^4 \Omega \text{ m}$. Example : germanium, silicon.

**Table 2.1 Electrical resistivities at room temperature
(NOT FOR EXAMINATION)**

Classification	Material	$\rho (\Omega \text{ m})$
conductors	silver	1.6×10^{-8}
	copper	1.7×10^{-8}
	aluminium	2.7×10^{-8}
	iron	10×10^{-8}
Semiconductors	germanium	0.46
	silicon	2300
Insulators	glass	$10^{10} - 10^{14}$
	wood	$10^8 - 10^{11}$
	quartz	10^{13}
	rubber	$10^{13} - 10^{16}$

2.2 Superconductivity

Ordinary conductors of electricity become better conductors at lower temperatures. The ability of certain metals, their compounds and alloys to conduct electricity with zero resistance at very low temperatures is called superconductivity. The materials which exhibit this property are called superconductors.

The phenomenon of superconductivity was first observed by Kammerlingh Onnes in 1911. He found that mercury suddenly showed

zero resistance at 4.2 K (Fig 2.3). The first theoretical explanation of superconductivity was given by Bardeen, Cooper and Schrieffer in 1957 and it is called the BCS theory.

The temperature at which electrical resistivity of the material suddenly drops to zero and the material changes from normal conductor to a superconductor is called the transition temperature or critical temperature T_C . At the transition temperature the following changes are observed :

- (i) The electrical resistivity drops to zero.
- (ii) The conductivity becomes infinity
- (iii) The magnetic flux lines are excluded from the material.

Applications of superconductors

(i) Superconductors form the basis of energy saving power systems, namely the superconducting generators, which are smaller in size and weight, in comparison with conventional generators.

(ii) Superconducting magnets have been used to levitate trains above its rails. They can be driven at high speed with minimal expenditure of energy.

(iii) Superconducting magnetic propulsion systems may be used to launch satellites into orbits directly from the earth without the use of rockets.

(iv) High efficiency ore-separating machines may be built using superconducting magnets which can be used to separate tumor cells from healthy cells by high gradient magnetic separation method.

(v) Since the current in a superconducting wire can flow without any change in magnitude, it can be used for transmission lines.

(vi) Superconductors can be used as memory or storage elements in computers.

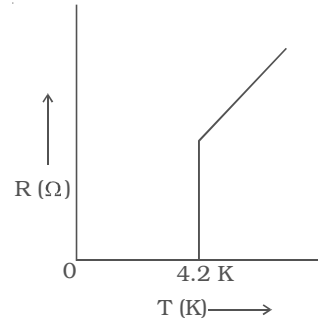


Fig 2.3 Superconductivity of mercury

2.3 Carbon resistors

The wire wound resistors are expensive and huge in size. Hence, carbon resistors are used. Carbon resistor consists of a ceramic core, on which a thin layer of crystalline carbon is deposited. These resistors are cheaper, stable and small in size. The resistance of a carbon resistor is indicated by the colour code drawn on it (Table 2.2). A three colour code carbon resistor is discussed here. The silver or gold ring at one end corresponds to the tolerance. It is a tolerable range (\pm) of the resistance. The tolerance of silver, gold, red and brown rings is 10%, 5%, 2% and 1% respectively. If there is no coloured ring at this end, the tolerance is 20%. The first two rings at the other end of tolerance ring are significant figures of resistance in ohm. The third ring indicates the powers of 10 to be multiplied or number of zeroes following the significant figure.

Table 2.2 Colour code for carbon resistors

Colour	Number
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9

Example :

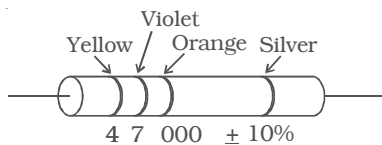


Fig 2.4 Carbon resistor colour code.

The first yellow ring in Fig 2.4 corresponds to 4. The next violet ring corresponds to 7. The third orange ring corresponds to 10^3 . The silver ring represents 10% tolerance. The total resistance is $47 \times 10^3 \pm 10\%$ i.e. 47 k Ω , 10%. Fig 2.5 shows 1 k Ω , 5% carbon resistor.

Presently four colour code carbon resistors are also used. For certain critical applications 1% and 2% tolerance resistors are used.

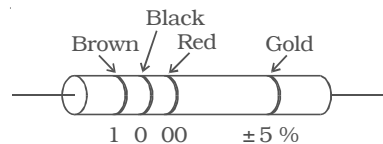


Fig 2.5 Carbon resistor

2.4 Combination of resistors

In simple circuits with resistors, Ohm's law can be applied to find the effective resistance. The resistors can be connected in series and parallel.

2.4.1 Resistors in series

Let us consider the resistors of resistances R_1 , R_2 , R_3 and R_4 connected in series as shown in Fig 2.6.

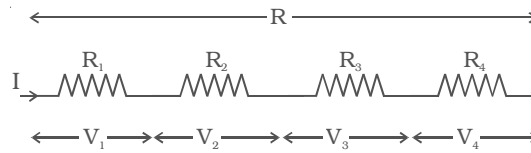


Fig 2.6 Resistors in series

When resistors are connected in series, the current flowing through each resistor is the same. If the potential difference applied between the ends of the combination of resistors is V , then the potential difference across each resistor R_1 , R_2 , R_3 and R_4 is V_1 , V_2 , V_3 and V_4 respectively.

$$\text{The net potential difference } V = V_1 + V_2 + V_3 + V_4$$

By Ohm's law

$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3, V_4 = IR_4 \text{ and } V = IR_S$$

where R_S is the equivalent or effective resistance of the series combination.

$$\text{Hence, } IR_S = IR_1 + IR_2 + IR_3 + IR_4 \text{ or } R_S = R_1 + R_2 + R_3 + R_4$$

Thus, the equivalent resistance of a number of resistors in series connection is equal to the sum of the resistance of individual resistors.

2.4.2 Resistors in parallel

Consider four resistors of resistances R_1 , R_2 , R_3 and R_4 are connected in parallel as shown in Fig 2.7. A source of emf V is connected to the parallel combination. When resistors are in parallel, the potential difference (V) across each resistor is the same.

A current I entering the combination gets divided into I_1 , I_2 , I_3 and I_4 through R_1 , R_2 , R_3 and R_4 respectively,

$$\text{such that } I = I_1 + I_2 + I_3 + I_4.$$

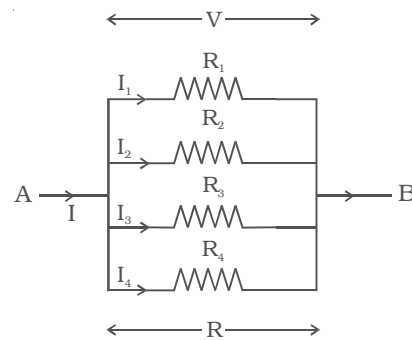


Fig 2.7 Resistors in parallel

By Ohm's law

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}, I_4 = \frac{V}{R_4} \text{ and } I = \frac{V}{R_p}$$

where R_p is the equivalent or effective resistance of the parallel combination.

$$\therefore \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the resistance of the individual resistors is equal to the reciprocal of the effective resistance of the combination.

2.5 Temperature dependence of resistance

The resistivity of substances varies with temperature. For conductors the resistance increases with increase in temperature. If R_0 is the resistance of a conductor at 0°C and R_t is the resistance of same conductor at $t^\circ\text{C}$, then

$$R_t = R_0 (1 + \alpha t)$$

where α is called the temperature coefficient of resistance.

$$\alpha = \frac{R_t - R_0}{R_0 t}$$

The temperature coefficient of resistance is defined as the ratio of increase in resistance per degree rise in temperature to its resistance at 0°C . Its unit is per $^\circ\text{C}$.

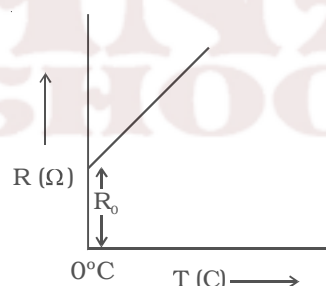


Fig 2.8 Variation of resistance with temperature

The variation of resistance with temperature is shown in Fig 2.8.

Metals have positive temperature coefficient of resistance, i.e., their resistance increases with increase in temperature. Insulators and semiconductors have negative temperature coefficient of resistance, i.e., their resistance decreases with increase in temperature. A material with a negative temperature coefficient is called a thermistor. The temperature coefficient is low for alloys.

2.6 Internal resistance of a cell

The electric current in an external circuit flows from the positive terminal to the negative terminal of the cell, through different circuit elements. In order to maintain continuity, the current has to flow through the electrolyte of the cell, from its negative terminal to positive terminal. During this process of flow of current inside the cell, a resistance is offered to current flow by the electrolyte of the cell. This is termed as the internal resistance of the cell.

A freshly prepared cell has low internal resistance and this increases with ageing.

Determination of internal resistance of a cell using voltmeter

The circuit connections are made as shown in Fig 2.9. With key K open, the emf of cell E is found by connecting a high resistance voltmeter across it. Since the high resistance voltmeter draws only a very feeble current for deflection, the circuit may be considered as an open circuit. Hence the voltmeter reading gives the emf of the cell. A small value of resistance R is included in the external circuit and key K is closed. The potential difference across R is equal to the potential difference across cell (V).

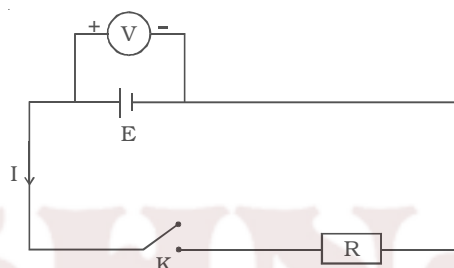


Fig 2.9 Internal resistance of a cell using voltmeter.

$$\text{The potential drop across R, } V = IR \quad \dots(1)$$

Due to internal resistance r of the cell, the voltmeter reads a value V , less than the emf of cell.

$$\text{Then } V = E - Ir \text{ or } Ir = E - V \quad \dots(2)$$

Dividing equation (2) by equation (1)

$$\frac{Ir}{IR} = \frac{E - V}{V} \text{ or } r = \left(\frac{E - V}{V} \right) R$$

Since E , V and R are known, the internal resistance r of the cell can be determined.

2.7 Kirchoff's law

Ohm's law is applicable only for simple circuits. For complicated circuits, Kirchoff's laws can be used to find current or voltage. There are two generalised laws : (i) Kirchoff's current law (ii) Kirchoff's voltage law

Kirchoff's first law (current law)

Kirchoff's current law states that the algebraic sum of the currents meeting at any junction in a circuit is zero.

The convention is that, the current flowing towards a junction is positive and the current flowing away from the junction is negative. Let 1,2,3,4 and 5 be the conductors meeting at a junction O in an electrical circuit (Fig 2.10). Let I_1 , I_2 , I_3 , I_4 and I_5 be the currents passing through the conductors respectively. According to Kirchoff's first law.

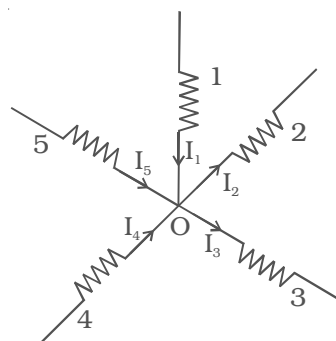


Fig 2.10 Kirchoff's current law

$$I_1 + (-I_2) + (-I_3) + I_4 + I_5 = 0 \quad \text{or} \quad I_1 + I_4 + I_5 = I_2 + I_3.$$

The sum of the currents entering the junction is equal to the sum of the currents leaving the junction. This law is a consequence of conservation of charges.

Kirchoff's second law (voltage law)

Kirchoff's voltage law states that the algebraic sum of the products of resistance and current in each part of any closed circuit is equal to the algebraic sum of the emf's in that closed circuit. This law is a consequence of conservation of energy.

In applying Kirchoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the problems, the current will be found to have negative sign. If the result is positive, then the assumed direction is the same as actual direction.

It should be noted that, once the particular direction has been assumed, the same should be used throughout the problem. However, in the application of Kirchoff's second law, we follow that the current in clockwise direction is taken as positive and the current in anticlockwise direction is taken as negative.

Let us consider the electric circuit given in Fig 2.11a.

Considering the closed loop ABCDEFA,

$$I_1 R_2 + I_3 R_4 + I_3 r_3 + I_3 R_5 + I_4 R_6 + I_1 r_1 + I_1 R_1 = E_1 + E_3$$

Both cells E_1 and E_3 send currents in clockwise direction.

For the closed loop ABEFA

$$I_1 R_2 + I_2 R_3 + I_2 r_2 + I_4 R_6 + I_1 r_1 + I_1 R_1 = E_1 - E_2$$

Negative sign in E_2 indicates that it sends current in the anticlockwise direction.

As an illustration of application of Kirchoff's second law, let us calculate the current in the following networks.

Illustration I

Applying first law to the Junction B, (Fig.2.11b)

$$I_1 - I_2 - I_3 = 0$$

$$\therefore I_3 = I_1 - I_2 \quad \dots(1)$$

For the closed loop ABEFA,

$$132 I_3 + 20 I_1 = 200 \quad \dots(2)$$

Substituting equation (1) in equation (2)

$$132 (I_1 - I_2) + 20 I_1 = 200$$

$$152 I_1 - 132 I_2 = 200 \quad \dots(3)$$

For the closed loop BCDEB,

$$60 I_2 - 132 I_3 = 100$$

substituting for I_3 ,

$$\therefore 60 I_2 - 132 (I_1 - I_2) = 100$$

$$- 132 I_1 + 192 I_2 = 100 \quad \dots(4)$$

Solving equations (3) and (4), we obtain

$$I_1 = 4.39 \text{ A and } I_2 = 3.54 \text{ A}$$

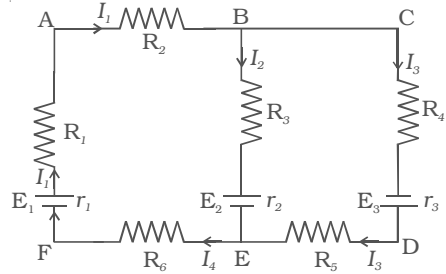


Fig 2.11a Kirchoff's laws

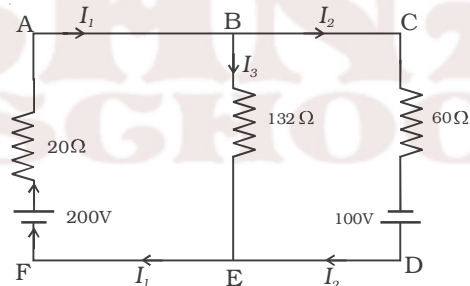


Fig 2.11b Kirchoff's laws

Illustration 2

Taking the current in the clockwise direction along ABCDA as positive (Fig 2.11c)

$$10I + 0.5I + 5I + 0.5I + 8I + 0.5I + 5I + 0.5I + 10I = 50 - 70 - 30 + 40$$

$$I (10 + 0.5 + 5 + 0.5 + 8 + 0.5 + 5 + 0.5 + 10) = -10$$

$$40 I = -10$$

$$\therefore I = \frac{-10}{40} = -0.25 \text{ A}$$

The negative sign indicates that the current flows in the anticlockwise direction.

2.7.1 Wheatstone's bridge

An important application of Kirchoff's law is the Wheatstone's bridge (Fig 2.12).

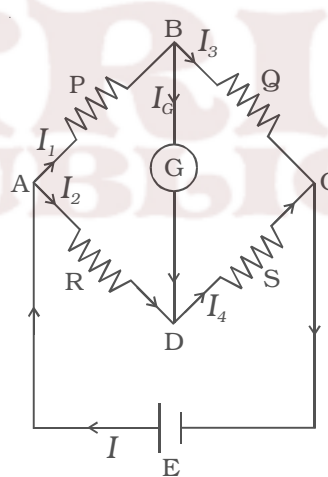


Fig 2.12

Wheatstone's bridge

Wheatstone's network consists of resistances P, Q, R and S connected to form a closed path. A cell of emf E is connected between points A and C. The current I from the cell is divided into I₁, I₂, I₃ and I₄ across the four branches. The current through the galvanometer is I_g. The resistance of galvanometer is G.

Applying Kirchoff's current law to junction B,

$$I_1 - I_g - I_3 = 0 \quad \dots(1)$$

Applying Kirchoff's current law to junction D

$$I_2 + I_g - I_4 = 0 \quad \dots(2)$$

Applying Kirchoff's voltage law to closed path ABDA

$$I_1 P + I_g G - I_2 R = 0 \quad \dots(3)$$

Applying Kirchoff's voltage law to closed path ABCDA

$$I_1 P + I_3 Q - I_4 S - I_2 R = 0 \quad \dots(4)$$

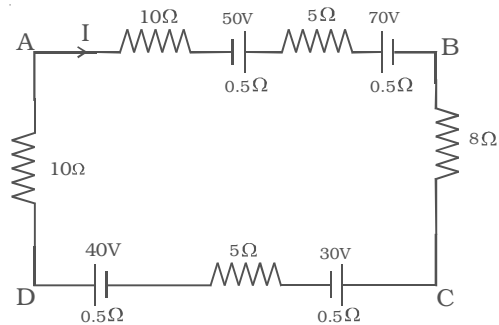


Fig 2.11c Kirchoff's laws

When the galvanometer shows zero deflection, the points B and D are at same potential and $I_g = 0$. Substituting $I_g = 0$ in equation (1), (2) and (3)

$$I_1 = I_3 \quad \dots(5)$$

$$I_2 = I_4 \quad \dots(6)$$

$$I_1 P = I_2 R \quad \dots(7)$$

Substituting the values of (5) and (6) in equation (4)

$$I_1 P + I_1 Q - I_2 S - I_2 R = 0$$

$$I_1 (P + Q) = I_2 (R + S) \quad \dots(8)$$

Dividing (8) by (7)

$$\frac{I_1(P+Q)}{I_1 P} = \frac{I_2(R+S)}{I_2 R}$$

$$\therefore \frac{P+Q}{P} = \frac{R+S}{R}$$

$$1 + \frac{Q}{P} = 1 + \frac{S}{R}$$

$$\therefore \frac{Q}{P} = \frac{S}{R} \quad \text{or} \quad \frac{P}{Q} = \frac{R}{S}$$

This is the condition for bridge balance. If P, Q and R are known, the resistance S can be calculated.

2.7.2 Metre bridge

Metre bridge is one form of Wheatstone's bridge. It consists of thick strips of copper, of negligible resistance, fixed to a wooden board. There are two gaps G_1 and G_2 between

these strips. A uniform manganin wire AC of length one metre whose temperature coefficient is low, is stretched along a metre scale and its ends are soldered to two copper strips. An unknown resistance P is connected in the gap G_1 and a standard resistance Q is connected in

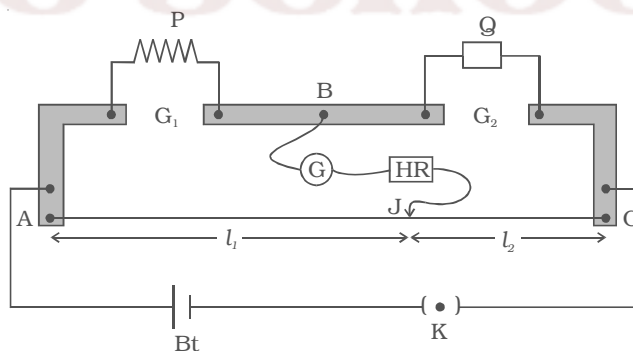


Fig 2.13 Metre bridge

the gap G_2 (Fig 2.13). A metal jockey J is connected to B through a galvanometer (G) and a high resistance (HR) and it can make contact at any point on the wire AC. Across the two ends of the wire, a Leclanche cell and a key are connected.

Adjust the position of metal jockey on metre bridge wire so that the galvanometer shows zero deflection. Let the point be J. The portions AJ and JC of the wire now replace the resistances R and S of Wheatstone's bridge. Then

$$\frac{P}{Q} = \frac{R}{S} = \frac{r.AJ}{r.JC}$$

where r is the resistance per unit length of the wire.

$$\therefore \frac{P}{Q} = \frac{AJ}{JC} = \frac{l_1}{l_2}$$

where $AJ = l_1$ and $JC = l_2$

$$\therefore P = Q \frac{l_1}{l_2}$$

Though the connections between the resistances are made by thick copper strips of negligible resistance, and the wire AC is also soldered to such strips a small error will occur in the value of $\frac{l_1}{l_2}$ due to the end resistance. This error can be eliminated, if another set of readings are taken with P and Q interchanged and the average value of P is found, provided the balance point J is near the mid point of the wire AC.

2.7.3 Determination of specific resistance

The specific resistance of the material of a wire is determined by knowing the resistance (P), radius (r) and length (L) of the wire using

the expression $\rho = \frac{P\pi r^2}{L}$

2.7.4 Determination of temperature coefficient of resistance

If R_1 and R_2 are the resistances of a given coil of wire at the temperatures t_1 and t_2 , then the temperature coefficient of resistance of the material of the coil is determined using the relation,

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

2.8 Potentiometer

The Potentiometer is an instrument used for the measurement of potential difference (Fig 2.14). It consists of a ten metre long uniform wire of manganin or constantan stretched in ten segments,

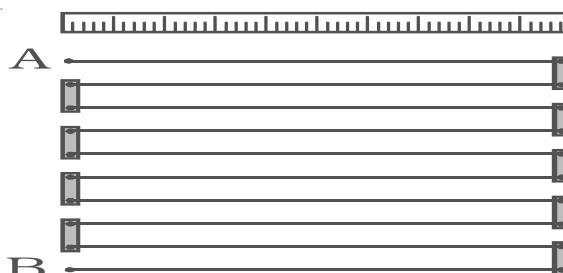


Fig 2.14 Potentiometer

each of one metre length. The segments are stretched parallel to each other on a horizontal wooden board. The ends of the wire are fixed to copper strips with binding screws. A metre scale is fixed on the board, parallel to the wire. Electrical contact with wires is established by pressing the jockey J.

2.8.1 Principle of potentiometer

A battery Bt is connected between the ends A and B of a potentiometer wire through a key K. A steady current I flows through the potentiometer wire (Fig 2.15). This forms the

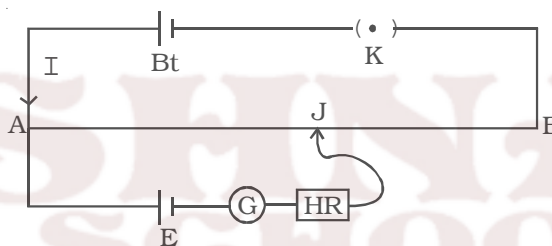


Fig 2.15 Principle of potentiometer

primary circuit. A primary cell is connected in series with the positive terminal A of the potentiometer, a galvanometer, high resistance and jockey. This forms the secondary circuit.

If the potential difference between A and J is equal to the emf of the cell, no current flows through the galvanometer. It shows zero deflection. AJ is called the balancing length. If the balancing length is l , the potential difference across $AJ = Irl$ where r is the resistance per unit length of the potentiometer wire and I the current in the primary circuit.

$$\therefore E = Irl$$

since I and r are constants, $E \propto l$

Hence emf of the cell is directly proportional to its balancing length. This is the principle of a potentiometer.

2.8.2 Comparison of emfs of two given cells using potentiometer

The potentiometer wire AB is connected in series with a battery (Bt), Key (K), rheostat (Rh) as shown in Fig 2.16. This forms the primary circuit. The end A of potentiometer is connected to the terminal C of a DPDT switch (six way key-double pole double throw). The terminal D is connected to the jockey (J) through a galvanometer (G) and high resistance (HR). The cell of emf E_1 is connected between terminals C_1 and D_1 and the cell of emf E_2 is connected between C_2 and D_2 of the DPDT switch.

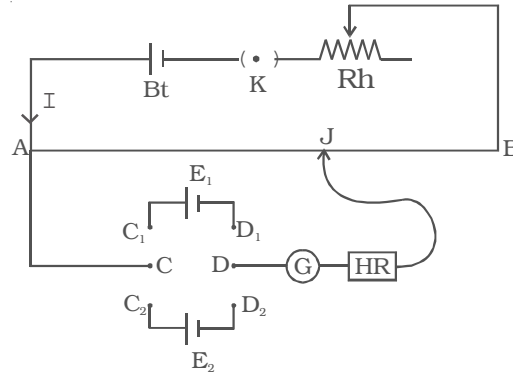


Fig 2.16 comparison of emf of two cells

Let I be the current flowing through the primary circuit and r be the resistance of the potentiometer wire per metre length.

The DPDT switch is pressed towards C_1, D_1 so that cell E_1 is included in the secondary circuit. The jockey is moved on the wire and adjusted for zero deflection in galvanometer. The balancing length is l_1 . The potential difference across the balancing length $l_1 = Irl_1$. Then, by the principle of potentiometer,

$$E_1 = Irl_1 \quad \dots(1)$$

The DPDT switch is pressed towards E_2 . The balancing length l_2 for zero deflection in galvanometer is determined. The potential difference across the balancing length is $l_2 = Irl_2$, then

$$E_2 = Irl_2 \quad \dots(2)$$

Dividing (1) and (2) we get

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

If emf of one cell (E_1) is known, the emf of the other cell (E_2) can be calculated using the relation.

$$E_2 = E_1 \frac{l_2}{l_1}$$

2.8.3 Comparison of emf and potential difference

1. The difference of potentials between the two terminals of a cell in an open circuit is called the electromotive force (emf) of a cell. The difference in potentials between any two points in a closed circuit is called potential difference.

2. The emf is independent of external resistance of the circuit, whereas potential difference is proportional to the resistance between any two points.

2.9 Electric energy and electric power.

If I is the current flowing through a conductor of resistance R in time t , then the quantity of charge flowing is, $q = It$. If the charge q , flows between two points having a potential difference V , then the work done in moving the charge is $= V \cdot q = V It$.

Then, electric power is defined as the rate of doing electric work.

$$\therefore \text{Power} = \frac{\text{Work done}}{\text{time}} = \frac{VIt}{t} = VI$$

Electric power is the product of potential difference and current strength.

$$\text{Since } V = IR, \text{ Power} = I^2R$$

Electric energy is defined as the capacity to do work. Its unit is joule. In practice, the electrical energy is measured by watt hour (Wh) or kilowatt hour (kWh). 1 kWh is known as one unit of electric energy.

$$(1 \text{ kWh} = 1000 \text{ Wh} = 1000 \times 3600 \text{ J} = 36 \times 10^5 \text{ J})$$

2.9.1 Wattmeter

A wattmeter is an instrument used to measure electrical power consumed i.e energy absorbed in unit time by a circuit. The wattmeter consists of a movable coil arranged between a pair of fixed coils in the form of a solenoid. A pointer is attached to the movable coil. The free end of the pointer moves over a circular scale. When current flows through the coils, the deflection of the pointer is directly proportional to the power.

2.10 Chemical effect of current

The passage of an electric current through a liquid causes chemical changes and this process is called electrolysis. The conduction

is possible, only in liquids wherein charged ions can be dissociated in opposite directions (Fig 2.17). Such liquids are called electrolytes. The plates through which current enters and leaves an electrolyte are known as electrodes. The electrode towards which positive ions travel is called the cathode and the other, towards which negative ions travel is called anode. The positive ions are called cations and are mostly formed from metals or hydrogen. The negative ions are called anions.

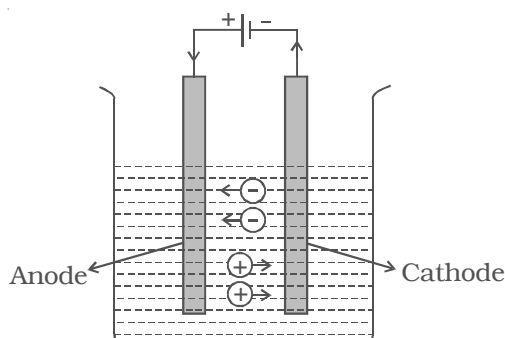


Fig 2.17 Conduction in liquids

2.10.1 Faraday's laws of electrolysis

The factors affecting the quantities of matter liberated during the process of electrolysis were investigated by Faraday.

First Law : The mass of a substance liberated at an electrode is directly proportional to the charge passing through the electrolyte.

If an electric current I is passed through an electrolyte for a time t , the amount of charge (q) passed is It . According to the law, mass of substance liberated (m) is

$$m \propto q \quad \text{or} \quad m = zIt$$

where Z is a constant for the substance being liberated called as electrochemical equivalent. Its unit is kg C^{-1} .

The electrochemical equivalent of a substance is defined as the mass of substance liberated in electrolysis when one coulomb charge is passed through the electrolyte.

Second Law : The mass of a substance liberated at an electrode by a given amount of charge is proportional to the *chemical equivalent of the substance.

If E is the chemical equivalent of a substance, from the second law

$$m \propto E$$

$$\text{*Chemical equivalent} = \frac{\text{Relative atomic mass}}{\text{Valency}} = \frac{\text{mass of the atom}}{1/12 \text{ of the mass C}^{12} \text{ atom} \times \text{valency}}$$

2.10.2 Verification of Faraday's laws of electrolysis

First Law : A battery, a rheostat, a key and an ammeter are connected in series to an electrolytic cell (Fig 2.18). The cathode is cleaned, dried, weighed and then inserted in the cell. A current I_1 is passed for a time t . The current is measured by the ammeter. The cathode is taken out, washed, dried and weighed again. Hence the mass m_1 of the substance deposited is obtained.

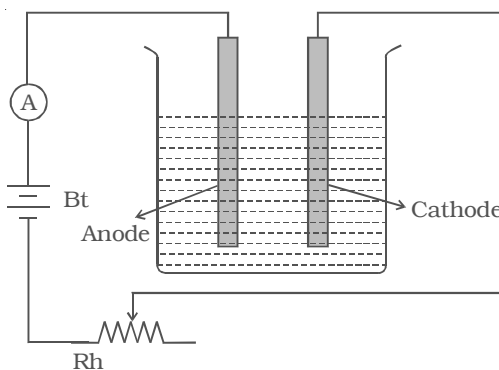


Fig 2.18 Verification of Faraday's first law

The cathode is reinserted in the cell and a different current I_2 is passed for the same time t . The mass m_2 of the deposit is obtained. It is found that

$$\frac{m_1}{m_2} = \frac{I_1}{I_2}$$

$$\therefore m \propto I \quad \dots(1)$$

The experiment is repeated for same current I but for different times t_1 and t_2 . If the masses of the deposits are m_3 and m_4 respectively, it is found that

$$\frac{m_3}{m_4} = \frac{t_1}{t_2}$$

$$\therefore m \propto t \quad \dots(2)$$

From relations (1) and (2)

$m \propto It$ or $m \propto q$ Thus, the first law is verified.

Second Law : Two electrolytic cells containing different electrolytes, CuSO_4 solution and AgNO_3 solution are connected in series with a battery, a rheostat and an ammeter (Fig 2.19). Copper electrodes are inserted in CuSO_4 and silver electrodes are inserted in AgNO_3 .

The cathodes are cleaned, dried, weighed and then inserted in the respective cells. The current is passed for some time. Then the cathodes are taken out, washed, dried and weighed. Hence the masses of copper and silver deposited are found as m_1 and m_2 .

It is found that $\frac{m_1}{m_2} = \frac{E_1}{E_2}$, where E_1 and E_2 are the chemical equivalents of copper and silver respectively.

$$m \propto E$$

Thus, the second law is verified.

2.11 Electric cells

The starting point to the development of electric cells is the classic experiment by Luige Galvani and his wife Lucia on a dissected frog hung from iron railings with brass hooks. It was observed that, whenever the leg of the frog touched the iron railings, it jumped and this led to the introduction of animal electricity. Later, Italian scientist and genius professor Alessandro Volta came up with an electrochemical battery. The battery Volta named after him consisted of a pile of copper and zinc discs placed alternately separated by paper and introduced in salt solution. When the end plates were connected to an electric bell, it continued to ring, opening a new world of electrochemical cells. His experiment established that, a cell could be made by using two dissimilar metals and a salt solution which reacts with atleast one of the metals as electrolyte.

2.11.1 Voltaic cell

The simple cell or voltaic cell consists of two electrodes, one of copper and the other of zinc dipped in a solution of dilute sulphuric acid in a glass vessel (Fig 2.20). On connecting the two electrodes externally, with a piece of wire, current flows

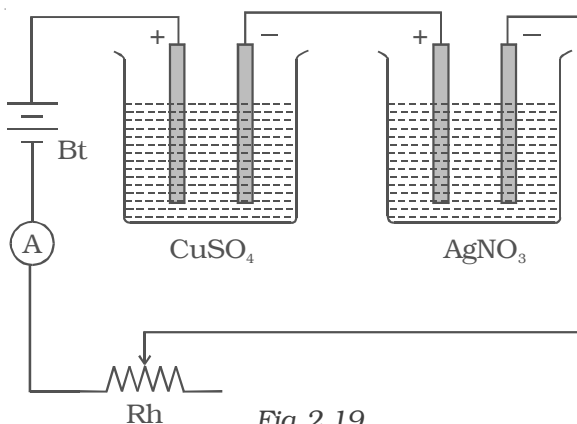


Fig 2.19

Verification of Faraday's second law

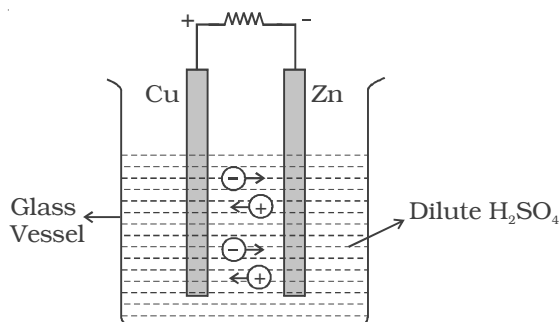


Fig 2.20 Voltaic cell

from copper to zinc outside the cell and from zinc to copper inside it. The copper electrode is the positive pole or copper rod of the cell and zinc is the negative pole or zinc rod of the cell. The electrolyte is dilute sulphuric acid.

The action of the cell is explained in terms of the motion of the charged ions. At the zinc rod, the zinc atoms get ionized and pass into solution as Zn^{++} ions. This leaves the zinc rod with two electrons more, making it negative. At the same time, two hydrogen ions ($2H^+$) are discharged at the copper rod, by taking these two electrons. This makes the copper rod positive. As long as excess electrons are available on the zinc electrode, this process goes on and a current flows continuously in external circuit. This simple cell is thus seen as a device which converts chemical energy into electrical energy. Due to opposite charges on the two plates, a potential difference is set up between copper and zinc, copper being at a higher potential than zinc. The difference of potential between the two electrodes is 1.08V.

2.11.2 Primary Cell

The cells from which the electric energy is derived by irreversible chemical actions are called primary cells. The primary cell is capable of giving an emf, when its constituents, two electrodes and a suitable electrolyte, are assembled together. The three main primary cells, namely Daniel Cell and Leclanche cell are discussed here. These cells cannot be recharged electrically.

2.11.3 Daniel cell

Daniel cell is a primary cell which cannot supply steady current for a long time. It consists of a copper vessel containing a strong solution of copper sulphate (Fig 2.21). A zinc rod is dipped in dilute sulphuric acid contained in a porous pot. The porous pot is placed inside the copper sulphate solution.

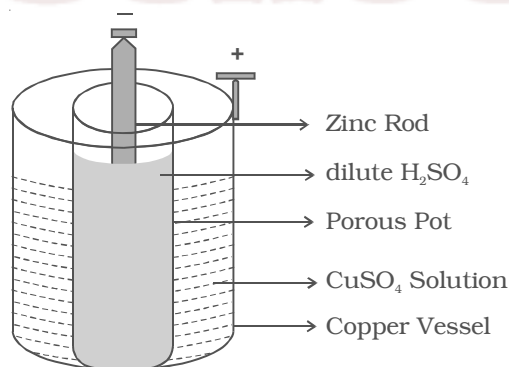


Fig 2.21 Daniel cell

The zinc rod reacting with dilute sulphuric acid produces Zn^{++} ions and 2 electrons.

Zn^{++} ions pass through the pores of the porous pot and reacts with copper sulphate solution, producing Cu^{++} ions. The Cu^{++} ions deposit on the copper vessel. When Daniel cell is connected in a circuit, the two electrons on the zinc rod pass through the external circuit and reach the copper vessel thus neutralizing the copper ions. This constitutes an electric current from copper to zinc. Daniel cell produces an emf of 1.08 volt.

2.11.4 Leclanche cell

A Leclanche cell consists of a carbon electrode packed in a porous pot containing manganese dioxide and charcoal powder (Fig 2.22). The porous pot is immersed in a saturated solution of ammonium chloride (electrolyte) contained in an outer glass vessel. A zinc rod is immersed in electrolytic solution.

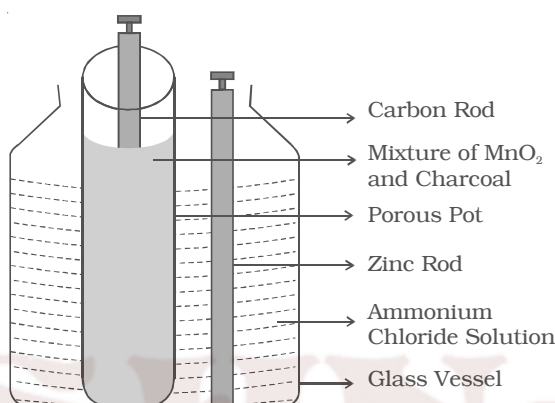
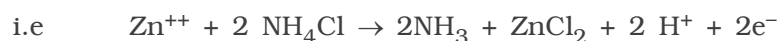


Fig 2.22 Leclanche cell

At the zinc rod, due to oxidation reaction Zn atom is converted into Zn^{++} ions and 2 electrons. Zn^{++} ions reacting with ammonium chloride produces zinc chloride and ammonia gas.



The ammonia gas escapes. The hydrogen ions diffuse through the pores of the porous pot and react with manganese dioxide. In this process the positive charge of hydrogen ion is transferred to carbon rod. When zinc rod and carbon rod are connected externally, the two electrons from the zinc rod move towards carbon and neutralizes the positive charge. Thus current flows from carbon to zinc.

Leclanche cell is useful for supplying intermittent current. The emf of the cell is about 1.5 V, and it can supply a current of 0.25 A.

2.11.5 Secondary Cells

The advantage of secondary cells is that they are rechargeable. The chemical reactions that take place in secondary cells are reversible. The active materials that are used up when the cell delivers current can be reproduced by passing current through the cell in opposite direction. The chemical process of obtaining current from a secondary cell is called discharge. The process of reproducing active materials is called charging. The most common secondary cells are lead acid accumulator and alkali accumulator.

2.11.6 Lead - Acid accumulator

The lead acid accumulator consists of a container made up of hard rubber or glass or celluloid. The container contains dilute sulphuric acid which acts as the electrolyte.

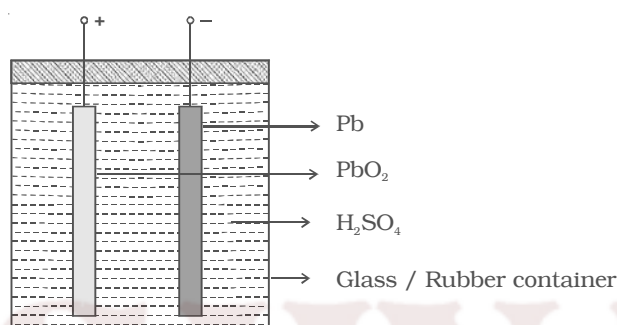


Fig 2.23 Lead - Acid accumulator

Spongy lead (Pb) acts as the negative electrode and lead oxide (PbO₂) acts as the positive electrode (Fig 2.23). The electrodes are separated by suitable insulating materials and assembled in a way to give low internal resistance.

When the cell is connected in a circuit, due to the oxidation reaction that takes place at the negative electrode, spongy lead reacting with dilute sulphuric acid produces lead sulphate and two electrons. The electrons flow in the external circuit from negative electrode to positive electrode where the reduction action takes place. At the positive electrode, lead oxide on reaction with sulphuric acid produces lead sulphate and the two electrons are neutralized in this process. This makes the conventional current to flow from positive electrode to negative electrode in the external circuit.

The emf of a freshly charged cell is 2.2 Volt and the specific gravity of the electrolyte is 1.28. The cell has low internal resistance and hence can deliver high current. As the cell is discharged by drawing current from it, the emf falls to about 2 volts. In the process of charging, the chemical reactions are reversed.

2.11.7 Applications of secondary cells

The secondary cells are rechargeable. They have very low internal resistance. Hence they can deliver a high current if required. They can be recharged a very large number of times without any deterioration in properties. These cells are huge in size. They are used in all automobiles like cars, two wheelers, trucks etc. The state of charging these cells is, simply monitoring the specific gravity of the electrolyte. It should lie between 1.28 to 1.12 during charging and discharging respectively.

Solved problems

- 2.1 If 6.25×10^{18} electrons flow through a given cross section in unit time, find the current. (Given : Charge of an electron is 1.6×10^{-19} C)

Data : $n = 6.25 \times 10^{18}$; $e = 1.6 \times 10^{-19}$ C ; $t = 1$ s ; $I = ?$

Solution : $I = \frac{q}{t} = \frac{ne}{t} = \frac{6.25 \times 10^{18} \times 1.6 \times 10^{-19}}{1} = 1$ A

- 2.2 A copper wire of 10^{-6} m² area of cross section, carries a current of 2 A. If the number of electrons per cubic metre is 8×10^{28} , calculate the current density and average drift velocity.

(Given $e = 1.6 \times 10^{-19}$ C)

Data : $A = 10^{-6}$ m² ; Current flowing $I = 2$ A ; $n = 8 \times 10^{28}$

$e = 1.6 \times 10^{-19}$ C ; $J = ?$; $v_d = ?$

Solution : Current density, $J = \frac{I}{A} = \frac{2}{10^{-6}} = 2 \times 10^6$ A/m²

$$J = n e v_d$$

$$\text{or } v_d = \frac{J}{ne} = \frac{2 \times 10^6}{8 \times 10^{28} \times 1.6 \times 10^{-19}} = 15.6 \times 10^{-5} \text{ m s}^{-1}$$

- 2.3 An incandescent lamp is operated at 240 V and the current is 0.5 A. What is the resistance of the lamp ?

Data : $V = 240$ V ; $I = 0.5$ A ; $R = ?$

Solution : From Ohm's law

$$V = IR \quad \text{or} \quad R = \frac{V}{I} = \frac{240}{0.5} = 480 \, \Omega$$

- 2.4 The resistance of a copper wire of length 5m is 0.5 Ω . If the diameter of the wire is 0.05 cm, determine its specific resistance.

Data : $l = 5\text{ m}$; $R = 0.5 \, \Omega$; $d = 0.05 \text{ cm} = 5 \times 10^{-4} \text{ m}$;

$$r = 2.5 \times 10^{-4} \text{ m} ; \rho = ?$$

Solution : $R = \frac{\rho l}{A}$ or $\rho = \frac{RA}{l}$

$$A = \pi r^2 = 3.14 \times (2.5 \times 10^{-4})^2 = 1.9625 \times 10^{-7} \text{ m}^2$$

$$\rho = \frac{0.5 \times 1.9625 \times 10^{-7}}{5}$$

$$\rho = 1.9625 \times 10^{-8} \, \Omega \text{ m}$$

- 2.5 The resistance of a nichrome wire at 0°C is 10 Ω . If its temperature coefficient of resistance is 0.004/°C, find its resistance at boiling point of water. Comment on the result.

Data : At 0°C, $R_0 = 10 \, \Omega$; $\alpha = 0.004/\text{°C}$; $t = 100^\circ\text{C}$;

$$\text{At } t^\circ\text{C, } R_t = ?$$

Solution : $R_t = R_0 (1 + \alpha t)$
 $= 10 (1 + (0.004 \times 100))$

$$R_t = 14 \, \Omega$$

As temperature increases the resistance of wire also increases.

- 2.6 Two wires of same material and length have resistances 5 Ω and 10 Ω respectively. Find the ratio of radii of the two wires.

Data : Resistance of first wire $R_1 = 5 \, \Omega$;

$$\text{Radius of first wire} = r_1$$

$$\text{Resistance of second wire } R_2 = 10 \, \Omega$$

$$\text{Radius of second wire} = r_2$$

$$\text{Length of the wires} = l$$

$$\text{Specific resistance of the material of the wires} = \rho$$

Solution : $R = \frac{\rho l}{A}; A = \pi r^2$

$$\therefore R_1 = \frac{\rho l}{\pi r_1^2}; R_2 = \frac{\rho l}{\pi r_2^2}$$

$$\frac{R_2}{R_1} = \frac{r_1^2}{r_2^2} \text{ or } \frac{r_1}{r_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{10}{5}} = \frac{\sqrt{2}}{1}$$

$$r_1 : r_2 = \sqrt{2} : 1$$

2.7 If a copper wire is stretched to make it 0.1% longer, what is the percentage change in resistance?

Data : Initial length of copper wire $l_1 = l$

Final length of copper wire after stretching

$$l_2 = l + 0.1\% \text{ of } l$$

$$= l + \frac{0.1}{100} l$$

$$= l (1 + 0.001)$$

$$l_2 = 1.001 l$$

During stretching, if length increases, area of cross section decreases.

$$\text{Initial volume} = A_1 l_1 = A_1 l$$

$$\text{Final volume} = A_2 l_2 = 1.001 A_2 l$$

$$\text{Resistance of wire before stretching} = R_1.$$

$$\text{Resistance after stretching} = R_2$$

Solution : Equating the volumes

$$A_1 l = 1.001 A_2 l$$

$$\text{(or)} \quad A_1 = 1.001 A_2$$

$$R = \frac{\rho l}{A}$$

$$R_1 = \frac{\rho l_1}{A_1} \text{ and } R_2 = \frac{\rho l_2}{A_2}$$

$$R_1 = \frac{\rho l}{1.001A_2} \quad \text{and} \quad R_2 = \frac{\rho 1.001l}{A_2}$$

$$\frac{R_2}{R_1} = (1.001)^2 = 1.002$$

$$\text{Change in resistance} = (1.002 - 1) = 0.002$$

$$\text{Change in resistance in percentage} = 0.002 \times 100 = 0.2\%$$

- 2.8 The resistance of a field coil measures $50 \, \Omega$ at 20°C and $65 \, \Omega$ at 70°C . Find the temperature coefficient of resistance.

Data : At $R_{20} = 50 \, \Omega$; 70°C , $R_{70} = 65 \, \Omega$; $\alpha = ?$

Solution : $R_t = R_o (1 + \alpha t)$

$$R_{20} = R_o (1 + \alpha 20)$$

$$50 = R_o (1 + \alpha 20) \quad \dots(1)$$

$$R_{70} = R_o (1 + \alpha 70)$$

$$65 = R_o (1 + \alpha 70) \quad \dots(2)$$

Dividing (2) by (1)

$$\frac{65}{50} = \frac{1 + 70\alpha}{1 + 20\alpha}$$

$$65 + 1300 \alpha = 50 + 3500 \alpha$$

$$2200 \alpha = 15$$

$$\alpha = 0.0068 / ^\circ\text{C}$$

- 2.9 An iron box of $400 \, \text{W}$ power is used daily for 30 minutes. If the cost per unit is 75 paise, find the weekly expense on using the iron box.

Data : Power of an iron box $P = 400 \, \text{W}$

rate / unit = $75 \, \text{p}$

consumption time $t = 30$ minutes / day

cost / week = ?

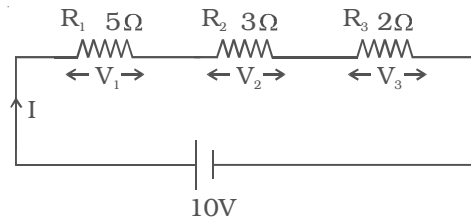
Solution :

Energy consumed in 30 minutes = Power \times time in hours

$$= 400 \times \frac{1}{2} = 200 \, \text{W h}$$

Energy consumed in one week = $200 \times 7 = 1400 \text{ Wh} = 1.4 \text{ unit}$
 Cost / week = Total units consumed \times rate/ unit
 = $1.4 \times 0.75 = \text{Rs.}1.05$

2.10 Three resistors are connected in series with 10 V supply as shown in the figure. Find the voltage drop across each resistor.



Data : $R_1 = 5\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$; $V = 10 \text{ volt}$
 Effective resistance of series combination,
 $R_s = R_1 + R_2 + R_3 = 10\Omega$

Solution : Current in circuit $I = \frac{V}{R_s} = \frac{10}{10} = 1\text{A}$

Voltage drop across R_1 , $V_1 = IR_1 = 1 \times 5 = 5\text{V}$

Voltage drop across R_2 , $V_2 = IR_2 = 1 \times 3 = 3\text{V}$

Voltage drop across R_3 , $V_3 = IR_3 = 1 \times 2 = 2\text{V}$

2.11 Find the current flowing across three resistors 3Ω , 5Ω and 2Ω connected in parallel to a 15 V supply. Also find the effective resistance and total current drawn from the supply.

Data : $R_1 = 3\Omega$, $R_2 = 5\Omega$, $R_3 = 2\Omega$; Supply voltage $V = 15 \text{ volt}$

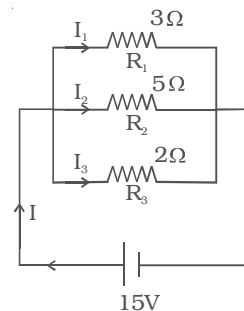
Solution :

Effective resistance of parallel combination

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{5} + \frac{1}{2}$$

$$R_p = 0.9677 \Omega$$

$$\text{Current through } R_1, I_1 = \frac{V}{R_1} = \frac{15}{3} = 5\text{A}$$

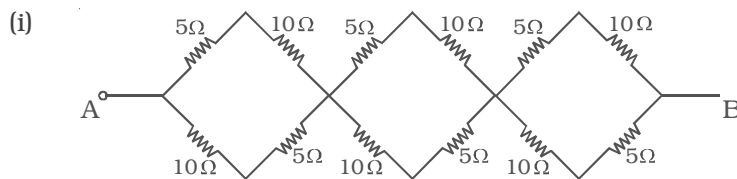


$$\text{Current through } R_2, I_2 = \frac{V}{R_2} = \frac{15}{5} = 3A$$

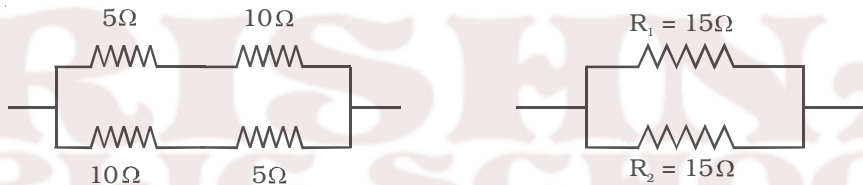
$$\text{Current through } R_3, I_3 = \frac{V}{R_3} = \frac{15}{2} = 7.5A$$

$$\text{Total current } I = \frac{V}{R_p} = \frac{15}{0.9677} = 15.5 A$$

2.12 In the given network, calculate the effective resistance between points A and B



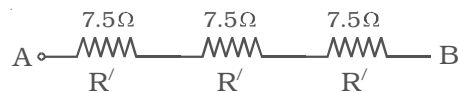
Solution : The network has three identical units. The simplified form of one unit is given below :



The equivalent resistance of one unit is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{15} + \frac{1}{15} \text{ or } R_p = 7.5 \Omega$$

Each unit has a resistance of 7.5 Ω. The total network reduces to



The combined resistance between points A and B is

$$R = R' + R' + R' (\because R_s = R_1 + R_2 + R_3)$$

$$R = 7.5 + 7.5 + 7.5 = 22.5 \Omega$$

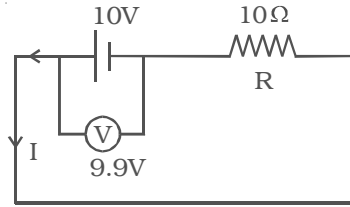
2.13 A 10 Ω resistance is connected in series with a cell of emf 10V. A voltmeter is connected in parallel to a cell, and it reads. 9.9 V. Find internal resistance of the cell.

Data : $R = 10 \Omega$; $E = 10 V$; $V = 9.9 V$; $r = ?$

Solution : $r = \left(\frac{E - V}{V} \right) R$

$$= \left(\frac{10 - 9.9}{9.9} \right) \times 10$$

$$= 0.101 \Omega$$



Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

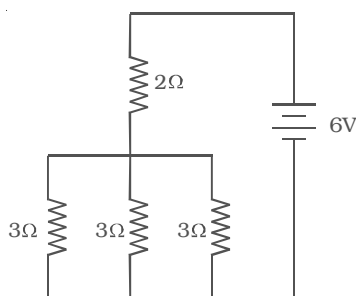
- 2.1 A charge of 60 C passes through an electric lamp in 2 minutes. Then the current in the lamp is
 (a) 30 A (b) 1 A (c) 0.5 A (d) 5 A
- 2.2 The material through which electric charge can flow easily is
 (a) quartz (b) mica (c) germanium (d) copper
- 2.3 The current flowing in a conductor is proportional to
 (a) drift velocity
 (b) 1/ area of cross section
 (c) 1/no of electrons
 (d) square of area of cross section.
- 2.4 A toaster operating at 240V has a resistance of 120Ω. The power is
 (a) 400 W (b) 2 W (c) 480 W (d) 240 W
- 2.5 If the length of a copper wire has a certain resistance R, then on doubling the length its specific resistance
 (a) will be doubled (b) will become 1/4th
 (c) will become 4 times (d) will remain the same.
- 2.6 When two 2Ω resistances are in parallel, the effective resistance is
 (a) 2 Ω (b) 4 Ω (c) 1 Ω (d) 0.5 Ω
- 2.7 In the case of insulators, as the temperature decreases, resistivity
 (a) decreases (b) increases

- (c) remains constant (d) becomes zero
- 2.8 If the resistance of a coil is $2\ \Omega$ at 0°C and $\alpha = 0.004\ /^\circ\text{C}$, then its resistance at 100°C is
 (a) $1.4\ \Omega$ (b) $0\ \Omega$ (c) $4\ \Omega$ (d) $2.8\ \Omega$
- 2.9 According to Faraday's law of electrolysis, when a current is passed, the mass of ions deposited at the cathode is independent of
 (a) current (b) charge (c) time (d) resistance
- 2.10 When n resistors of equal resistances (R) are connected in series, the effective resistance is
 (a) n/R (b) R/n (c) $1/nR$ (d) nR
- 2.11 Why is copper wire not suitable for a potentiometer?
- 2.12 Explain the flow of charges in a metallic conductor.
- 2.13 Distinguish between drift velocity and mobility. Establish a relation between drift velocity and current.
- 2.14 State Ohm's law.
- 2.15 Define resistivity of a material. How are materials classified based on resistivity?
- 2.16 Write a short note on superconductivity. List some applications of superconductors.
- 2.17 The colours of a carbon resistor is orange, orange, orange. What is the value of resistor?
- 2.18 Explain the effective resistance of a series network and parallel network.
- 2.19 Discuss the variation of resistance with temperature with an expression and a graph.
- 2.20 Explain the determination of the internal resistance of a cell using voltmeter.
- 2.21 State and explain Kirchoff's laws for electrical networks.
- 2.22 Describe an experiment to find unknown resistance and temperature coefficient of resistance using metre bridge?
- 2.23 Define the term specific resistance. How will you find this using a metre bridge?

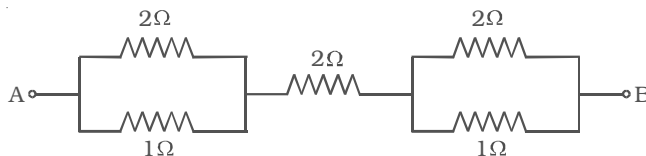
- 2.24 Explain the principle of a potentiometer. How can emf of two cells be compared using potentiometer?
- 2.25 Distinguish between electric power and electric energy
- 2.26 State and Explain Faraday's laws of electrolysis. How are the laws verified experimentally?
- 2.27 Explain the reactions at the electrodes of (i) Daniel cell (ii) Leclanche cell
- 2.28 Explain the action of the following secondary cell.
(i) lead acid accumulator
- 2.29 Why automobile batteries have low internal resistance?

Problems

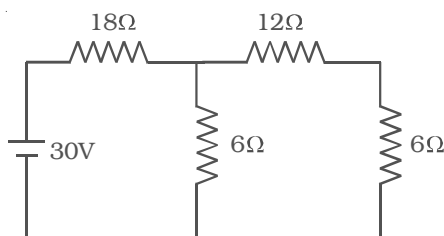
- 2.30 What is the drift velocity of an electron in a copper conductor having area $10 \times 10^{-6} \text{m}^2$, carrying a current of 2 A. Assume that there are 10×10^{28} electrons / m^3 .
- 2.31 How much time 10^{20} electrons will take to flow through a point, so that the current is 200 mA? ($e = 1.6 \times 10^{-19} \text{ C}$)
- 2.32 A manganin wire of length 2m has a diameter of 0.4 mm with a resistance of 70Ω . Find the resistivity of the material.
- 2.33 The effective resistances are 10Ω , 2.4Ω when two resistors are connected in series and parallel. What are the resistances of individual resistors?
- 2.34 In the given circuit, what is the total resistance and current supplied by the battery.



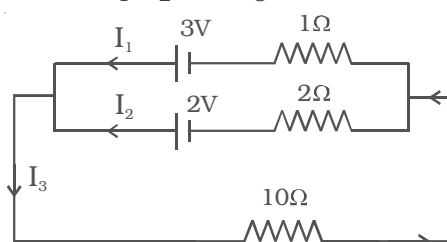
- 2.35 Find the effective resistance between A and B in the given circuit



2.36 Find the voltage drop across $18\ \Omega$ resistor in the given circuit



2.37 Calculate the current I_1 , I_2 and I_3 in the given electric circuit.



2.38 The resistance of a platinum wire at 0°C is $4\ \Omega$. What will be the resistance of the wire at 100°C if the temperature coefficient of resistance of platinum is $0.0038 / ^\circ\text{C}$.

2.39 A cell has a potential difference of $6\ \text{V}$ in an open circuit, but it falls to $4\ \text{V}$ when a current of $2\ \text{A}$ is drawn from it. Find the internal resistance of the cell.

2.40 In a Wheatstone's bridge, if the galvanometer shows zero deflection, find the unknown resistance. Given $P = 1000\ \Omega$, $Q = 10000\ \Omega$ and $R = 20\ \Omega$

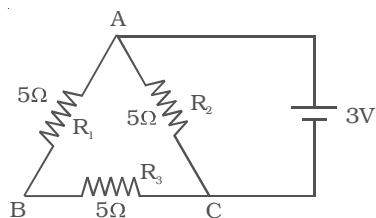
2.41 An electric iron of resistance $80\ \Omega$ is operated at $200\ \text{V}$ for two hours. Find the electrical energy consumed.

2.42 In a house, electric kettle of $1500\ \text{W}$ is used everyday for 45 minutes, to boil water. Find the amount payable per month (30 days) for usage of this, if cost per unit is Rs. 3.25

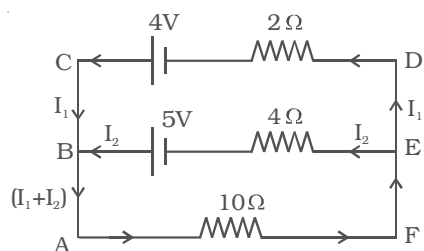
2.43 A $1.5\ \text{V}$ carbon - zinc dry cell is connected across a load of $1000\ \Omega$. Calculate the current and power supplied to it.

2.44 In a metre bridge, the balancing length for a $10\ \Omega$ resistance in left gap is $51.8\ \text{cm}$. Find the unknown resistance and specific resistance of a wire of length $108\ \text{cm}$ and radius $0.2\ \text{mm}$.

2.45 Find the electric current flowing through the given circuit connected to a supply of 3 V.



2.46 In the given circuit, find the current through each branch of the circuit and the potential drop across the 10 Ω resistor.



Answers

- | | | | |
|------------------------------------|-----------------|-----------------------------------------------------|----------------|
| 2.1 (c) | 2.2 (d) | 2.3 (a) | 2.4 (c) |
| 2.5 (d) | 2.6 (c) | 2.7 (b) | 2.8 (d) |
| 2.9 (d) | 2.10 (d) | | |
| 2.17 33 kΩ | | 2.30 $1.25 \times 10^{-5} \text{ m s}^{-1}$ | |
| 2.31 80s | | 2.32 $4.396 \mu\Omega \text{ m}$ | |
| 2.33 6 Ω and 4Ω | | 2.34 3 Ω and 2A | |
| 2.35 3.33 Ω | | 2.36 24 V | |
| 2.37 0.5 A, -0.25 A, 0.25 A | | 2.38 5.52 Ω | |
| 2.39 1 Ω | | 2.40 200 Ω | |
| 2.41 1 kWh | | 2.42 Rs. 110 | |
| 2.43 1.5 mA; 2.25 mW | | 2.44 $1.082 \times 10^{-6} \Omega \text{ m}$ | |
| 2.45 0.9 A | | 2.46 0.088A, 0.294A, 3.82 V | |

3. Effects of electric current

The ideas of electric current, electromotive force having been already discussed in the preceding chapter, we shall discuss in this chapter the physical consequences of electric current. Living in an electrical and interestingly in an electronic age, we are familiar with many practical applications of electric current, such as bulbs, electroplating, electric fans, electric motors etc. In a source of emf, a part of the energy may go into useful work like in an electric motor. The remaining part of the energy is dissipated in the form of heat in the resistors. This is the heating effect of current. Just as current produces thermal energy, thermal energy may also be suitably used to produce an emf. This is thermoelectric effect. This effect is not only a cause but also a consequence of current. A steady electric current produces a magnetic field in surrounding space. This important physical consequence of current is magnetic effect of electric current.

3.1 Heating effect : Joule's law

In a conductor, the free electrons are always at random motion making collisions with ions or atoms of the conductor. When a voltage V is applied between the ends of the conductor, resulting in the flow of current I , the free electrons are accelerated. Hence the electrons gain energy at the rate of VI per second. The lattice ions or atoms receive this energy VI from the colliding electrons in random bursts. This increase in energy is nothing but the thermal energy of the lattice. Thus for a steady current I , the amount of heat produced in time t is

$$H = Vit \quad \dots(1)$$

For a resistance R ,

$$H = I^2Rt \quad \dots(2) \text{ and}$$

$$H = \frac{V^2}{R} t \quad \dots(3)$$

The above relations were experimentally verified by Joule and are known as Joule's law of heating. By equation (2) Joule's law implies

that the heat produced is (i) directly proportional to the square of the current for a given R (ii) directly proportional to resistance R for a given I and (iii) directly proportional to the time of passage of current. Also by equation (3), the heat produced is inversely proportional to resistance R for a given V.

3.1.1 Verification of Joule's law

Joule's law is verified using Joule's calorimeter. It consists of a resistance coil R enclosed inside a copper calorimeter (Fig 3.1).

The ends of the coil are connected to two terminals, fixed to the lid of the calorimeter. A stirrer and a thermometer T are inserted through two holes in the lid. Two thirds of the volume of the calorimeter is filled with water. The calorimeter is enclosed in a wooden box to minimise loss of heat.

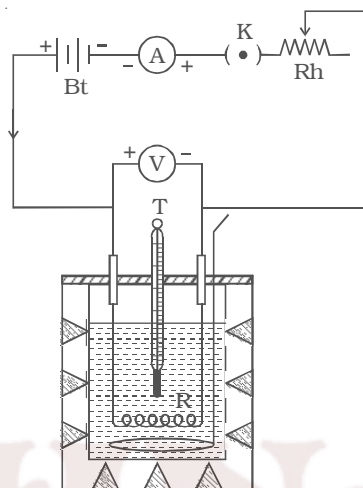


Fig 3.1 Joule's calorimeter

A battery (Bt), a key (K), a rheostat (Rh) and an ammeter (A) are connected in series with the calorimeter. A voltmeter (V) is connected across the ends of the coil R.

(i) Law of current

The initial temperature of water is measured as θ_1 . Let W be the heat capacity of the calorimeter and contents. Now a current of I_1 is passed for a time of t (about 20 minutes). The final temperature (θ_2) (after applying necessary correction) is noted. The quantity of heat gained by calorimeter and the contents is calculated as $H_1 = W (\theta_2 - \theta_1)$. Water is then cooled to θ_1 . The experiment is repeated by passing currents $I_2, I_3 \dots$ etc., through the same coil for the same interval of time t and the corresponding quantities of heat H_2, H_3 etc. are calculated. It is found that

$$\frac{H_1}{I_1^2} = \frac{H_2}{I_2^2} = \frac{H_3}{I_3^2}$$

$$\text{i.e. } \frac{H}{I^2} = \text{a constant}$$

$$\text{i.e. } H \propto I^2$$

i.e. Hence, law of current is verified.

(ii) Law of resistance

The same amount of current I is passed for the same time t through different coils of resistances R_1, R_2, R_3 etc. The corresponding quantities of heat gained H_1, H_2, H_3 etc. are calculated. It is found that,

$$\frac{H_1}{R_1} = \frac{H_2}{R_2} = \frac{H_3}{R_3}$$

$$\frac{H}{R} = \text{constant}$$

i.e. $H \propto R$. Hence, law of resistance is verified.

(iii) Law of time

The same amount of current I is passed through the same resistance R for different intervals of time t_1, t_2, t_3 etc. The corresponding quantities of heat gained H_1, H_2, H_3 etc. are calculated. It is found that

$$\frac{H_1}{t_1} = \frac{H_2}{t_2} = \frac{H_3}{t_3}$$

$$\frac{H}{t} = \text{constant}$$

i.e. $H \propto t$. Hence, law of time is verified.

3.1.2 Some applications of Joule heating

(i) Electric heating device

Electric iron, electric heater, electric toaster are some of the appliances that work on the principle of heating effect of current. In these appliances, Nichrome which is an alloy of nickel and chromium is used as the heating element for the following reasons.

- (1) It has high specific resistance
- (2) It has high melting point
- (3) It is not easily oxidized

(ii) Fuse wire

Fuse wire is an alloy of lead 37% and tin 63%. It is connected in series in an electric circuit. It has high resistance and low melting point. When large current flows through a circuit due to short circuiting, the fuse wire melts due to heating and hence the circuit becomes open. Therefore, the electric appliances are saved from damage.

(iii) Electric bulb

Since the resistance of the filament in the bulb is high, the quantity of heat produced is also high. Therefore, the filament is heated to incandescence and emits light. Tungsten with a high melting point (3380°C) is used as the filament. The filament is usually enclosed in a glass bulb containing some inert gas at low pressure.

Electric arc and electric welding also work on the principle of heating effect of current.

In some cases such as transformers and dynamos, Joule heating effect is undesirable. These devices are designed in such a way as to reduce the loss of energy due to heating.

3.1.3 Seebeck effect

In 1821, German Physicist Thomas Johann Seebeck discovered that in a circuit consisting of two dissimilar metals like iron and copper, an emf is developed when the junctions are maintained at different temperatures.

Two dissimilar metals connected to form two junctions is called thermocouple. The emf developed in the circuit is thermo electric emf. The current through the circuit is called thermoelectric current. This

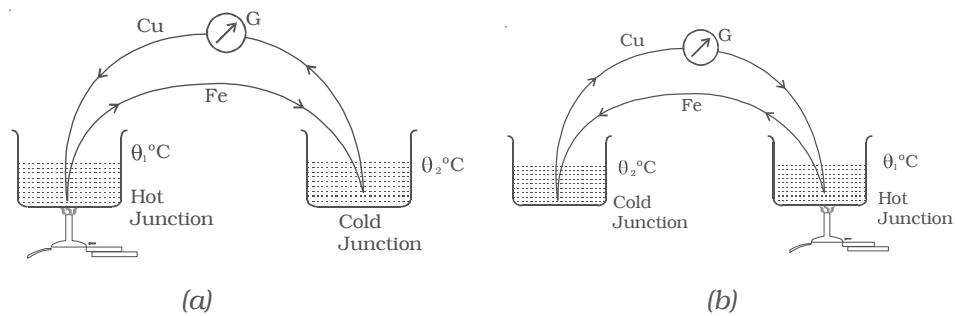


Fig 3.2 Seebeck effect

effect is called thermoelectric effect or Seebeck effect. If the hot and cold junctions are interchanged, the direction of current also reverses. Hence Seebeck effect is reversible. In a Cu-Fe thermocouple (Fig 3.2a), the direction of the current is from copper to iron at the hot junction (Fig 3.2b).

The magnitude and sign of thermo emf depends on the materials of the two conductors and the temperatures of the hot and cold junctions. Seebeck after studying the thermoelectric properties of different pairs of metals, arranged them in a series called thermoelectric series. The direction of the current at the hot junction is from the metal occurring earlier in the series to the one occurring later in the series. The magnitude of thermoemf is larger for metals appearing farther apart in the series. The thermo-electric series of metals is :

Bi, Ni, Pd, Pt, Cu, Mn, Hg, Pb, Sn, Au, Ag, Zn, Cd, Fe, Sb.

The position of the metal in the series depends upon the temperature. The thermoemf of any thermocouple has the temperature dependence given by the relation,

$$V = \alpha \theta + \frac{1}{2} \beta \theta^2,$$

where θ is the temperature difference between the junctions and α and β are constants depending on the nature of the materials.

3.1.4 Neutral and Inversion temperature

The graph showing the variation of thermoemf with temperature of the hot junction, taking the temperature of the cold junction (θ_c) as origin is shown in Fig 3.3. For small difference in temperature between the junctions, the graph is a straight line. For large difference in temperature, the graph is a parabola.

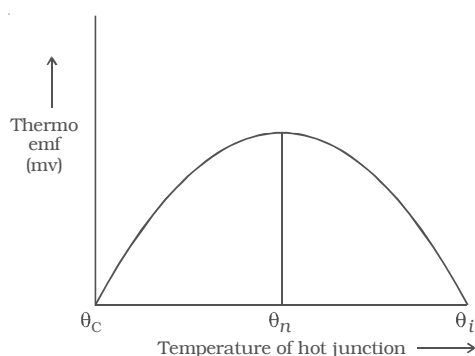


Fig 3.3 Graph showing the variation of thermo emf with temperature

Keeping the temperature of the cold junction constant, the temperature of the hot junction is gradually increased. The thermo emf rises to a maximum at a

temperature (θ_n) called neutral temperature and then gradually decreases and eventually becomes zero at a particular temperature (θ_i) called temperature of inversion. Beyond the temperature of inversion, the thermoemf changes sign and then increases.

For a given thermocouple, the neutral temperature is a constant, but the temperature of inversion depends upon the temperature of cold junction. These temperatures are related by the expression

$$\frac{\theta_c + \theta_i}{2} = \theta_n$$

3.1.5 Peltier effect

In 1834, a French scientist Peltier discovered that when electric current is passed through a circuit consisting of two dissimilar metals, heat is evolved at one junction and absorbed at the other junction. This is called Peltier effect. Peltier effect is the converse of Seebeck effect.

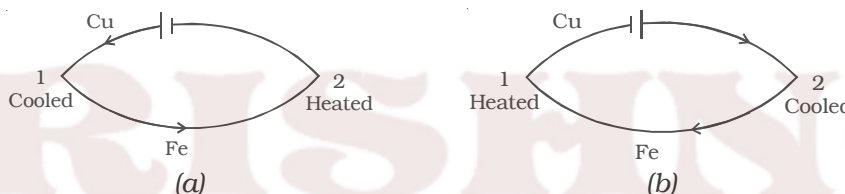


Fig 3.4 Peltier effect

In a Cu-Fe thermocouple, at the junction 1 (Fig 3.4a) where the current flows from Cu to Fe, heat is absorbed (so, it gets cooled) and at the junction 2 where the current flows from Fe to Cu heat is liberated (so, it gets heated). When the direction of the current is reversed (Fig 3.4b) junction 1 gets heated and the junction 2 gets cooled. Hence Peltier effect is reversible.

Peltier Co-efficient (π)

The amount of heat energy absorbed or evolved at one of the junctions of a thermocouple when one ampere current flows for one second (one coulomb) is called Peltier coefficient. It is denoted by π . Its unit is volt. If H is the quantity of heat absorbed or evolved at one junction then $H = \pi It$

The Peltier coefficient at a junction is the Peltier emf at that junction. The Peltier coefficient depends on the pair of metals in contact and the temperature of the junction.

3.1.6 Thomson effect

Thomson suggested that when a current flows through unequally heated conductors, heat energy is absorbed or evolved throughout the body of the metal.

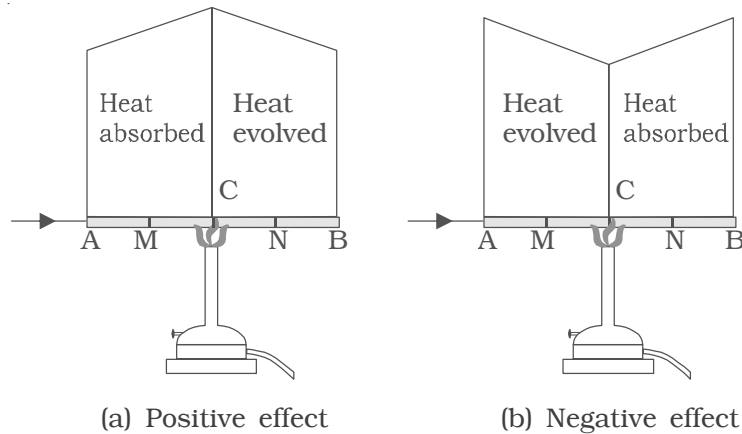


Fig. 3.5 Thomson effect

Consider a copper bar AB heated in the middle at the point C and current flowing as shown in Fig. 3.5a. When no current is flowing, the point M and N equidistant from C are at the same temperature. When current is passed from A to B, N shows higher temperature compared to M. Similarly, B will show higher temperature as compared to A. It means from A to C heat is absorbed and from C to B heat is evolved. This is known as positive Thomson effect. Similar effect is observed in the case of Sb, Ag, Zn, Cd, etc. When the current is passed from B to A, M will show higher temperature as compared to N.

In the case of Iron (fig. 3.5b), when it is heated at the point C and current is flowing from A to B, M shows higher temperature as compared to N. It means from A to C, heat is evolved and from C to B heat is absorbed. This is negative Thomson effect. Similar effect is observed in the case of Pt, Bi, Co, Ni, Hg, etc.

If we take a bar of lead and heat it at the middle point C, the point M and N equidistant from C show the same temperature when current is flowing from A to B or from B to A. Therefore, in the case of lead, Thomson effect is nil. Due to this reason, lead is used as one of the metals to form a thermo couple with other metals for the purpose of drawing thermo electric diagrams.

Thomson coefficient (σ)

The amount of heat energy absorbed or evolved when one ampere current flows for one second (one coulomb) in a metal between two points which differ in temperature by 1°C is called Thomson coefficient. It is denoted by σ . Its unit is volt per $^{\circ}\text{C}$.

3.1.7 Thermopile

Thermopile is a device used to detect thermal radiation. It works on the principle of Seebeck effect.

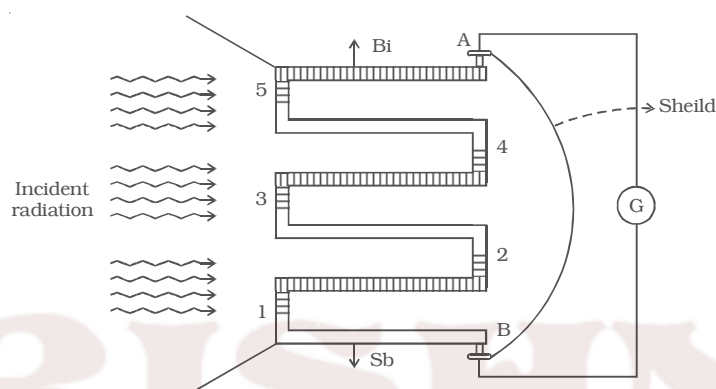


Fig 3.6 Thermopile

Since a single thermocouple gives a very small emf, a large number of thermocouples are connected in series. The ends are connected to a galvanometer G (Fig. 3.6). One set of junctions (1,3,5) is blackened to absorb completely the thermal radiation falling on it. The other set of junctions (2,4) called cold junction is shielded from the radiation.

When thermal radiation falls on one set of junctions (1, 3, 5) a difference in temperature between the junctions is created and a large thermo emf is produced. The deflection in the galvanometer is proportional to the intensity of radiation.

3.2 Magnetic effect of current

In 1820, Danish Physicist, Hans Christian Oersted observed that current through a wire caused a deflection in a nearby magnetic needle. This indicates that magnetic field is associated with a current carrying conductor.

3.2.1 Magnetic field around a straight conductor carrying current

A smooth cardboard with iron filings spread over it, is fixed in a horizontal plane with the help of a clamp. A straight wire passes through a hole made at the centre of the cardboard (Fig 3.7).

A current is passed through the wire by connecting its ends to a battery. When the cardboard is gently tapped, it is found that the iron filings arrange themselves along concentric circles. This clearly shows that magnetic field is developed around a current carrying conductor.

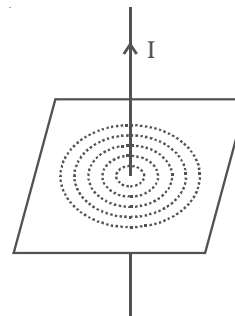


Fig 3.7 Magnetic field around a straight conductor carrying current

To find the direction of the magnetic field, let us imagine, a straight wire passes through the plane of the paper and perpendicular to it. When a compass needle is placed, it comes to rest in such a way that its axis is always tangential to a circular field around the conductor. When the current is inwards (Fig 3.8a) the direction of the magnetic field around the conductor looks clockwise.

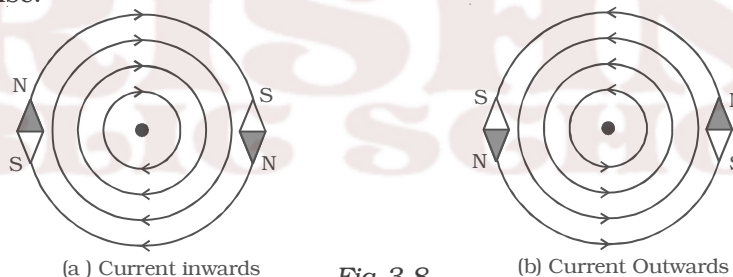


Fig 3.8

When the direction of the current is reversed, that it is outwards, (Fig 3.8b) the direction of the magnetic pole of the compass needle also changes showing the reversal of the direction of the magnetic field. Now, it is anticlockwise around the conductor. This proves that the direction of the magnetic field also depends on the direction of the current in the conductor. This is given by Maxwell's rule.

Maxwells's right hand cork screw rule

If a right handed cork screw is rotated to advance along the direction of the current through a conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.

3.2.2 Magnetic field due to a circular loop carrying current

A cardboard is fixed in a horizontal plane. A circular loop of wire passes through two holes in the cardboard as shown in Fig 3.9. Iron filings are sprinkled over the cardboard. Current is passed through the loop and the cardboard is gently

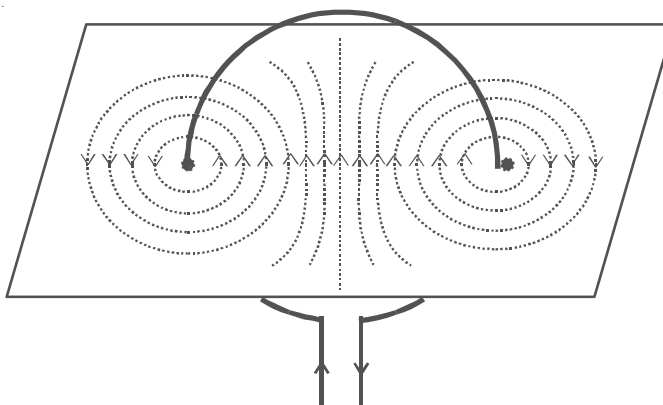


Fig 3.9 Magnetic field due to a circular loop carrying current

tapped. It is observed that the iron filings arrange themselves along the resultant magnetic field. The magnetic lines of force are almost circular around the wire where it passes through the cardboard. At the centre of the loop, the line of force is almost straight and perpendicular to the plane of the circular loop.

3.3 Biot - Savart Law

Biot and Savart conducted many experiments to determine the factors on which the magnetic field due to current in a conductor depends.

The results of the experiments are summarized as Biot-Savart law.

Let us consider a conductor XY carrying a current I (Fig 3.10). $AB = dl$ is a small element of the conductor. P is a point at a distance r from the mid point O of AB. According to Biot and Savart, the magnetic induction dB at P due to the element of length dl is

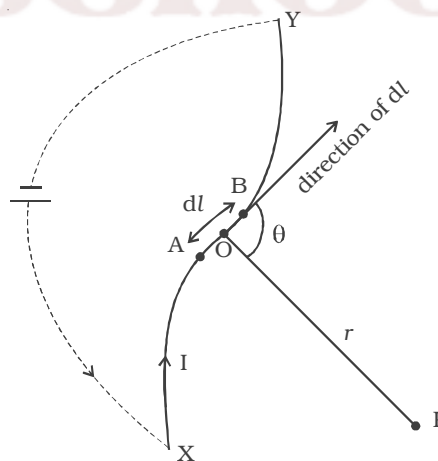


Fig 3.10 Biot - Savart Law

- (i) directly proportional to the current (I)
- (ii) directly proportional to the length of the element (dl)
- (iii) directly proportional to the sine of the angle between dl and the line joining element dl and the point P (sin θ)
- (iv) inversely proportional to the square of the distance of the point from the element ($\frac{1}{r^2}$)

$$\therefore dB \propto \frac{I dl \sin \theta}{r^2}$$

$$dB = K \frac{I dl \sin \theta}{r^2}, \text{ K is the constant of proportionality}$$

The constant $K = \frac{\mu}{4\pi}$ where μ is the permeability of the medium.

$$dB = \frac{\mu}{4\pi} \frac{I dl \sin \theta}{r^2}$$

$\mu = \mu_r \mu_0$ where μ_r is the relative permeability of the medium and μ_0 is the permeability of free space. $\mu_0 = 4\pi \times 10^{-7}$ henry/metre. For air $\mu_r = 1$.

$$\text{So, in air medium } dB = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot dl \sin \theta}{r^2}$$

$$\text{In vector form, } \vec{dB} = \frac{\mu_0}{4\pi} \frac{\vec{Idl} \times \vec{r}}{r^3} \quad \text{or} \quad \vec{dB} = \frac{\mu_0}{4\pi} \frac{\vec{Idl} \times \hat{r}}{r^2}$$

The direction of dB is perpendicular to the plane containing current element Idl and r (i.e plane of the paper) and acts inwards. The unit of magnetic induction is tesla (or) weber m⁻².

3.3.1 Magnetic induction due to infinitely long straight conductor carrying current

XY is an infinitely long straight conductor carrying a current I (Fig 3.11). P is a point at a distance a from the conductor. AB is a small element of length dl. θ is the angle between the current element I dl and the line joining the element dl and the point P. According to Biot-Savart law, the magnetic induction at the point P due to the current element Idl is

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \cdot \sin \theta}{r^2} \quad \dots(1)$$

AC is drawn perpendicular to BP from A.

$$\angle OPA = \phi, \quad \angle APB = d\phi$$

$$\text{In } \triangle ABC, \sin \theta = \frac{AC}{AB} = \frac{AC}{dl}$$

$$\therefore AC = dl \sin \theta \quad \dots(2)$$

$$\text{From } \triangle APC, AC = rd\phi \quad \dots(3)$$

$$\text{From equations (2) and (3), } rd\phi = dl \sin \theta \quad \dots(4)$$

substituting equation (4) in equation (1)

$$dB = \frac{\mu_0}{4\pi} \frac{I rd\phi}{r^2} = \frac{\mu_0}{4\pi} \frac{I d\phi}{r} \quad \dots(5)$$

$$\text{In } \triangle OPA, \cos \phi = \frac{a}{r}$$

$$\therefore r = \frac{a}{\cos \phi} \quad \dots(6)$$

substituting equation (6) in equation (5)

$$dB = \frac{\mu_0}{4\pi} \frac{I}{a} \cos \phi d\phi$$

The total magnetic induction at P due to the conductor XY is

$$B = \int_{-\phi_1}^{\phi_2} dB = \int_{-\phi_1}^{\phi_2} \frac{\mu_0 I}{4\pi a} \cos \phi d\phi$$

$$B = \frac{\mu_0 I}{4\pi a} [\sin \phi_1 + \sin \phi_2]$$

For infinitely long conductor, $\phi_1 = \phi_2 = 90^\circ$

$$\therefore B = \frac{\mu_0 I}{2\pi a}$$

If the conductor is placed in a medium of permeability μ ,

$$B = \frac{\mu I}{2\pi a}$$

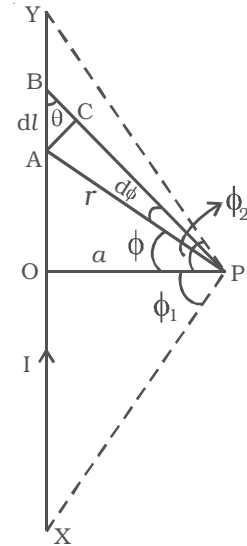


Fig 3.11 Straight conductor

3.3.2 Magnetic induction along the axis of a circular coil carrying current

Let us consider a circular coil of radius 'a' with a current I as shown in Fig 3.12. P is a point along the axis of the coil at a distance x from the centre O of the coil.

AB is an infinitesimally small element of length dl . C is the mid point of AB and $CP = r$

According to Biot - Savart law, the magnetic induction at P due to the element dl is

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin \theta}{r^2}, \text{ where } \theta \text{ is the angle between } Idl \text{ and } r$$

Here, $\theta = 90^\circ$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

The direction of dB is perpendicular to the current element Idl and CP . It is therefore along PR perpendicular to CP .

Considering the diametrically opposite element $A'B'$, the magnitude of dB at P due to this element is the same as that for AB but its direction is along PM . Let the angle between the axis of the coil and the line joining the element (dl) and the point (P) be α .

dB is resolved into two components :- $dB \sin \alpha$ along OP and $dB \cos \alpha$ perpendicular to OP . $dB \cos \alpha$ components due to two opposite elements cancel each other whereas $dB \sin \alpha$ components get added up. So, the total magnetic induction at P due to the entire coil is

$$\begin{aligned} B &= \int dB \sin \alpha = \int \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \frac{a}{r} = \frac{\mu_0}{4\pi} \frac{Ia}{r^3} \int dl \\ &= \frac{\mu_0 Ia}{4\pi r^3} 2\pi a \\ &= \frac{\mu_0 Ia^2}{2(a^2+x^2)^{\frac{3}{2}}} \quad (\because r^2 = a^2 + x^2) \end{aligned}$$

If the coil contains n turns, the magnetic induction is

$$B = \frac{\mu_0 nIa^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

At the centre of the coil, $x = 0$

$$B = \frac{\mu_0 nI}{2a}$$

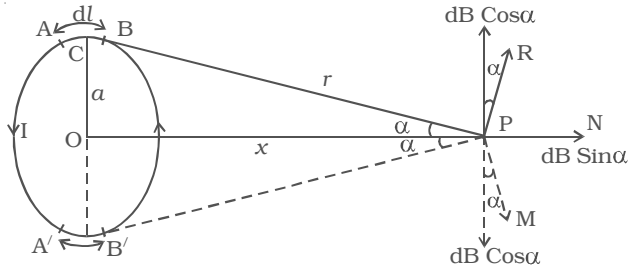


Fig. 3.12 Circular coil

3.3.3 Tangent galvanometer

Tangent galvanometer is a device used for measuring current. It works on the principle of tangent law. A magnetic needle suspended at a point where there are two crossed fields at right angles to each other will come to rest in the direction of the resultant of the two fields.

Construction

It consists of a circular coil of wire wound over a non magnetic frame of brass or wood. The vertical frame is mounted on a horizontal circular turn table provided with three levelling screws. The vertical frame can be rotated about its vertical diameter. There is a small upright projection at the centre of the turn table on which a compass box is supported.

The compass box consists of a small pivoted magnet to which a thin long aluminium pointer is fixed at right angles. The aluminium pointer can move over a circular scale graduated in degrees. The scale consists of four quadrants. The compass box is supported such that the centre of the pivoted magnetic needle coincides with the centre of the coil. Since the magnetic field at the centre of the coil is uniform over a very small area, a small magnetic needle is used so that it remains in an uniform field even in deflected position. Usually the coil consists of three sections of 2, 5 and 50 turns, which are of different thickness, used for measuring currents of different strength.

Theory

When the plane of the coil is placed parallel to the horizontal component of Earth's magnetic induction (B_H) and a current is passed



Fig 3.13 Tangent galvanometer
(This diagram need not be drawn in the examination)

through the coil, there will be two magnetic fields acting perpendicular to each other : (1) the magnetic induction (B) due to the current in the coil acting normal to the plane of the coil and (2) the horizontal component of Earth's magnetic induction (B_h) (Fig 3.14).

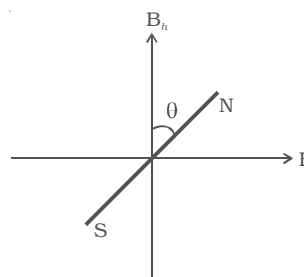


Fig 3.14 Tangent law

Due to these two crossed fields, the pivoted magnetic needle is deflected through an angle θ . According to tangent Law,

$$B = B_h \tan \theta \quad \dots(1)$$

If a current I passes through the coil of n turns and of radius a , the magnetic induction at the centre of the coil is

$$B = \frac{\mu_0 n I}{2a} \quad \dots(2)$$

Substituting equation (2) in equation (1)

$$\frac{\mu_0 n I}{2a} = B_h \tan \theta$$

$$\therefore I = \frac{2aB_h}{\mu_0 n} \tan \theta$$

$$I = K \tan \theta \quad \dots(3)$$

where $K = \frac{2aB_h}{\mu_0 n}$ is called the reduction factor of the tangent galvanometer. It is a constant at a place. Using this equation, current in the circuit can be determined.

Since the tangent galvanometer is most sensitive at a deflection of 45° , the deflection has to be adjusted to be between 30° and 60° .

3.4 Ampere's Circuital Law

Biot - Savart law expressed in an alternative way is called Ampere's circuital law.

The magnetic induction due to an infinitely long straight current carrying conductor is

$$B = \frac{\mu_0 I}{2\pi a}$$

$$B (2\pi a) = \mu_0 I$$

$B (2\pi a)$ is the product of the magnetic field and the circumference of the circle of radius ' a ' on which the magnetic field is constant. If L

is the perimeter of the closed curve and I_0 is the net current enclosed by the closed curve, then the above equation may be expressed as,

$$BL = \mu_0 I_0 \quad \dots(1)$$

In a more generalized way, Ampere's circuital law is written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_0 \quad \dots(2)$$

The line integral does not depend on the shape of the path or the position of the wire within the magnetic field. If the current in the wire is in the opposite direction, the integral would have the opposite sign. If the closed path does not encircle the wire (if a wire lies outside the path), the line integral of the field of that wire is zero. Although derived for the case of a number of long straight parallel conductors, the law is true for conductors and paths of any shape. Ampere's circuital law is hence defined using equation (1) as follows :

The line integral $\oint \vec{B} \cdot d\vec{l}$ for a closed curve is equal to μ_0 times the net current I_0 through the area bounded by the curve.

3.4.1 Solenoid

A long closely wound helical coil is called a solenoid. Fig 3.15 shows a section of stretched out solenoid. The magnetic field due to the solenoid is the vector sum of the magnetic fields due to current through individual turns of the solenoid. The magnetic fields associated with each single turn are almost concentric circles and hence tend to cancel between the turns. At the interior mid point, the field is strong and along the axis of the solenoid (i.e) the field is parallel to the axis. For a point such as P, the field due to the upper part of the solenoid turns tend to cancel the field due to the lower part of the solenoid turns, acting in opposite directions. Hence the field outside the solenoid is nearly zero. The direction of the magnetic field due to circular closed loops (solenoid) is given by right hand palm-rule.

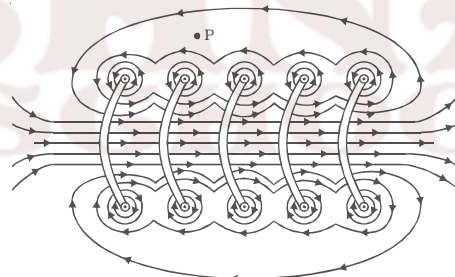


Fig 3.15 Magnetic field due to a current carrying solenoid.

Right hand palm rule

The coil is held in the right hand so that the fingers point in the direction of the current in the windings. The extended thumb, points in the direction of the magnetic field.



Fig 3.16 Right hand palm rule

3.4.2 Magnetic induction due to a long solenoid carrying current.

Let us consider an infinitely long solenoid having n turns per unit length carrying a current of I . For such an ideal solenoid (whose length is very large compared to its radius), the magnetic field at points outside the solenoid is zero.

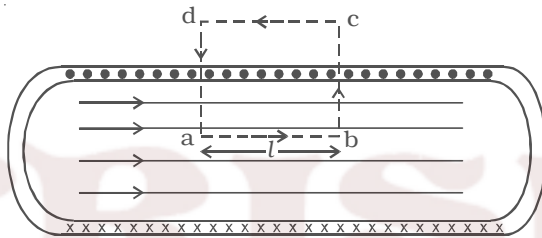


Fig 3.17 Magnetic field due to a long solenoid.

A long solenoid appears like a long cylindrical metal sheet (Fig 3.17). The upper view of dots is like a uniform current sheet coming out of the plane of the paper. The lower row of crosses is like a uniform current sheet going into the plane of the paper.

To find the magnetic induction (B) at a point inside the solenoid, let us consider a rectangular Amperean loop $abcd$. The line integral $\oint \vec{B} \cdot d\vec{l}$ for the loop $abcd$ is the sum of four integrals.

$$\therefore \oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

If l is the length of the loop, the first integral on the right side is Bl . The second and fourth integrals are equal to zero because \vec{B} is at right angles for every element $d\vec{l}$ along the path. The third integral is zero since the magnetic field at points outside the solenoid is zero.

$$\therefore \oint \vec{B} \cdot d\vec{l} = Bl \quad \dots(1)$$

Since the path of integration includes nl turns, the net current enclosed by the closed loop is

$$I_o = Inl \quad \dots(2)$$

Ampere's circuital law for a closed loop is

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_o \quad \dots(3)$$

Substituting equations (1) and (2) in equation (3)

$$Bl = \mu_o Inl$$

$$\therefore B = \mu_o nI \quad \dots(4)$$

The solenoid is commonly used to obtain uniform magnetic field. By inserting a soft iron core inside the solenoid, a large magnetic field is produced

$$B = \mu nI = \mu_o \mu_r nI \quad \dots(5)$$

when a current carrying solenoid is freely suspended, it comes to rest like a suspended bar magnet pointing along north-south. The magnetic polarity of the current carrying solenoid is given by End rule.

End rule

When looked from one end, if the current through the solenoid is along clockwise direction Fig 3.18a, the nearer end corresponds to south pole and the other end is north pole.



Fig 3.18 End rule

When looked from one end, if the current through the solenoid is along anti-clock wise direction, the nearer end corresponds to north pole and the other end is south pole (Fig 3.18b)

3.5 Magnetic Lorentz force

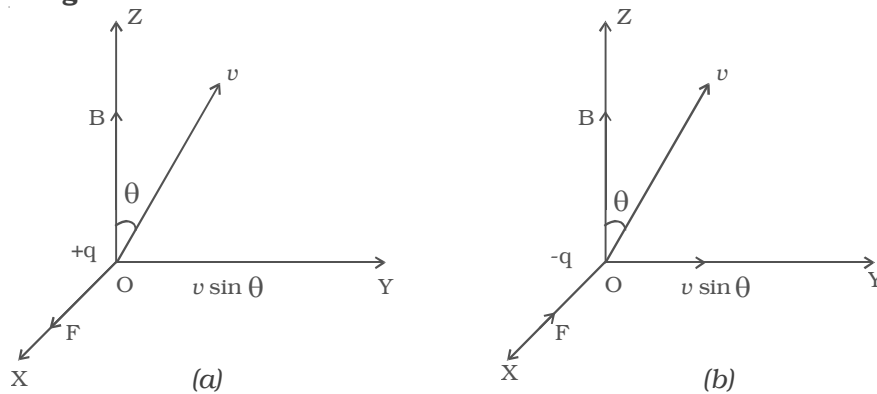


Fig 3.19 Lorentz force

Let us consider a uniform magnetic field of induction B acting along the Z -axis. A particle of charge $+q$ moves with a velocity v in YZ plane making an angle θ with the direction of the field (Fig 3.19a). Under the influence of the field, the particle experiences a force F .

H.A.Lorentz formulated the special features of the force F (Magnetic Lorentz force) as under :

(i) the force F on the charge is zero, if the charge is at rest. (i.e) the moving charges alone are affected by the magnetic field.

(ii) the force is zero, if the direction of motion of the charge is either parallel or anti-parallel to the field and the force is maximum, when the charge moves perpendicular to the field.

(iii) the force is proportional to the magnitude of the charge (q)

(iv) the force is proportional to the magnetic induction (B)

(v) the force is proportional to the speed of the charge (v)

(vi) the direction of the force is oppositely directed for charges of opposite sign (Fig 3.19b).

All these results are combined in a single expression as

$$\vec{F} = q (\vec{v} \times \vec{B})$$

The magnitude of the force is

$$F = Bqv \sin \theta$$

Since the force always acts perpendicular to the direction of motion of the charge, the force does not do any work.

In the presence of an electric field E and magnetic field B , the total force on a moving charged particle is

$$\vec{F} = q [(\vec{v} \times \vec{B}) + \vec{E}]$$

3.5.1 Motion of a charged particle in a uniform magnetic field.

Let us consider a uniform magnetic field of induction B acting along the Z -axis. A particle of charge q and mass m moves in XY plane. At a point P , the velocity of the particle is v . (Fig 3.20)

The magnetic Lorentz force on the particle is $\vec{F} = q (\vec{v} \times \vec{B})$. Hence \vec{F} acts along PO perpendicular to the plane containing \vec{v} and \vec{B} . Since the force acts perpendicular to its velocity, the force does not do any work. So, the magnitude of the velocity remains constant and only

its direction changes. The force F acting towards the point O acts as the centripetal force and makes the particle to move along a circular path. At points Q and R , the particle experiences force along QO and RO respectively.

Since \vec{v} and \vec{B} are at right angles to each other

$$F = Bqv \sin 90^\circ = Bqv$$

This magnetic Lorentz force provides the necessary centripetal force.

$$Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq} \quad \dots(1)$$

It is evident from this equation, that the radius of the circular path is proportional to (i) mass of the particle and (ii) velocity of the particle

From equation (1), $\frac{v}{r} = \frac{Bq}{m}$

$$\omega = \frac{Bq}{m} \quad \dots(2)$$

This equation gives the angular frequency of the particle inside the magnetic field.

Period of rotation of the particle,

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi m}{Bq} \quad \dots(3)$$

From equations (2) and (3), it is evident that the angular frequency and period of rotation of the particle in the magnetic field do not depend upon (i) the velocity of the particle and (ii) radius of the circular path.

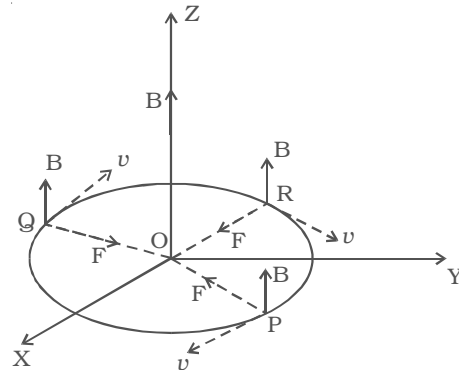


Fig 3.20 Motion of a charged particle

3.5.2 Cyclotron

Cyclotron is a device used to accelerate charged particles to high energies. It was devised by Lawrence.

Principle

Cyclotron works on the principle that a charged particle moving normal to a magnetic field experiences magnetic Lorentz force due to which the particle moves in a circular path.

Construction

It consists of a hollow metal cylinder divided into two sections D_1 and D_2 called Dees, enclosed in an evacuated chamber (Fig 3.21). The Dees are kept separated and a source of ions is placed at the centre in the gap between the Dees. They are placed between the pole pieces of a strong electromagnet. The magnetic field acts perpendicular to the plane of the Dees. The Dees are connected to a high frequency oscillator.

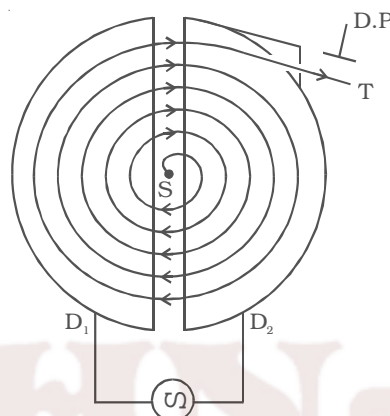


Fig 3.21 Cyclotron

Working

When a positive ion of charge q and mass m is emitted from the source, it is accelerated towards the Dee having a negative potential at that instant of time. Due to the normal magnetic field, the ion experiences magnetic Lorentz force and moves in a circular path. By the time the ion arrives at the gap between the Dees, the polarity of the Dees gets reversed. Hence the particle is once again accelerated and moves into the other Dee with a greater velocity along a circle of greater radius. Thus the particle moves in a spiral path of increasing radius and when it comes near the edge, it is taken out with the help of a deflector plate (D.P). The particle with high energy is now allowed to hit the target T.

When the particle moves along a circle of radius r with a velocity v , the magnetic Lorentz force provides the necessary centripetal force.

$$Bqv = \frac{mv^2}{r}$$

$$\therefore \frac{v}{r} = \frac{Bq}{m} = \text{constant} \quad \dots(1)$$

The time taken to describe a semi-circle

$$t = \frac{\pi r}{v} \quad \dots(2)$$

Substituting equation (1) in (2),

$$t = \frac{\pi m}{Bq} \quad \dots(3)$$

It is clear from equation (3) that the time taken by the ion to describe a semi-circle is independent of

(i) the radius (r) of the path and (ii) the velocity (v) of the particle

Hence, period of rotation $T = 2t$

$$\therefore T = \frac{2\pi m}{Bq} = \text{constant} \quad \dots(4)$$

So, in a uniform magnetic field, the ion traverses all the circles in exactly the same time. The frequency of rotation of the particle,

$$v = \frac{1}{T} = \frac{Bq}{2\pi m} \quad \dots(5)$$

If the high frequency oscillator is adjusted to produce oscillations of frequency as given in equation (5), resonance occurs.

Cyclotron is used to accelerate protons, deuterons and α - particles.

Limitations

(i) Maintaining a uniform magnetic field over a large area of the Dees is difficult.

(ii) At high velocities, relativistic variation of mass of the particle upsets the resonance condition.

(iii) At high frequencies, relativistic variation of mass of the electron is appreciable and hence electrons cannot be accelerated by cyclotron.

3.6 Force on a current carrying conductor placed in a magnetic field.

Let us consider a conductor PQ of length l and area of cross section A . The conductor is placed in a uniform magnetic field of induction B making an angle θ with the field [Fig 3.22]. A current I flows along PQ. Hence, the electrons are drifted along QP with drift velocity v_d . If n is the number of free electrons per unit volume in the conductor, then the current is

$$I = nAv_d e$$

Multiplying both sides by the length l of the conductor,

$$\therefore Il = nAv_d e l$$

Therefore the current element,

$$\vec{Il} = -nAv_d e l \vec{v}_d \quad \dots(1)$$

The negative sign in the equation indicates that the direction of current is opposite to the direction of drift velocity of the electrons.

Since the electrons move under the influence of magnetic field, the magnetic Lorentz force on a moving electron,

$$\vec{f} = -e (\vec{v}_d \times \vec{B}) \quad \dots(2)$$

The negative sign indicates that the charge of the electron is negative.

The number of free electrons in the conductor

$$N = nAl \quad \dots(3)$$

The magnetic Lorentz force on all the moving free electrons

$$\vec{F} = N\vec{f}$$

Substituting equations (2) and (3) in the above equation

$$\begin{aligned} \vec{F} &= nAl \{ -e (\vec{v}_d \times \vec{B}) \} \\ \vec{F} &= -nAl e \vec{v}_d \times \vec{B} \quad \dots(4) \end{aligned}$$

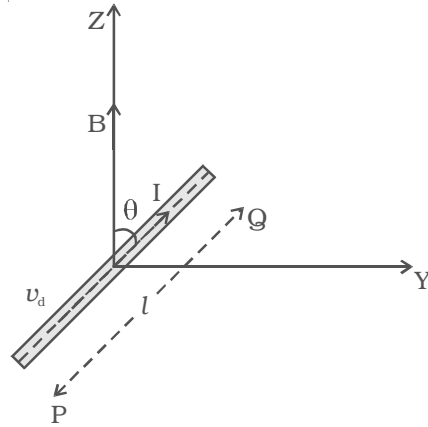


Fig 3.22 Force on a current carrying conductor placed in a magnetic field

Substituting equation (1) in equation (4)

$$\vec{F} = \vec{I}l \times \vec{B}$$

This total force on all the moving free electrons is the force on the current carrying conductor placed in the magnetic field.

Magnitude of the force

The magnitude of the force is $F = BIl \sin \theta$

(i) If the conductor is placed along the direction of the magnetic field, $\theta = 0^\circ$, Therefore force $F = 0$.

(ii) If the conductor is placed perpendicular to the magnetic field, $\theta = 90^\circ$, $F = BIl$. Therefore the conductor experiences maximum force.

Direction of force

The direction of the force on a current carrying conductor placed in a magnetic field is given by Fleming's Left Hand Rule.

The forefinger, the middle finger and the thumb of the left hand are stretched in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, the middle finger points in the direction of the current, then the thumb points in the direction of the force on the conductor.

3.6.1 Force between two long parallel current-carrying conductors

AB and CD are two straight very long parallel conductors placed in air at a distance a . They carry currents I_1 and I_2 respectively. (Fig 3.23). The magnetic induction due to current I_1 in AB at a distance a is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \quad \dots(1)$$

This magnetic field acts perpendicular to the plane of the paper and inwards. The conductor CD with current I_2 is situated in this magnetic field. Hence, force on a segment of length l of CD due to magnetic field B_1 is

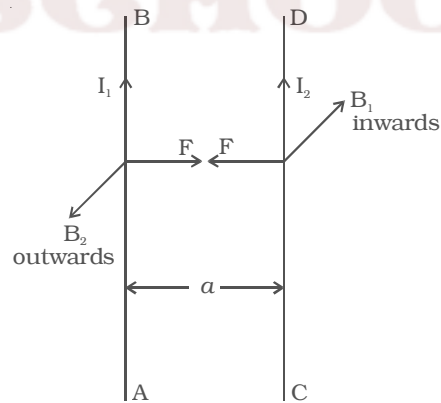


Fig. 3.23 Force between two long parallel current-carrying conductors

$$F = B_1 I_2 l$$

substituting equation (1)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \quad \dots(2)$$

By Fleming's Left Hand Rule, F acts towards left. Similarly, the magnetic induction due to current I_2 flowing in CD at a distance a is

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \quad \dots(3)$$

This magnetic field acts perpendicular to the plane of the paper and outwards. The conductor AB with current I_1 , is situated in this field. Hence force on a segment of length l of AB due to magnetic field B_2 is

$$F = B_2 I_1 l$$

substituting equation (3)

$$\therefore F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \quad \dots(4)$$

By Fleming's left hand rule, this force acts towards right. These two forces given in equations (2) and (4) attract each other. Hence, two parallel wires carrying currents in the same direction attract each other and if they carry currents in the opposite direction, repel each other.

Definition of ampere

The force between two parallel wires carrying currents on a segment of length l is

$$F = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

\therefore Force per unit length of the conductor is

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

If $I_1 = I_2 = 1\text{A}$, $a = 1\text{m}$

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{1 \times 1}{1} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ Nm}^{-1}$$

The above conditions lead the following definition of ampere.

Ampere is defined as that constant current which when flowing through two parallel infinitely long straight conductors of negligible cross section and placed in air or vacuum at a distance of one metre apart, experience a force of 2×10^{-7} newton per unit length of the conductor.

3.7 Torque experienced by a current loop in a uniform magnetic field

Let us consider a rectangular loop PQRS of length l and breadth b (Fig 3.24). It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction B . Let θ be the angle between the normal to the plane of the loop and the direction of the magnetic field.

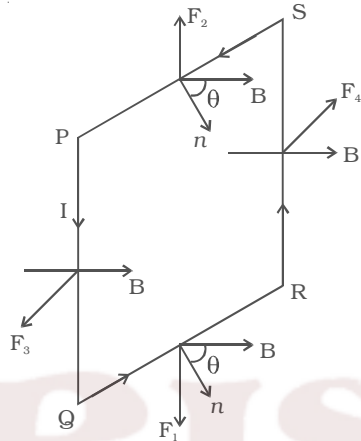


Fig 3.24 Torque on a current loop placed in a magnetic field

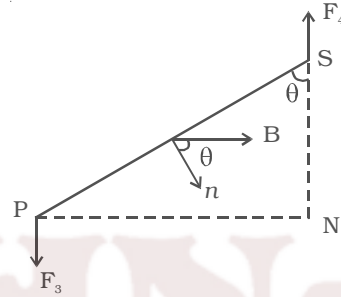


Fig 3.25 Torque

Force on the arm QR, $\vec{F}_1 = I(\overline{QR}) \times \vec{B}$

Since the angle between \overline{QR} and \vec{B} is $(90^\circ - \theta)$,

Magnitude of the force $F_1 = B I b \sin(90^\circ - \theta)$

ie. $F_1 = B I b \cos \theta$

Force on the arm SP, $\vec{F}_2 = I(\overline{SP}) \times \vec{B}$

Since the angle between \overline{SP} and \vec{B} is $(90^\circ + \theta)$,

Magnitude of the force $F_2 = B I b \cos \theta$

The forces F_1 and F_2 are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is zero.

Force on the arm PQ, $\vec{F}_3 = I(\overline{PQ}) \times \vec{B}$

Since the angle between \overline{PQ} and \vec{B} is 90° ,

Magnitude of the force $F_3 = BIl \sin 90^\circ = BIl$

F_3 acts perpendicular to the plane of the paper and outwards.

Force on the arm RS, $\vec{F}_4 = I(\overline{RS}) \times \vec{B}$

Since the angle between $\overline{I(RS)}$ and \vec{B} is 90° ,

Magnitude of the force $F_4 = BIl \sin 90^\circ = BIl$

F_4 acts perpendicular to the plane of the paper and inwards.

The forces F_3 and F_4 are equal in magnitude, opposite in direction and have different lines of action. So, they constitute a couple.

Hence, Torque = $BIl \times PN = BIl \times PS \times \sin \theta$ (Fig 3.25)
= $BIl \times b \sin \theta = BIA \sin \theta$

If the coil contains n turns, $\tau = nBIA \sin \theta$

So, the torque is maximum when the coil is parallel to the magnetic field and zero when the coil is perpendicular to the magnetic field.

3.7.1 Moving coil galvanometer

Moving coil galvanometer is a device used for measuring the current in a circuit.

Principle

Moving coil galvanometer works on the principle that a current carrying coil placed in a magnetic field experiences a torque.

Construction

It consists of a rectangular coil of a large number of turns of thin insulated copper wire wound over a light metallic frame (Fig 3.26). The coil is suspended between the pole pieces of a horse-shoe magnet by a fine phosphor – bronze strip from a movable torsion head. The lower end of the coil is connected to a hair spring (HS) of phosphor bronze having only a few turns. The other end of the spring is connected to a binding screw. A soft iron cylinder is placed symmetrically inside the coil. The hemispherical magnetic poles produce a radial magnetic field in which the plane of the coil is parallel to the magnetic field in all its positions (Fig 3.27).

A small plane mirror (m) attached to the suspension wire is used along with a lamp and scale arrangement to measure the deflection of the coil.

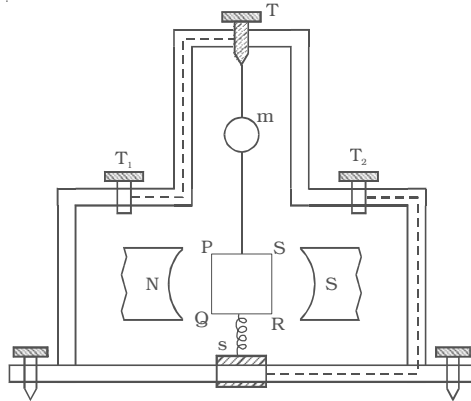


Fig 3.26 Moving coil galvanometer

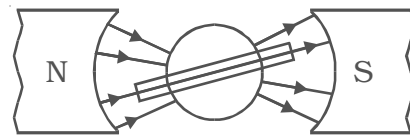


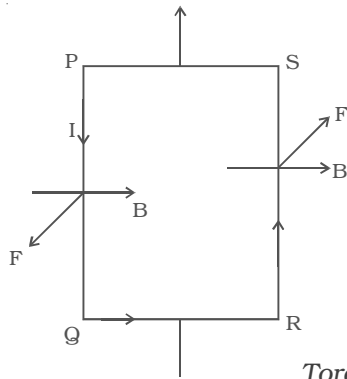
Fig 3.27 Radial magnetic field

Theory

Let PQRS be a single turn of the coil (Fig 3.28). A current I flows through the coil. In a radial magnetic field, the plane of the coil is always parallel to the magnetic field. Hence the sides QR and SP are always parallel to the field. So, they do not experience any force. The sides PQ and RS are always perpendicular to the field.

$PQ = RS = l$, length of the coil and $PS = QR = b$, breadth of the coil

Force on PQ, $F = BI(PQ) = BIl$. According to Fleming's left hand rule, this force is normal to the plane of the coil and acts outwards.



Torque on the coil

Fig 3.28

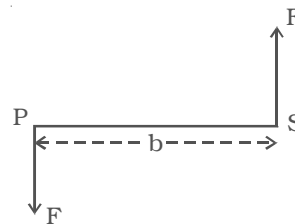


Fig 3.29

Force on RS, $F = BI (RS) = BIl$.

This force is normal to the plane of the coil and acts inwards. These two equal, oppositely directed parallel forces having different lines of action constitute a couple and deflect the coil. If there are n turns in the coil,

$$\begin{aligned}\text{moment of the deflecting couple} &= n BIl \times b \text{ (Fig 3.29)} \\ &= nBIA\end{aligned}$$

When the coil deflects, the suspension wire is twisted. On account of elasticity, a restoring couple is set up in the wire. This couple is proportional to the twist. If θ is the angular twist, then,

$$\text{moment of the restoring couple} = C\theta$$

where C is the restoring couple per unit twist

At equilibrium, deflecting couple = restoring couple

$$nBIA = C\theta$$

$$\therefore I = \frac{C}{nBA} \theta$$

$$I = K \theta \text{ where } K = \frac{C}{nBA} \text{ is the galvanometer constant.}$$

i.e $I \propto \theta$. Since the deflection is directly proportional to the current flowing through the coil, the scale is linear and is calibrated to give directly the value of the current.

3.7.2 Pointer type moving coil galvanometer

The suspended coil galvanometers are very sensitive. They can measure current of the order of 10^{-8} ampere. Hence these galvanometers have to be carefully handled. So, in the laboratory, for experiments like Wheatstone's bridge, where sensitivity is not required, pointer type galvanometers are used. In this type of galvanometer, the coil is pivoted on ball bearings. A lighter aluminium pointer attached to the coil moves over a scale when current is passed. The restoring couple is provided by a spring.

3.7.3 Current sensitivity of a galvanometer.

The current sensitivity of a galvanometer is defined as the deflection produced when unit current passes through the

galvanometer. A galvanometer is said to be sensitive if it produces large deflection for a small current.

$$\begin{aligned} \text{In a galvanometer, } I &= \frac{C}{nBA} \theta \\ \therefore \text{Current sensitivity } \frac{\theta}{I} &= \frac{nBA}{C} \quad \dots(1) \end{aligned}$$

The current sensitivity of a galvanometer can be increased by

- (i) increasing the number of turns
- (ii) increasing the magnetic induction
- (iii) increasing the area of the coil

(iv) decreasing the couple per unit twist of the suspension wire.

This explains why phosphor-bronze wire is used as the suspension wire which has small couple per unit twist.

3.7.4 Voltage sensitivity of a galvanometer

The voltage sensitivity of a galvanometer is defined as the deflection per unit voltage.

$$\therefore \text{Voltage sensitivity } \frac{\theta}{V} = \frac{\theta}{IG} = \frac{nBA}{CG} \quad \dots(2)$$

where G is the galvanometer resistance.

An interesting point to note is that, increasing the current sensitivity does not necessarily, increase the voltage sensitivity. When the number of turns (n) is doubled, current sensitivity is also doubled (equation 1). But increasing the number of turns correspondingly increases the resistance (G). Hence voltage sensitivity remains unchanged.

3.7.5 Conversion of galvanometer into an ammeter

A galvanometer is a device used to detect the flow of current in an electrical circuit. Eventhough the deflection is directly proportional to the current, the galvanometer scale is not marked in ampere. Being a very sensitive instrument, a large current cannot be passed through the galvanometer, as it may damage the coil. However, a galvanometer is converted into an ammeter by connecting a low resistance in parallel with it. As a result, when large current flows in a circuit, only a small fraction of the current passes through the galvanometer and the remaining larger portion of the current passes through the low

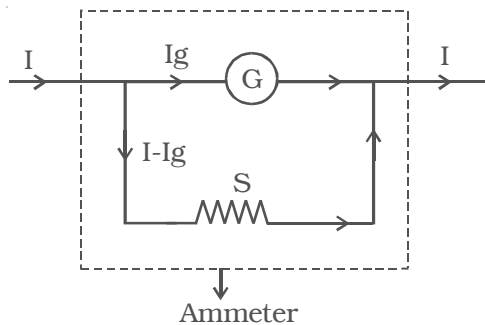


Fig 3.30 Conversion of galvanometer into an ammeter

resistance. The low resistance connected in parallel with the galvanometer is called shunt resistance. The scale is marked in ampere.

The value of shunt resistance depends on the fraction of the total current required to be passed through the galvanometer. Let I_g be the maximum current that can be

passed through the galvanometer. The current I_g will give full scale deflection in the galvanometer.

Galvanometer resistance = G

Shunt resistance = S

Current in the circuit = I

\therefore Current through the shunt resistance = $I_s = (I - I_g)$

Since the galvanometer and shunt resistance are parallel, potential is common.

$\therefore I_g \cdot G = (I - I_g)S$

$$S = G \frac{I_g}{I - I_g} \quad \dots(1)$$

The shunt resistance is very small because I_g is only a fraction of I .

The effective resistance of the ammeter R_a is (G in parallel with S)

$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S}$$

$$\therefore R_a = \frac{GS}{G + S}$$

R_a is very low and this explains why an ammeter should be connected in series. When connected in series, the ammeter does not appreciably change the resistance and current in the circuit. Hence an ideal ammeter is one which has zero resistance.

3.7.6 Conversion of galvanometer into a voltmeter

Voltmeter is an instrument used to measure potential difference between the two ends of a current carrying conductor.

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with it. The scale is calibrated in volt. The value of the resistance connected in series decides the range of the voltmeter.

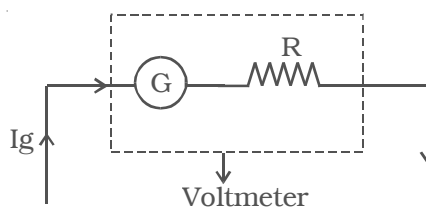


Fig 3.31 Conversion of galvanometer into voltmeter

Galvanometer resistance = G

The current required to produce full scale deflection in the galvanometer = I_g

Range of voltmeter = V

Resistance to be connected in series = R

Since R is connected in series with the galvanometer, the current through the galvanometer,

$$I_g = \frac{V}{R + G}$$
$$\therefore R = \frac{V}{I_g} - G$$

From the equation the resistance to be connected in series with the galvanometer is calculated.

The effective resistance of the voltmeter is

$$R_v = G + R$$

R_v is very large, and hence a voltmeter is connected in parallel in a circuit as it draws the least current from the circuit. In other words, the resistance of the voltmeter should be very large compared to the resistance across which the voltmeter is connected to measure the potential difference. Otherwise, the voltmeter will draw a large current from the circuit and hence the current through the remaining part of the circuit decreases. In such a case the potential difference measured by the voltmeter is very much less than the actual potential difference. The error is eliminated only when the voltmeter has a high resistance. An ideal voltmeter is one which has infinite resistance.

3.8 Current loop as a magnetic dipole

Ampere found that the distribution of magnetic lines of force around a finite current carrying solenoid is similar to that produced by a bar magnet. This is evident from the fact that a compass needle when moved around these two bodies show similar deflections. After noting the close resemblance between these two, Ampere demonstrated that a simple current loop behaves like a bar magnet and put forward that all the magnetic phenomena is due to circulating electric current. This is Ampere's hypothesis.

The magnetic induction at a point along the axis of a circular coil carrying current is

$$B = \frac{\mu_0 n I a^2}{2 (a^2 + x^2)^{\frac{3}{2}}}$$

The direction of this magnetic field is along the axis and is given by right hand rule. For points which are far away from the centre of the coil, $x \gg a$, a^2 is small and it is neglected. Hence for such points,

$$B = \frac{\mu_0 n I a^2}{2 x^3}$$

If we consider a circular loop, $n = 1$, its area $A = \pi a^2$

$$\therefore B = \frac{\mu_0 I A}{2 \pi x^3} \quad \dots(1)$$

The magnetic induction at a point along the axial line of a short bar magnet is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$$

$$B = \frac{\mu_0}{2\pi} \cdot \frac{M}{x^3} \quad \dots(2)$$

Comparing equations (1) and (2), we find that

$$M = IA \quad \dots(3)$$

Hence a current loop is equivalent to a magnetic dipole of moment $M = IA$

The magnetic moment of a current loop is defined as the product of the current and the loop area. Its direction is perpendicular to the plane of the loop.

3.9 The magnetic dipole moment of a revolving electron

According to Neil Bohr's atom model, the negatively charged electron is revolving around a positively charged nucleus in a circular orbit of radius r . The revolving electron in a closed path constitutes an electric current. The motion of the electron in anticlockwise direction produces conventional current in clockwise direction.

Current, $i = \frac{e}{T}$ where T is the period of revolution of the electron.

If v is the orbital velocity of the electron, then

$$T = \frac{2\pi r}{v}$$

$$\therefore i = \frac{ev}{2\pi r}$$

Due to the orbital motion of the electron, there will be orbital magnetic moment μ_l

$\mu_l = iA$, where A is the area of the orbit

$$\mu_l = \frac{ev}{2\pi r} \cdot \pi r^2$$

$$\mu_l = \frac{evr}{2}$$

If m is the mass of the electron

$$\mu_l = \frac{e}{2m} (mvr)$$

mvr is the angular momentum (l) of the electron about the central nucleus.

$$\mu_l = \frac{e}{2m} l \quad \dots (1)$$

$\frac{\mu_l}{l} = \frac{e}{2m}$ is called gyromagnetic ratio and is a constant. Its value is $8.8 \times 10^{10} \text{ C kg}^{-1}$. Bohr hypothesised that the angular momentum has only discrete set of values given by the equation.

$$l = \frac{nh}{2\pi} \quad \dots(2) \text{ where } n \text{ is a natural number}$$

and h is the Planck's constant = $6.626 \times 10^{-34} \text{ Js}$.

substituting equation (2) in equation (1)

$$\mu_l = \frac{e}{2m} \cdot \frac{nh}{2\pi} = \frac{neh}{4\pi m}$$

The minimum value of magnetic moment is

$$(\mu_l)_{min} = \frac{eh}{4\pi m}, n = 1$$

The value of $\frac{eh}{4\pi m}$ is called Bohr magneton

By substituting the values of e , h and m , the value of Bohr magneton is found to be $9.27 \times 10^{-24} \text{ Am}^2$

In addition to the magnetic moment due to its orbital motion, the electron possesses magnetic moment due to its spin. Hence the resultant magnetic moment of an electron is the vector sum of its orbital magnetic moment and its spin magnetic moment.

Solved problems

3.1 In a Joule's calorimeter experiment, the temperature of a given quantity of water increases by 5°C when current passes through the resistance coil for 30 minutes and the potential difference across the coil is 6 volt. Find the rise in temperature of water, if the current passes for 20 minutes and the potential difference across the coil is 9 volt.

Data : $V_1 = 6\text{V}$, $t_1 = 30 \times 60 \text{ s}$, $\theta_2 - \theta_1 = d\theta = 5^\circ\text{C}$
 $V_2 = 9\text{V}$, $t_2 = 20 \times 60 \text{ s}$, $d\theta' = ?$

Solution : $\frac{V_1^2}{R} t_1 = w d\theta$

$$\frac{V_2^2}{R} t_2 = w d\theta'$$

$$\frac{V_2^2}{V_1^2} \frac{t_2}{t_1} = \frac{d\theta'}{d\theta}$$

$$\begin{aligned} \therefore d\theta' &= \frac{V_2^2}{V_1^2} \cdot \frac{t_2}{t_1} \cdot d\theta \\ &= \frac{(9)^2}{(6)^2} \times \frac{20 \times 60}{30 \times 60} \times 5 \end{aligned}$$

$$\therefore d\theta' = 7.5^\circ\text{C}.$$

3.2 Calculate the resistance of the filament of a 100 W, 220 V electric bulb.

Data : $P = 100 \text{ W}$, $V = 220 \text{ V}$, $R = ?$

Solution : $P = \frac{V^2}{R}$

$$\therefore R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484 \Omega$$

3.3 A water heater is marked 1500 W, 220 V. If the voltage drops to 180 V, calculate the power consumed by the heater.

Data : $P_1 = 1500 \text{ W}$, $V_1 = 220 \text{ V}$, $V_2 = 180 \text{ V}$, $P_2 = ?$

Solution : (i) $P_1 = \frac{V_1^2}{R}$

$$\therefore R = \frac{V_1^2}{P_1} = \frac{(220)^2}{1500} = 32.26 \Omega$$

$$\therefore P_2 = \frac{V_2^2}{R} = \frac{(180)^2}{32.26}$$

$$\therefore P_2 = 1004 \text{ Watt}$$

Aliter

$$P_1 = \frac{V_1^2}{R}, P_2 = \frac{V_2^2}{R}$$

$$\therefore \frac{P_1}{P_2} = \frac{V_1^2}{V_2^2}$$

$$\therefore P_2 = P_1 \times \frac{V_2^2}{V_1^2} = 1500 \times \frac{(180)^2}{(220)^2}$$

$$\therefore P_2 = 1004 \text{ Watt.}$$

3.4 A long straight wire carrying current produces a magnetic induction of $4 \times 10^{-6} \text{ T}$ at a point, 15 cm from the wire. Calculate the current through the wire.

Data : $B = 4 \times 10^{-6} \text{ T}$, $a = 15 \times 10^{-2} \text{ m}$, $I = ?$

Solution : $B = \frac{\mu_0 I}{2\pi a}$

$$\therefore I = \frac{B \times 2\pi a}{\mu_0} = \frac{4 \times 10^{-6} \times 2\pi \times 15 \times 10^{-2}}{4\pi \times 10^{-7}}$$

$$\therefore I = 3 \text{ A}$$

- 3.5 A circular coil of 200 turns and of radius 20 cm carries a current of 5A. Calculate the magnetic induction at a point along its axis, at a distance three times the radius of the coil from its centre.

Data : $n = 200$; $a = 20\text{cm} = 2 \times 10^{-1}\text{m}$; $I = 5\text{A}$; $x = 3a$; $B = ?$

Solution : $B = \frac{\mu_0 n I a^2}{2(a^2 + x^2)^{3/2}}$

$$B = \frac{\mu_0 n I a^2}{2(a^2 + 9a^2)^{3/2}} = \frac{\mu_0 n I a^2}{2(10a^2)^{3/2}} = \frac{\mu_0 n I}{a \times 20\sqrt{10}}$$

$$B = \frac{\mu_0 n I \sqrt{10}}{a \times 200} = \frac{4\pi \times 10^{-7} \times 200 \times 5 \times \sqrt{10}}{2 \times 10^{-1} \times 200}$$

$$B = 9.9 \times 10^{-5} \text{ T}$$

- 3.6 A current of 4A flows through 5 turn coil of a tangent galvanometer having a diameter of 30 cm. If the horizontal component of Earth's magnetic induction is $4 \times 10^{-5} \text{ T}$, find the deflection produced in the coil

Data : $n = 5$; $I = 4\text{A}$; $d = 3 \times 10^{-1} \text{ m}$; $B_h = 4 \times 10^{-5} \text{ T}$;

$$a = 1.5 \times 10^{-1} \text{ m}; \theta = ?$$

Solution : $I = \frac{2aB_h \tan \theta}{\mu_0 n}$

$$\therefore \tan \theta = \frac{\mu_0 n I}{2aB_h} = \frac{4\pi \times 10^{-7} \times 5 \times 4}{2 \times 1.5 \times 10^{-1} \times 4 \times 10^{-5}}$$

$$\tan \theta = 2.093$$

$$\therefore \theta = 64^\circ 28'$$

- 3.7 In a tangent galvanometer, a current of 1A produces a deflection of 30° . Find the current required to produce a deflection of 60° .

Data : $I_1 = 1\text{A}$; $\theta_1 = 30^\circ$; $\theta_2 = 60^\circ$; $I_2 = ?$

Solution : $I_1 = k \tan \theta_1$; $I_2 = k \tan \theta_2$

$$\therefore \frac{I_2}{I_1} = \frac{\tan \theta_2}{\tan \theta_1}$$

$$I_2 = I_1 \times \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{1 \times \sqrt{3}}{\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{3} \sqrt{3} = 3\text{A}$$

$$I_2 = 3\text{A}$$

- 3.8 A solenoid is 2m long and 3 cm in diameter. It has 5 layers of windings of 1000 turns each and carries a current of 5A. Find the magnetic induction at its centre along its axis.

Data : $l = 2\text{m}$, $N = 5 \times 1000$ turns, $I = 5\text{A}$, $B = ?$

Solution : $B = \mu_0 nI = \mu_0 \frac{N}{l} \cdot I$

$$B = \frac{4\pi \times 10^{-7} \times 5000 \times 5}{2}$$

$$B = 1.57 \times 10^{-2} \text{ T}$$

- 3.9 An α -particle moves with a speed of $5 \times 10^5 \text{ ms}^{-1}$ at an angle of 30° with respect to a magnetic field of induction 10^{-4} T . Find the force on the particle. [α particle has a +ve charge of $2e$]

Data : $B = 10^{-4} \text{ T}$, $q = 2e$, $v = 5 \times 10^5 \text{ ms}^{-1}$, $\theta = 30^\circ$, $F = ?$

Solution $F = Bqv \sin \theta$
 $= B(2e) v \sin 30^\circ$

$$= 10^{-4} \times 2 \times 1.6 \times 10^{-19} \times 5 \times 10^5 \times \frac{1}{2}$$

$$F = 8 \times 10^{-18} \text{ N}$$

- 3.10 A stream of deuterons is projected with a velocity of 10^4 ms^{-1} in XY - plane. A uniform magnetic field of induction 10^{-3} T acts along the Z-axis. Find the radius of the circular path of the particle. (Mass of deuteron is $3.32 \times 10^{-27} \text{ kg}$ and charge of deuteron is $1.6 \times 10^{-19} \text{ C}$)

Data : $v = 10^4 \text{ ms}^{-1}$, $B = 10^{-3} \text{ T}$, $m = 3.32 \times 10^{-27} \text{ kg}$

$e = 1.6 \times 10^{-19} \text{ C}$, $r = ?$

Solution : $Bev = \frac{mv^2}{r}$

$$\therefore r = \frac{mv}{Be} = \frac{3.32 \times 10^{-27} \times 10^4}{10^{-3} \times 1.6 \times 10^{-19}} = 2.08 \times 10^{-1}$$

$$r = 0.208 \text{ m}$$

3.11 A uniform magnetic field of induction 0.5 T acts perpendicular to the plane of the Dees of a cyclotron. Calculate the frequency of the oscillator to accelerate protons. (mass of proton = 1.67×10^{-27} kg)

Data : $B = 0.5$ T, $m_p = 1.67 \times 10^{-27}$ kg, $q = 1.6 \times 10^{-19}$ C, $v = ?$

Solution: $v = \frac{Bq}{2\pi m_p}$

$$= \frac{0.5 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 1.67 \times 10^{-27}} = 0.763 \times 10^7 = 7.63 \times 10^6 \text{ Hz}$$

$\therefore v = 7.63 \text{ MHz}$

3.12 A conductor of length 50 cm carrying a current of 5A is placed perpendicular to a magnetic field of induction 2×10^{-3} T. Find the force on the conductor.

Data : $l = 50$ cm = 5×10^{-1} m, $I = 5$ A, $B = 2 \times 10^{-3}$ T; $\theta = 90^\circ$, $F = ?$

Solution: $F = BIl \sin\theta$

$$= 2 \times 10^{-3} \times 5 \times 5 \times 10^{-1} \times \sin 90^\circ$$

$\therefore F = 5 \times 10^{-3}$ N

3.13 Two parallel wires each of length 5m are placed at a distance of 10 cm apart in air. They carry equal currents along the same direction and experience a mutually attractive force of 3.6×10^{-4} N. Find the current through the conductors.

Data : $I_1 = I_2 = I$,
 $l = 5$ m, $a = 10^{-1}$ m,
 $F = 3.6 \times 10^{-4}$ N, $I = ?$

Solution: $F = \frac{\mu_0 I_1 I_2 l}{2\pi a}$

$$F = \frac{2 \times 10^{-7} I^2 l}{a}$$

$$\therefore I^2 = \frac{F \cdot a}{2 \times 10^{-7} l} = \frac{3.6 \times 10^{-4} \times 10^{-1}}{2 \times 10^{-7} \times 5} = 36$$

$\therefore I = 6$ A

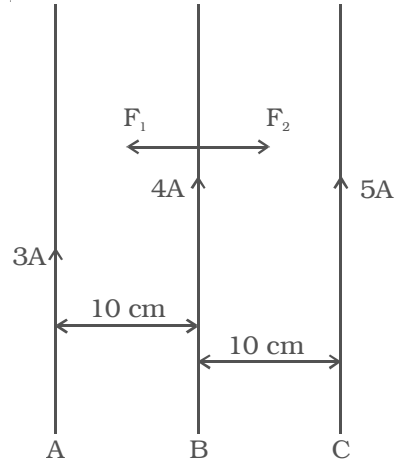
3.14 A, B and C are three parallel conductors each of length 10 m, carrying currents as shown in the figure. Find the magnitude and direction of the resultant force on the conductor B.

Solution : Between the wires A and B, force of attraction exists.

F_1 acts towards left

$$F_1 = \frac{2 \times 10^{-7} I_1 I_2 l}{a} = \frac{2 \times 10^{-7} \times 3 \times 4 \times 10}{10^{-1}}$$

$$F_1 = 24 \times 10^{-5} \text{ N}$$



Between the wires B and C, force of attraction exists

F_2 acts towards right

$$F_2 = \frac{2 \times 10^{-7} I_1 I_2 l}{a} = \frac{2 \times 10^{-7} \times 4 \times 5 \times 10}{10^{-1}}$$

$$F_2 = 40 \times 10^{-5} \text{ N}$$

$$F_2 - F_1 = 16 \times 10^{-5} \text{ N}$$

The wire B is attracted towards C with a net force of $16 \times 10^{-5} \text{ N}$.

3.15 A rectangular coil of area $20 \text{ cm} \times 10 \text{ cm}$ with 100 turns of wire is suspended in a radial magnetic field of induction $5 \times 10^{-3} \text{ T}$. If the galvanometer shows an angular deflection of 15° for a current of 1 mA , find the torsional constant of the suspension wire.

Data : $n = 100$, $A = 20 \text{ cm} \times 10 \text{ cm} = 2 \times 10^{-1} \times 10^{-1} \text{ m}^2$

$B = 5 \times 10^{-3} \text{ T}$, $\theta = 15^\circ$, $I = 1 \text{ mA} = 10^{-3} \text{ A}$, $C = ?$

$$\text{Solution : } \theta = 15^\circ = \frac{\pi}{180} \times 15 = \frac{\pi}{12} \text{ rad}$$

$$nBIA = C\theta$$

$$\therefore C = \frac{nBIA}{\theta} = \frac{10^2 \times 5 \times 10^{-3} \times 10^{-3} \times 2 \times 10^{-1} \times 10^{-1}}{\left(\frac{\pi}{12}\right)}$$

$$C = 3.82 \times 10^{-5} \text{ N m rad}^{-1}$$

3.16 A moving coil galvanometer of resistance 20Ω produces full scale deflection for a current of 50 mA . How you will convert the galvanometer into (i) an ammeter of range 20 A and (ii) a voltmeter of range 120 V .

Data : $G = 20 \Omega$; $I_g = 50 \times 10^{-3} \text{ A}$; $I = 20 \text{ A}$, $S = ?$
 $V = 120 \text{ V}$, $R = ?$

Solution : (i) $S = G \cdot \frac{I_g}{I - I_g} = \frac{20 \times 50 \times 10^{-3}}{20 - 50 \times 10^{-3}} = \frac{1}{20 - 0.05}$

$$S = 0.05 \Omega$$

A shunt of 0.05Ω should be connected in parallel

(ii) $R = \frac{V}{I_g} - G$

$$= \frac{120}{50 \times 10^{-3}} - 20 = 2400 - 20 = 2380 \Omega$$

$$R = 2380 \Omega$$

A resistance of 2380Ω should be connected in series with the galvanometer.

3.17 The deflection in a galvanometer falls from 50 divisions to 10 divisions when 12Ω resistance is connected across the galvanometer. Calculate the galvanometer resistance.

Data : $\theta_1 = 50 \text{ divs}$, $\theta_g = 10 \text{ divs}$, $S = 12 \Omega$ $G = ?$

Solution : $I \propto \theta_1$

$$I_g \propto \theta_g$$

In a parallel circuit potential is common.

$$\therefore G \cdot I_g = S (I - I_g)$$

$$\therefore G = \frac{S (I - I_g)}{I_g} = \frac{12 (50 - 10)}{10}$$

$$\therefore G = 48 \Omega$$

3.18 In a hydrogen atom electron moves in an orbit of radius 0.5 \AA making 10^{16} revolutions per second. Determine the magnetic moment associated with orbital motion of the electron.

Data : $r = 0.5 \text{ \AA} = 0.5 \times 10^{-10} \text{ m}$, $n = 10^{16} \text{ s}^{-1}$

Solution :

Orbital magnetic moment $\mu_l = i.A \dots(1)$

$$i = \frac{e}{T} = e.n \quad \dots(2)$$

$$A = \pi r^2 \quad \dots(3)$$

substituting equation (2), (3) in (1)

$$\begin{aligned} \mu_l &= e.n. \pi r^2 \\ &= 1.6 \times 10^{-19} \times 10^{16} \times 3.14 (0.5 \times 10^{-10})^2 \\ &= 1.256 \times 10^{-23} \\ \therefore \mu_l &= 1.256 \times 10^{-23} \text{ Am}^2 \end{aligned}$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

3.1 Joule's law of heating is

$$(a) H = \frac{I^2}{R} t$$

$$(b) H = V^2 Rt$$

$$(c) H = VI t$$

$$(d) H = IR^2 t$$

3.2 Nichrome wire is used as the heating element because it has

(a) low specific resistance (b) low melting point

(c) high specific resistance (d) high conductivity

3.3 Peltier coefficient at a junction of a thermocouple depends on

(a) the current in the thermocouple

(b) the time for which current flows

(c) the temperature of the junction

(d) the charge that passes through the thermocouple

3.4 In a thermocouple, the temperature of the cold junction is 20°C , the neutral temperature is 270°C . The temperature of inversion is

(a) 520°C

(b) 540°C

(c) 500°C

(d) 510°C

3.5 Which of the following equations represents Biot-savart law?

(a) $dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$ (b) $\vec{dB} = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$
(c) $\vec{dB} = \frac{\mu_0}{4\pi} \frac{Idl \times \vec{r}}{r^2}$ (d) $\vec{dB} = \frac{\mu_0}{4\pi} \frac{Idl \times \vec{r}}{r^3}$

3.6 Magnetic induction due to an infinitely long straight conductor placed in a medium of permeability μ is

(a) $\frac{\mu_0 I}{4\pi a}$ (b) $\frac{\mu_0 I}{2\pi a}$
(c) $\frac{\mu I}{4\pi a}$ (d) $\frac{\mu I}{2\pi a}$

3.7 In a tangent galvanometer, for a constant current, the deflection is 30° . The plane of the coil is rotated through 90° . Now, for the same current, the deflection will be

(a) 30° (b) 60°
(c) 90° (d) 0°

3.8 The period of revolution of a charged particle inside a cyclotron does not depend on

(a) the magnetic induction (b) the charge of the particle
(c) the velocity of the particle (d) the mass of the particle

3.9 The torque on a rectangular coil placed in a uniform magnetic field is large, when

(a) the number of turns is large
(b) the number of turns is less
(c) the plane of the coil is perpendicular to the field
(d) the area of the coil is small

3.10 Phosphor – bronze wire is used for suspension in a moving coil galvanometer, because it has

(a) high conductivity (b) high resistivity
(c) large couple per unit twist (d) small couple per unit twist

3.11 Of the following devices, which has small resistance?

(a) moving coil galvanometer (b) ammeter of range 0 – 1A
(c) ammeter of range 0–10 A (d) voltmeter

- 3.12 A galvanometer of resistance $G \Omega$ is shunted with $S \Omega$. The effective resistance of the combination is R_a . Then, which of the following statements is true?
- G is less than S
 - S is less than R_a but greater than G .
 - R_a is less than both G and S
 - S is less than both G and R_a
- 3.13 An ideal voltmeter has
- zero resistance
 - finite resistance less than G but greater than Zero
 - resistance greater than G but less than infinity
 - infinite resistance
- 3.14 State Joule's law
- 3.15 Explain Joule's calorimeter experiment to verify Joule's laws of heating.
- 3.16 Define Peltier coefficient
- 3.17 Define Thomson coefficient
- 3.18 State Biot - Savart law
- 3.19 Obtain an expression for the magnetic induction at a point due to an infinitely long straight conductor carrying current.
- 3.20 Deduce the relation for the magnetic induction at a point along the axis of a circular coil carrying current.
- 3.21 Explain in detail the principle, construction and theory of a tangent galvanometer.
- 3.22 What is Ampere's circuital law?
- 3.23 Applying Ampere's circuital law, find the magnetic induction due to a straight solenoid.
- 3.24 Define ampere
- 3.25 Deduce an expression for the force on a current carrying conductor placed in a magnetic field.
- 3.26 Explain in detail the principle, construction and the theory of moving coil galvanometer.

- 3.27 Explain how you will convert a galvanometer into (i) an ammeter and (ii) a voltmeter.

Problems

- 3.28 In a thermocouple, the temperature of the cold junction is -20°C and the temperature of inversion is 600°C . If the temperature of the cold junction is 20°C , find the temperature of inversion.
- 3.29 Find the magnetic induction at a point, 10 cm from a long straight wire carrying a current of 10A
- 3.30 A circular coil of radius 20 cm has 100 turns wire and it carries a current of 5A. Find the magnetic induction at a point along its axis at a distance of 20 cm from the centre of the coil.
- 3.31 Three tangent galvanometers have turns ratio of 2:3:5. When connected in series in a circuit, they show deflections of 30° , 45° and 60° respectively. Find the ratio of their radii.
- 3.32 A straight wire of length one metre and of resistance $2\ \Omega$ is connected across a battery of emf 12V. The wire is placed normal to a magnetic field of induction $5 \times 10^{-3}\ \text{T}$. Find the force on the wire.
- 3.33 A circular coil of 50 turns and radius 25 cm carries a current of 6A. It is suspended in a uniform magnetic field of induction $10^{-3}\ \text{T}$. The normal to the plane of the coil makes an angle of 60° with the field. Calculate the torque of the coil.
- 3.34 A uniform magnetic field 0.5 T is applied normal to the plane of the Dees of a Cyclotron. Calculate the period of the alternating potential to be applied to the Dees to accelerate deuterons (mass of deuteron = $3.3 \times 10^{-27}\ \text{kg}$ and its charge = $1.6 \times 10^{-19}\ \text{C}$).
- 3.35 A rectangular coil of 500 turns and of area $6 \times 10^{-4}\ \text{m}^2$ is suspended inside a radial magnetic field of induction $10^{-4}\ \text{T}$ by a suspension wire of torsional constant $5 \times 10^{-10}\ \text{Nm per degree}$. calculate the current required to produce a deflection of 10° .
- 3.36 Two straight infinitely long parallel wires carrying equal currents and placed at a distance of 20 cm apart in air experience a mutually attractive force of $4.9 \times 10^{-5}\ \text{N}$ per unit length of the wire. Calculate the current.
- 3.37 A long solenoid of length 3m has 4000 turns. Find the current through the solenoid if the magnetic field produced at the centre of the solenoid along its axis is $8 \times 10^{-3}\ \text{T}$.

- 3.38 A galvanometer has a resistance of 100Ω . A shunt resistance 1Ω is connected across it. What part of the total current flows through the galvanometer?
- 3.39 A galvanometer has a resistance of 40Ω . It shows full scale deflection for a current of 2 mA . How you will convert the galvanometer into a voltmeter of range 0 to 20V ?
- 3.40 A galvanometer with 50 divisions on the scale requires a current sensitivity of 0.1 mA/division . The resistance of the galvanometer is 40Ω . If a shunt resistance 0.1Ω is connected across it, find the maximum value of the current that can be measured using this ammeter.

Answers

3.1 (c) **3.2** (c) **3.3** (c) **3.4** (a) **3.5** (d)

3.6 (d) **3.7** (d) **3.8** (c) **3.9** (a) **3.10** (d)

3.11 (c) **3.12** (c) **3.13** (d)

3.28 560° C **3.29** $2 \times 10^{-5} \text{ T}$

3.30 $5.55 \times 10^{-4} \text{ T}$ **3.31** $6 : 3\sqrt{3} : 5$

3.32 $3 \times 10^{-2} \text{ N}$ **3.33** $5.1 \times 10^{-2} \text{ Nm}$

3.34 $2.6 \times 10^{-7} \text{ s}$ **3.35** 0.166 mA

3.36 7 A **3.37** 4.77 A

3.38 $1/101$ **3.39** 9960Ω in series

3.40 2 A

4. Electromagnetic Induction and Alternating Current

In the year 1820, Hans Christian Oersted demonstrated that a current carrying conductor is associated with a magnetic field. Thereafter, attempts were made by many to verify the reverse effect of producing an induced emf by the effect of magnetic field.

4.1 Electromagnetic induction

Michael Faraday demonstrated the reverse effect of Oersted experiment. He explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction. The discovery of this phenomenon brought about a revolution in the field of power generation.

4.1.1 Magnetic flux

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing a closed area (A) (Fig 4.1). If θ is the angle between the direction of the field and normal to the area, then

$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA \cos \theta$$

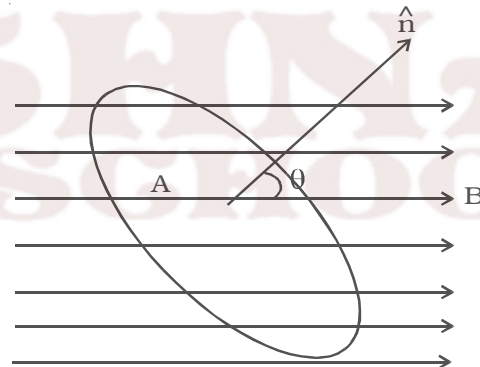


Fig 4.1 Magnetic flux

4.1.2 Induced emf and current – Electromagnetic induction.

Whenever there is a change in the magnetic flux linked with a closed circuit an emf is produced. This emf is known as the induced emf and the current that flows in the closed circuit is called induced current. The phenomenon of producing an induced emf due to the changes in the magnetic flux associated with a closed circuit is known as electromagnetic induction.

Faraday discovered the electromagnetic induction by conducting several experiments.

Fig 4.2 consists of a cylindrical coil C made up of several turns of insulated copper wire connected in series to a sensitive galvanometer G. A strong bar magnet NS with its north pole pointing towards the coil is moved up and down. The following inferences were made by Faraday.

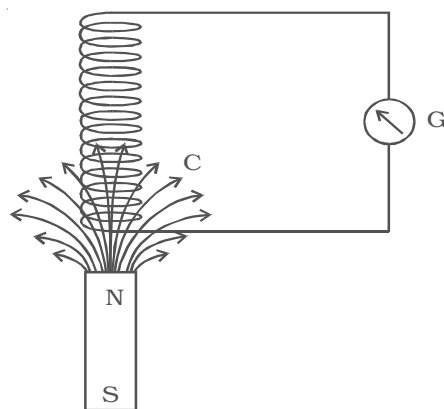


Fig 4.2 Electromagnetic Induction

(i) Whenever there is a relative motion between the coil and the magnet, the galvanometer shows deflection indicating the flow of induced current.

(ii) The deflection is momentary. It lasts so long as there is relative motion between the coil and the magnet.

(iii) The direction of the flow of current changes if the magnet is moved towards and withdrawn from it.

(iv) The deflection is more when the magnet is moved faster, and less when the magnet is moved slowly.

(v) However, on reversing the magnet (i.e) south pole pointing towards the coil, same results are obtained, but current flows in the opposite direction.

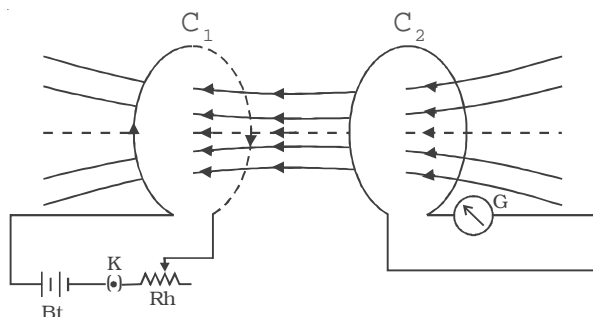


Fig 4.3 Electromagnetic Induction

Faraday demonstrated the electromagnetic induction by another experiment also.

Fig 4.3 shows two coils C_1 and C_2 placed close to each other.

The coil C_1 is connected to a battery Bt

through a key K and a rheostat. Coil C_2 is connected to a sensitive galvanometer G and kept close to C_1 . When the key K is pressed, the galvanometer connected with the coil C_2 shows a

sudden momentary deflection. This indicates that a current is induced in coil C_2 . This is because when the current in C_1 increases from zero to a certain steady value, the magnetic flux linked with the coil C_1 increases. Hence, the magnetic flux linked with the coil C_2 also increases. This causes the deflection in the galvanometer.

On releasing K, the galvanometer shows deflection in the opposite direction. This indicates that a current is again induced in the coil C_2 . This is because when the current in C_1 decreases from maximum to zero value, the magnetic flux linked with the coil C_1 decreases. Hence, the magnetic flux linked with the coil C_2 also decreases. This causes the deflection in the galvanometer in the opposite direction.

4.1.3 Faraday's laws of electromagnetic induction

Based on his studies on the phenomenon of electromagnetic induction, Faraday proposed the following two laws.

First law

Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

Second law

The magnitude of emf induced in a closed circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

Let ϕ_1 be the magnetic flux linked with the coil initially and ϕ_2 be the magnetic flux linked with the coil after a time t . Then

$$\text{Rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t}$$

According to Faraday's second law, the magnitude of induced emf is, $e \propto \frac{\phi_2 - \phi_1}{t}$. If $d\phi$ is the change in magnetic flux in a time dt , then the above equation can be written as $e \propto \frac{d\phi}{dt}$

4.1.4 Lenz's law

The Russian scientist H.F. Lenz in 1835 discovered a simple law giving the direction of the induced current produced in a circuit. Lenz's law states that the induced current produced in a circuit always flows in such a direction that it opposes the change or cause that produces it.

If the coil has N number of turns and ϕ is the magnetic flux linked with each turn of the coil then, the total magnetic flux linked with the coil at any time is $N\phi$

$$\therefore e = - \frac{d}{dt} (N\phi) \quad e = - \frac{Nd\phi}{dt} = - \frac{N(\phi_2 - \phi_1)}{t}$$

Lenz's law - a consequence of conservation of energy

Copper coils are wound on a cylindrical cardboard and the two ends of the coil are connected to a sensitive galvanometer. A magnet is moved towards the coil (Fig 4.4). The upper face of the coil acquires north polarity.

Consequently work has to be done to move the magnet further against the force of repulsion. When we withdraw the magnet away from the coil, its upper face acquires south polarity. Now the workdone is against the force of attraction. When the magnet is moved, the number of magnetic lines of force linking the coil changes, which causes an induced current to flow through the coil. The direction of the induced current, according to

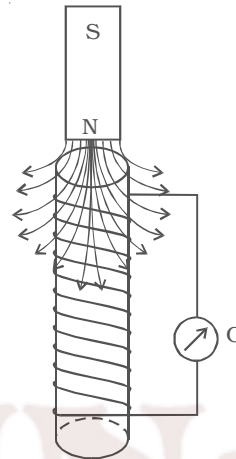


Fig 4.4 Lenz's law

Lenz's law is always to oppose the motion of the magnet. The workdone in moving the magnet is converted into electrical energy. This energy is dissipated as heat energy in the coil. If on the contrary, the direction of the current were to help the motion of the magnet, it would start moving faster increasing the change of magnetic flux linking the coil. This results in the increase of induced current. Hence kinetic energy and electrical energy would be produced without any external work being done, but this is impossible. Therefore, the induced current always flows in such a direction to oppose the cause. Thus it is proved that Lenz's law is the consequence of conservation of energy.

4.1.5 Fleming's right hand rule

The forefinger, the middle finger and the thumb of the right hand are held in the three mutually perpendicular directions. If the forefinger points along the direction of the magnetic field and the thumb is along the direction of motion of the conductor, then the middle finger points in the direction of the induced current. This rule is also called generator rule.

4.2. Self Induction

The property of a coil which enables to produce an opposing induced emf in it when the current in the coil changes is called self induction.

A coil is connected in series with a battery and a key (K) (Fig. 4.5). On pressing the key, the current through the coil increases to a maximum value and correspondingly the magnetic flux linked with the coil also increases. An induced current flows through the coil which according to Lenz's law opposes the further growth of current in the coil.

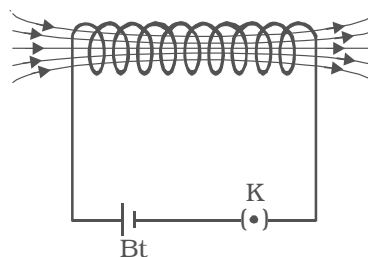


Fig 4.5 Self Induction

On releasing the key, the current through the coil decreases to a zero value and the magnetic flux linked with the coil also decreases. According to Lenz's law, the induced current will oppose the decay of current in the coil.

4.2.1 Coefficient of self induction

When a current I flows through a coil, the magnetic flux (ϕ) linked with the coil is proportional to the current.

$$\phi \propto I \quad \text{or} \quad \phi = LI$$

where L is a constant of proportionality and is called coefficient of self induction or self inductance.

If $I = 1\text{A}$, $\phi = L \times 1$, then $L = \phi$. Therefore, coefficient of self induction of a coil is numerically equal to the magnetic flux linked with a coil when unit current flows through it. According to laws of electromagnetic induction.

$$e = - \frac{d\phi}{dt} = - \frac{d}{dt}(LI) \quad \text{or} \quad e = - L \frac{dI}{dt}$$

$$\text{If } \frac{dI}{dt} = 1 \text{ A s}^{-1}, \text{ then } L = -e$$

The coefficient of self induction of a coil is numerically equal to the opposing emf induced in the coil when the rate of change of current through the coil is unity. The unit of self inductance is henry (H).

One henry is defined as the self-inductance of a coil in which a change in current of one ampere per second produces an opposing emf of one volt.

4.2.2 Self inductance of a long solenoid

Let us consider a solenoid of N turns with length l and area of cross section A . It carries a current I . If B is the magnetic field at any point inside the solenoid, then

Magnetic flux per turn = $B \times$ area of each turn

$$\text{But, } B = \frac{\mu_0 NI}{l}$$

$$\text{Magnetic flux per turn} = \frac{\mu_0 NIA}{l}$$

Hence, the total magnetic flux (ϕ) linked with the solenoid is given by the product of flux through each turn and the total number of turns.

$$\phi = \frac{\mu_0 NIA}{l} \times N$$

$$\text{i.e. } \phi = \frac{\mu_0 N^2 IA}{l} \quad \dots(1)$$

If L is the coefficient of self induction of the solenoid, then

$$\phi = LI \quad \dots(2)$$

From equations (1) and (2)

$$LI = \frac{\mu_0 N^2 IA}{l}$$

$$\therefore L = \frac{\mu_0 N^2 A}{l}$$

If the core is filled with a magnetic material of permeability μ ,

$$\text{then, } L = \frac{\mu N^2 A}{l}$$

4.2.3 Energy associated with an inductor

Whenever current flows through a coil, the self-inductance opposes the growth of the current. Hence, some work has to be done by external agencies in establishing the current. If e is the induced emf then,

$$e = -L \frac{dI}{dt}$$

The small amount of work dw done in a time interval dt is

$$\begin{aligned} dw &= e \cdot I \cdot dt \\ &= -L \frac{dI}{dt} I \cdot dt \end{aligned}$$

The total work done when the current increases from 0 to maximum value (I_0) is

$$w = \int dw = \int_0^{I_0} -L I \, dI$$

This work done is stored as magnetic potential energy in the coil.

\therefore Energy stored in the coil

$$= -L \int_0^{I_0} I \, dI = -\frac{1}{2} L I_0^2$$

Negative sign is consequence of Lenz's Law. Hence, quantitatively,

the energy stored in an inductor is $\frac{1}{2} L I_0^2$

4.2.4 Mutual induction

Whenever there is a change in the magnetic flux linked with a coil, there is also a change of flux linked with the neighbouring coil, producing an induced emf in the second coil. This phenomenon of producing an induced emf in a coil due to the change in current in the other coil is known as mutual induction.

P and S are two coils placed close to each other (Fig. 4.6). P is connected to a battery through a key K. S is connected to

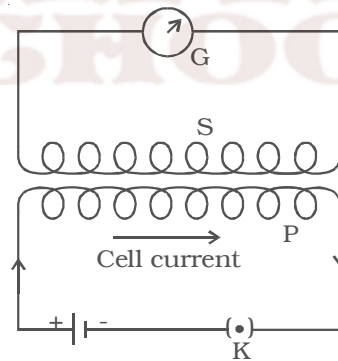


Fig 4.6 Mutual induction

a galvanometer G. On pressing K, current in P starts increasing from zero to a maximum value. As the flow of current increases, the magnetic flux linked with P increases. Therefore, magnetic flux linked with S also increases producing an induced emf in S. Now, the galvanometer shows the deflection. According to Lenz's law the induced current in S would oppose the increase in current in P by flowing in

a direction opposite to the current in P, thus delaying the growth of current to the maximum value. When the key 'K' is released, current starts decreasing from maximum to zero value, consequently magnetic flux linked with P decreases. Therefore magnetic flux linked with S also decreases and hence, an emf is induced in S. According to Lenz's law, the induced current in S flows in such a direction so as to oppose the decrease in current in P thus prolonging the decay of current.

4.2.5 Coefficient of mutual induction

I_P is the current in coil P and ϕ_s is the magnetic flux linked with coil S due to the current in coil P.

$$\therefore \phi_s \propto I_P \quad \text{or} \quad \phi_s = M I_P$$

where M is a constant of proportionality and is called the coefficient of mutual induction or mutual inductance between the two coils.

$$\text{If } I_P = 1\text{A, then, } M = \phi_s$$

Thus, coefficient of mutual induction of two coils is numerically equal to the magnetic flux linked with one coil when unit current flows through the neighbouring coil. If e_s is the induced emf in the coil (S) at any instant of time, then from the laws of electromagnetic induction,

$$e_s = -\frac{d\phi_s}{dt} = -\frac{d}{dt} (M I_P) = -M \frac{dI_P}{dt}$$

$$\therefore M = -\left(\frac{e_s}{\frac{dI_P}{dt}}\right)$$

$$\text{If } \frac{dI_P}{dt} = 1 \text{ A s}^{-1}, \text{ then, } M = -e_s$$

Thus, the coefficient of mutual induction of two coils is numerically equal to the emf induced in one coil when the rate of change of current through the other coil is unity. The unit of coefficient of mutual induction is henry.

One henry is defined as the coefficient of mutual induction between a pair of coils when a change of current of one ampere per second in one coil produces an induced emf of one volt in the other coil.

The coefficient of mutual induction between a pair of coils depends on the following factors

(i) Size and shape of the coils, number of turns and permeability of material on which the coils are wound.

(ii) proximity of the coils

Two coils P and S have their axes perpendicular to each other (Fig. 4.7a). When a current is passed through coil P, the magnetic flux linked with S is small and hence, the coefficient of mutual induction between the two coils is small.

The two coils are placed in such a way that they have a common axis (Fig. 4.7b). When current is passed through the coil P the magnetic flux linked with coil S is large and hence, the coefficient of mutual induction between the two coils is large.

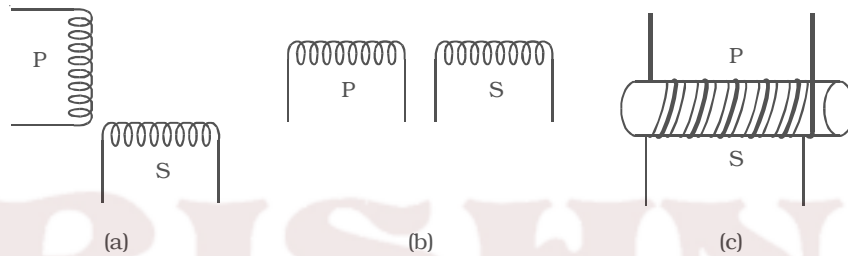


Fig 4.7 Mutual induction

If the two coils are wound on a soft iron core (Fig 4.7c) the mutual induction is very large.

4.2.6 Mutual induction of two long solenoids.

S_1 and S_2 are two long solenoids each of length l . The solenoid S_2 is wound closely over the solenoid S_1 (Fig 4.8).

N_1 and N_2 are the number of turns in the solenoids S_1 and S_2 respectively. Both the solenoids are considered to have the same area of cross section A as they are closely wound together. I_1 is the current flowing through the solenoid S_1 . The magnetic field B_1 produced at any point inside the solenoid S_1 due to the current I_1 is



Fig 4.8 Mutual induction between two long solenoids

$$B_1 = \mu_0 \frac{N_1}{l} I_1 \quad \dots(1)$$

The magnetic flux linked with each turn of S_2 is equal to $B_1 A$.

Total magnetic flux linked with solenoid S_2 having N_2 turns is

$$\phi_2 = B_1 A N_2$$

Substituting for B_1 from equation (1)

$$\begin{aligned}\phi_2 &= \left(\mu_o \frac{N_1}{l} I_1 \right) A N_2 \\ \phi_2 &= \frac{\mu_o N_1 N_2 A I_1}{l} \quad \dots(2)\end{aligned}$$

$$\text{But } \phi_2 = M I_1 \quad \dots(3)$$

where M is the coefficient of mutual induction between S_1 and S_2 .

From equations (2) and (3)

$$\begin{aligned}M I_1 &= \frac{\mu_o N_1 N_2 A I_1}{l} \\ M &= \frac{\mu_o N_1 N_2 A}{l}\end{aligned}$$

If the core is filled with a magnetic material of permeability μ ,

$$M = \frac{\mu N_1 N_2 A}{l}$$

4.3 Methods of producing induced emf

We know that the induced emf is given by the expression

$$e = - \frac{d\phi}{dt} = - \frac{d}{dt} (NBA \cos \theta)$$

Hence, the induced emf can be produced by changing

- (i) the magnetic induction (B)
- (ii) area enclosed by the coil (A) and
- (iii) the orientation of the coil (θ) with respect to the magnetic field.

4.3.1 Emf induced by changing the magnetic induction.

The magnetic induction can be changed by moving a magnet either towards or away from a coil and thus an induced emf is produced in the coil.

The magnetic induction can also be changed in one coil by changing the current in the neighbouring coil thus producing an induced emf.

$$\therefore e = - N A \cos \theta \left(\frac{dB}{dt} \right)$$

4.3.2 Emf induced by changing the area enclosed by the coil

PQRS is a conductor bent in the shape as shown in the Fig 4.9. L_1M_1 is a sliding conductor of length l resting on the arms PQ and RS. A uniform magnetic field 'B' acts perpendicular to the plane of the conductor. The closed area of the conductor is L_1QRM_1 . When L_1M_1 is moved through a distance dx in time dt , the new area is L_2QRM_2 . Due to the change in area $L_2L_1M_1M_2$, there is a change in the flux linked with the conductor. Therefore, an induced emf is produced.

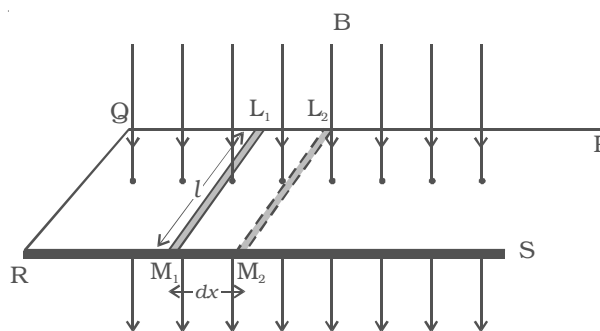


Fig 4.9 Emf induced by changing the area

Change in area $dA = \text{Area } L_2L_1M_1M_2$

$$\therefore dA = l dx$$

Change in the magnetic flux, $d\phi = B \cdot dA = Bl dx$

But
$$e = - \frac{d\phi}{dt}$$

$$\therefore e = - \frac{Bl dx}{dt} = - Blv$$

where v is the velocity with which the sliding conductor is moved.

4.3.3 Emf induced by changing the orientation of the coil

PQRS is a rectangular coil of N turns and area A placed in a uniform magnetic field B (Fig 4.10). The coil is rotated with an angular velocity ω in the clockwise direction about an axis perpendicular to the direction of the magnetic field. Suppose, initially the coil is in vertical position, so that the angle between normal to the plane of the coil and magnetic field is zero. After a time t , let $\theta (= \omega t)$ be the angle through which the coil is rotated. If ϕ is the flux linked with the coil at this instant, then

$$\phi = NBA \cos \theta$$

The induced emf is,

$$e = -\frac{d\phi}{dt} = -NBA \frac{d}{dt} \cos(\omega t)$$

$$\therefore e = NBA\omega \sin \omega t \dots(1)$$

The maximum value of the induced emf is, $E_0 = NAB\omega$

Hence, the induced emf can be represented as $e = E_0 \sin \omega t$

The induced emf e varies sinusoidally with time t and the frequency

being ν cycles per second $\left(\nu = \frac{\omega}{2\pi}\right)$.

(i) When $\omega t = 0$, the plane of the coil is perpendicular to the field B and hence $e = 0$.

(ii) When $\omega t = \pi/2$, the plane of the coil is parallel to B and hence $e = E_0$

(iii) When $\omega t = \pi$, the plane of the coil is at right angle to B and hence $e = 0$.

(iv) When $\omega t = 3\pi/2$, the plane of the coil is again parallel to B and the induced emf is $e = -E_0$.

(v) When $\omega t = 2\pi$, the plane of the coil is again perpendicular to B and hence $e = 0$.

If the ends of the coil are connected to an external circuit through a resistance R , current flows through the circuit, which is also sinusoidal in nature.

4.4 AC generator (Dynamo) – Single phase

The ac generator is a device used for converting mechanical energy into electrical energy. The generator was originally designed by a Yugoslav scientist Nikola Tesla.

Principle

It is based on the principle of electromagnetic induction,

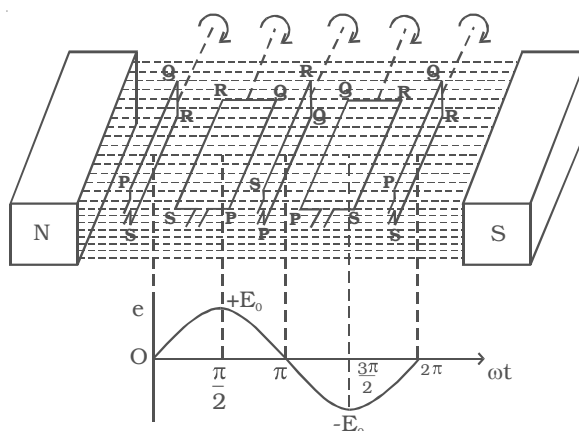


Fig 4.10 Induced emf by changing the orientation of the coil

according to which an emf is induced in a coil when it is rotated in a uniform magnetic field.

Essential parts of an AC generator

(i) Armature

Armature is a rectangular coil consisting of a large number of loops or turns of insulated copper wire wound over a laminated soft iron core or ring. The soft iron core not only increases the magnetic flux but also serves as a support for the coil

(ii) Field magnets

The necessary magnetic field is provided by permanent magnets in the case of low power dynamos. For high power dynamos, field is provided by electro magnet. Armature rotates between the magnetic poles such that the axis of rotation is perpendicular to the magnetic field.

(iii) Slip rings

The ends of the armature coil are connected to two hollow metallic rings R_1 and R_2 called slip rings. These rings are fixed to a shaft, to which the armature is also fixed. When the shaft rotates, the slip rings along with the armature also rotate.

(iv) Brushes

B_1 and B_2 are two flexible metallic plates or carbon brushes. They provide contact with the slip rings by keeping themselves pressed against the ring. They are used to pass the current from the armature to the external power line through the slip rings.

Working

Whenever, there is a change in orientation of the coil, the magnetic flux linked with the coil changes, producing an induced emf in the coil. The direction of the induced current is given by Fleming's right hand rule.

Suppose the armature ABCD is initially in the vertical position. It is rotated in the anticlockwise direction. The side AB of the coil moves downwards and the side DC moves

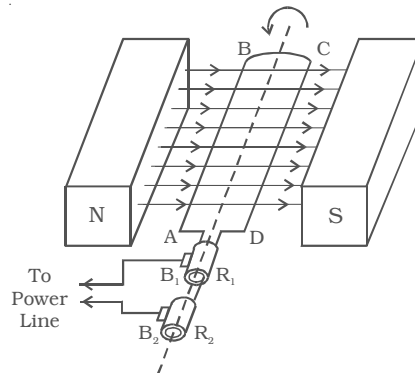


Fig 4.11 AC dynamo

upwards (Fig. 4.11). Then according to Flemings right hand rule the current induced in arm AB flows from B to A and in CD it flows from D to C. Thus the current flows along DCBA in the coil. In the external circuit the current flows from B_1 to B_2 .

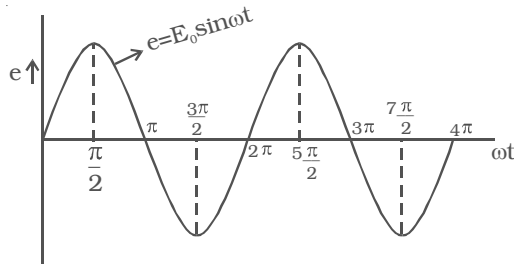


Fig 4.12 emf varies sinusoidally

On further rotation, the arm AB of the coil moves upwards and DC moves downwards. Now the current in the coil flows along ABCD. In the external circuit the current flows from B_2 to B_1 . As the rotation of the coil continues, the induced current in the external circuit keeps changing its direction for every half a rotation of the coil. Hence the induced current is alternating in nature (Fig 4.12). As the armature completes ν rotations in one second, alternating current of frequency ν cycles per second is produced. The induced emf at any instant is given by $e = E_0 \sin \omega t$

The peak value of the emf, $E_0 = NBA\omega$

where N is the number of turns of the coil,

A is the area enclosed by the coil,

B is the magnetic field and

ω is the angular velocity of the coil

4.4.1 AC generator (Alternator) - Three phase

A single phase a.c. generator or alternator has only one armature winding. If a number of armature windings are used in the alternator it is known as polyphase alternator. It produces voltage waves equal to the number of windings or phases. Thus a polyphase system consists of a numerous windings which are placed on the same axis but displaced from one another by equal angle which depends on the number of phases. Three phase alternators are widely preferred for transmitting large amount of power with less cost and high efficiency.

Generation of three phase emf

In a three - phase a.c. generator three coils are fastened rigidly together and displaced from each other by 120° . It is made to rotate about a fixed axis in a uniform magnetic field. Each coil is provided with a separate set of slip rings and brushes.

An emf is induced in each of the coils with a phase difference of 120° . Three coils $a_1 a_2$, $b_1 b_2$ and $c_1 c_2$ are mounted on the same axis but displaced from each other by 120° , and the coils rotate in the

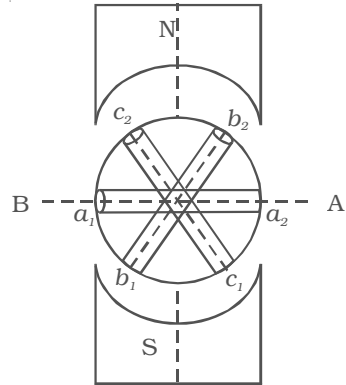


Fig 4.13a Section of 3 phase ac generator

anticlockwise direction in a magnetic field (Fig 4.13a).

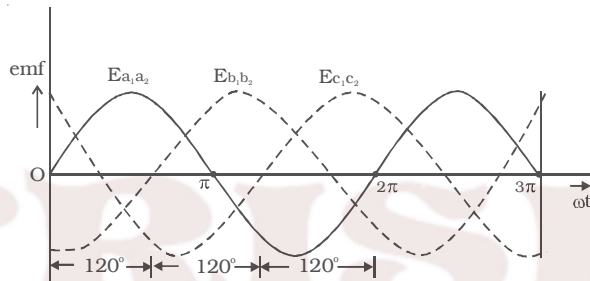


Fig 4.13b Three phase emf

$b_1 b_2$ is 120° behind coil $a_1 a_2$, so that emf induced in this coil is approaching its maximum negative value and the coil $c_1 c_2$ is 240° behind the coil $a_1 a_2$, so the emf induced in this coil has passed its positive maximum value and is decreasing. Thus the emfs induced in all the three coils are equal in magnitude and of same frequency. The emfs induced in the three coils are ;

$$\begin{aligned} e_{a_1 a_2} &= E_0 \sin \omega t \\ e_{b_1 b_2} &= E_0 \sin (\omega t - 2\pi/3) \\ e_{c_1 c_2} &= E_0 \sin (\omega t - 4\pi/3) \end{aligned}$$

The emfs induced and phase difference in the three coils $a_1 a_2$, $b_1 b_2$ and $c_1 c_2$ are shown in Fig 4.13b & Fig 4.13c.

When the coil $a_1 a_2$ is in position AB, emf induced in this coil is zero and starts increasing in the positive direction. At the same instant the coil

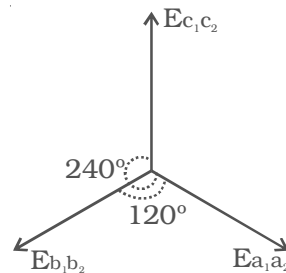


Fig 4.13c Angular displacement between the armature

4.5 Eddy currents

Foucault in the year 1895 observed that when a mass of metal moves in a magnetic field or when the magnetic field through a stationary mass of metal is altered, induced current is produced in the metal. This induced current flows in the metal in the form of closed loops resembling 'eddies' or whirl pool. Hence this current is called eddy current. The direction of the eddy current is given by Lenz's law.

When a conductor in the form of a disc or a metallic plate as shown in Fig 4.14, swings between the poles of a magnet, eddy currents are set up inside the plate. This current acts in a direction so as to oppose the

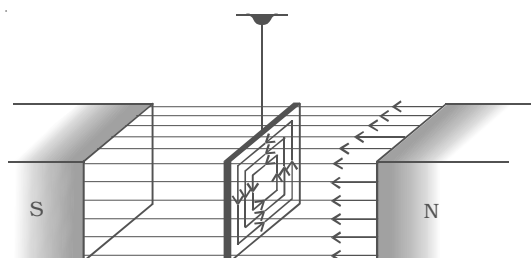


Fig 4.14 Eddy current

motion of the conductor with a strong retarding force, that the conductor almost comes to rest. If the metallic plate with holes drilled in it is made to swing inside the magnetic field, the effect of eddy current is greatly reduced consequently the plate swings freely inside the field. Eddy current can be minimised by using thin laminated sheets instead of solid metal.

Applications of Eddy current

(i) Dead beat galvanometer

When current is passed through a galvanometer, the coil oscillates about its mean position before it comes to rest. To bring the coil to rest immediately, the coil is wound on a metallic frame. Now, when the coil oscillates, eddy currents are set up in the metallic frame, which opposes further oscillations of the coil. This inturn enables the coil to attain its equilibrium position almost instantly. Since the oscillations of the coil die out instantaneously, the galvanometer is called dead beat galvanometer.

(ii) Induction furnace

In an induction furnace, high temperature is produced by generating eddy currents. The material to be melted is placed in a varying magnetic field of high frequency. Hence a strong eddy current is developed inside the metal. Due to the heating effect of the current, the metal melts.

(iii) Induction motors

Eddy currents are produced in a metallic cylinder called rotor, when it is placed in a rotating magnetic field. The eddy current initially tries to decrease the relative motion between the cylinder and the rotating magnetic field. As the magnetic field continues to rotate, the metallic cylinder is set into rotation. These motors are used in fans.

(iv) Electro magnetic brakes

A metallic drum is coupled to the wheels of a train. The drum rotates along with the wheel when the train is in motion. When the brake is applied, a strong magnetic field is developed and hence, eddy currents are produced in the drum which oppose the motion of the drum. Hence, the train comes to rest.

(v) Speedometer

In a speedometer, a magnet rotates according to the speed of the vehicle. The magnet rotates inside an aluminium cylinder (drum) which is held in position with the help of hair springs. Eddy currents are produced in the drum due to the rotation of the magnet and it opposes the motion of the rotating magnet. The drum in turn experiences a torque and gets deflected through a certain angle depending on the speed of the vehicle. A pointer attached to the drum moves over a calibrated scale which indicates the speed of the vehicle.

4.6 Transformer

Transformer is an electrical device used for converting low alternating voltage into high alternating voltage and vice versa. It transfers electric power from one circuit to another. The transformer is based on the principle of electromagnetic induction.

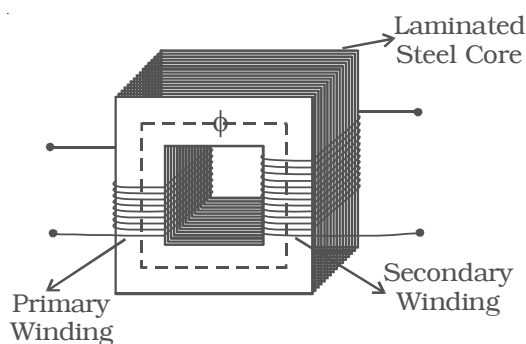


Fig 4.15 Transformer

A transformer consists of primary and secondary coils insulated from each other, wound on a soft iron core (Fig 4.15). To minimise eddy

currents a laminated iron core is used. The a.c. input is applied across the primary coil. The continuously varying current in the primary coil produces a varying magnetic flux in the primary coil, which in turn produces a varying magnetic flux in the secondary. Hence, an induced emf is produced across the secondary.

Let E_p and E_s be the induced emf in the primary and secondary coils and N_p and N_s be the number of turns in the primary and secondary coils respectively. Since same flux links with the primary and secondary, the emf induced per turn of the two coils must be the same

$$\begin{aligned} \text{(i.e.) } \frac{E_p}{N_p} &= \frac{E_s}{N_s} \\ \text{or } \frac{E_s}{E_p} &= \frac{N_s}{N_p} \quad \dots(1) \end{aligned}$$

For an ideal transformer, input power = output power

$$E_p I_p = E_s I_s$$

where I_p and I_s are currents in the primary and secondary coils.

$$\text{(i.e.) } \frac{E_s}{E_p} = \frac{I_p}{I_s} \quad \dots(2)$$

From equations (1) and (2)

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = k$$

where k is called transformer ratio.

(for step up transformer $k > 1$ and

for step down transformer $k < 1$)

In a step up transformer $E_s > E_p$ implying that $I_s < I_p$. Thus a step up transformer increases the voltage by decreasing the current, which is in accordance with the law of conservation of energy. Similarly a step down transformer decreases the voltage by increasing the current.

Efficiency of a transformer

Efficiency of a transformer is defined as the ratio of output power to the input power.

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{E_s I_s}{E_p I_p}$$

The efficiency $\eta = 1$ (ie. 100%), only for an ideal transformer where there is no power loss. But practically there are numerous factors leading to energy loss in a transformer and hence the efficiency is always less than one.

Energy losses in a transformer

(1) Hysteresis loss

The repeated magnetisation and demagnetisation of the iron core caused by the alternating input current, produces loss in energy called hysteresis loss. This loss can be minimised by using a core with a material having the least hysteresis loss. Alloys like mumetal and silicon steel are used to reduce hysteresis loss.

(2) Copper loss

The current flowing through the primary and secondary windings lead to Joule heating effect. Hence some energy is lost in the form of heat. Thick wires with considerably low resistance are used to minimise this loss.

(3) Eddy current loss (Iron loss)

The varying magnetic flux produces eddy current in the core. This leads to the wastage of energy in the form of heat. This loss is minimised by using a laminated core made of stelloy, an alloy of steel.

(4) Flux loss

The flux produced in the primary coil is not completely linked with the secondary coil due to leakage. This results in the loss of energy. This loss can be minimised by using a shell type core.

In addition to the above losses, due to the vibration of the core, sound is produced, which causes a loss in the energy.

4.6.1 Long distance power transmission

The electric power generated in a power station situated in a remote place is transmitted to different regions for domestic and industrial use. For long distance transmission, power lines are made of

conducting material like aluminium. There is always some power loss associated with these lines.

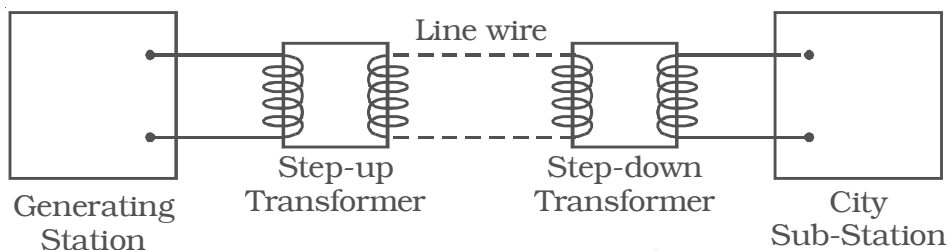


Fig 4.16 Distance transmission of power

If I is the current through the wire and R the resistance, a considerable amount of electric power I^2R is dissipated as heat. Hence, the power at the receiving end will be much lesser than the actual power generated. However, by transmitting the electrical energy at a higher voltage, the power loss can be controlled as is evident from the following two cases.

Case (i) A power of 11,000 W is transmitted at 220 V.

$$\text{Power } P = VI$$

$$\therefore I = \frac{P}{V} = \frac{11,000}{220} = 50\text{A}$$

If R is the resistance of line wires,

$$\text{Power loss} = I^2R = 50^2R = 2500(R) \text{ watts}$$

Case (ii) 11,000 W power is transmitted at 22,000 V

$$\therefore I = \frac{P}{V} = \frac{11,000}{22,000} = 0.5 \text{ A}$$

$$\text{Power loss} = I^2R = (0.5)^2 R = 0.25(R) \text{ watts}$$

Hence it is evident that if power is transmitted at a higher voltage the loss of energy in the form of heat can be considerably reduced.

For transmitting electric power at 11,000 W at 220 V the current capacity of line wires has to be 50 A and if transmission is done at 22,000 V, it is only 0.5 A. Thus, for carrying larger current (50A) thick wires have to be used. This increases the cost of transmission. To support these thick wires, stronger poles have to be erected which further adds on to the cost. On the other hand if transmission is done at high voltages, the wires required are of lower current carrying capacity. So thicker wires can be replaced by thin wires, thus reducing the cost of transmission considerably.

For example, 400MW power produced at 15,000 V in the power station at Neyveli, is stepped up by a step-up transformer to 230,000 V before transmission. The power is then transmitted through the transmission lines which forms a part of the grid. The grid connects different parts of the country. Outside the city, the power is stepped down to 110,000 V by a step-down transformer. Again the power is stepped down to 11,000 V by a transformer. Before distribution to the user, the power is stepped down to 230 V or 440 V depending upon the need of the user.

4.7 Alternating current

As we have seen earlier a rotating coil in a magnetic field, induces an alternating emf and hence an alternating current. Since the emf induced in the coil varies in magnitude and direction periodically, it is called an alternating emf. The significance of an alternating emf is that it can be changed to lower or higher voltages conveniently and efficiently using a transformer. Also the frequency of the induced emf can be altered by changing the speed of the coil. This enables us to utilize the whole range of electromagnetic spectrum for one purpose or the other. For example domestic power in India is supplied at a frequency of 50 Hz. For transmission of audio and video signals, the required frequency range of radio waves is between 100 KHz and 100 MHz. Thus owing to its wide applicability most of the countries in the world use alternating current.

4.7.1 Measurement of AC

Since alternating current varies continuously with time, its average value over one complete cycle is zero. Hence its effect is measured by rms value of a.c.

RMS value of a.c.

The rms value of alternating current is defined as that value of the steady current, which when passed through a resistor for a given time, will generate the same amount of heat as generated by an alternating current when passed through the same resistor for the same time.

The rms value is also called effective value of an a.c. and is denoted by I_{rms} or I_{eff} .

when an alter-nating current $i = I_0 \sin \omega t$ flows through a resistor of

resistance R , the amount of heat produced in the resistor in a small time dt is

$$dH = i^2 R dt$$

The total amount of heat produced in the resistance in one complete cycle is

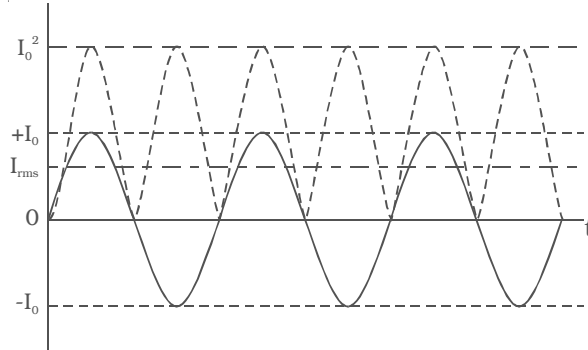


Fig 4.17 Variation I , I^2 and I_{rms} with time

$$\begin{aligned} H &= \int_0^T i^2 R dt = \int_0^T (I_o^2 \sin^2 \omega t) R dt \\ &= I_o^2 R \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt = \frac{I_o^2 R}{2} \left[\int_0^T dt - \int_0^T \cos 2\omega t dt \right] \\ &= \frac{I_o^2 R}{2} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T = \frac{I_o^2 R}{2} \left[T - \frac{\sin 4\pi}{2\omega} \right] \quad \left\{ \because T = \frac{2\pi}{\omega} \right\} \\ H &= \frac{I_o^2 RT}{2} \end{aligned}$$

But this heat is also equal to the heat produced by rms value of AC in the same resistor (R) and in the same time (T),

$$(i.e) H = I_{rms}^2 RT$$

$$\therefore I_{rms}^2 RT = \frac{I_o^2 RT}{2}$$

$$I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707 I_o$$

Similarly, it can be calculated that

$$E_{rms} = \frac{E_o}{\sqrt{2}}$$

Thus, the rms value of an a.c is 0.707 times the peak value of the a.c. In other words it is 70.7 % of the peak value.

4.7.2 AC Circuit with resistor

Let an alternating source of emf be connected across a resistor of resistance R.

The instantaneous value of the applied emf is

$$e = E_0 \sin \omega t \quad \dots(1)$$

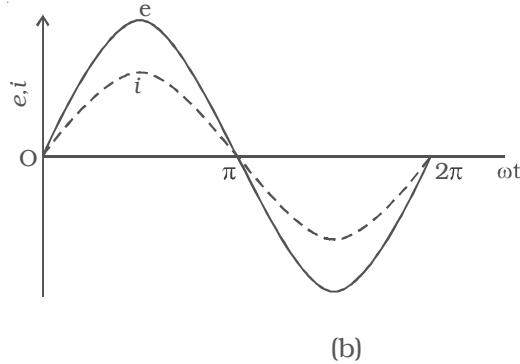
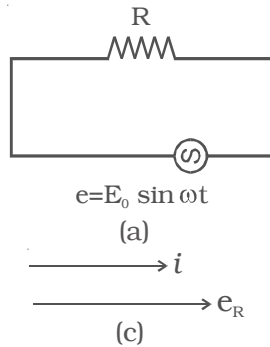


Fig 4.18 a.c. circuit with a resistor

If i is the current through the circuit at the instant t , the potential drop across R is, $e = i R$

Potential drop must be equal to the applied emf.

Hence, $iR = E_0 \sin \omega t$

$$i = \frac{E_0}{R} \sin \omega t ; \quad i = I_0 \sin \omega t \quad \dots(2)$$

where $I_0 = \frac{E_0}{R}$, is the peak value of a.c in the circuit. Equation

(2) gives the instantaneous value of current in the circuit containing R . From the expressions of voltage and current given by equations (1) and (2) it is evident that in a resistive circuit, the applied voltage and current are in phase with each other (Fig 4.18b).

Fig 4.18c is the phasor diagram representing the phase relationship between the current and the voltage.

4.7.3 AC Circuit with an inductor

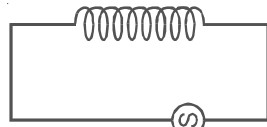
Let an alternating source of emf be applied to a pure inductor of inductance L . The inductor has a negligible resistance and is wound on a laminated iron core. Due to an alternating emf that is applied to the inductive coil, a self induced emf is generated which opposes the applied voltage. (eg) Choke coil.

The instantaneous value of applied emf is given by

$$e = E_0 \sin \omega t \quad \dots(1)$$

$$\text{Induced emf } e' = -L \cdot \frac{di}{dt}$$

where L is the self inductance of the coil. In an ideal inductor circuit induced emf is equal and opposite to the applied voltage.



$$e = E_0 \sin \omega t$$

(a)

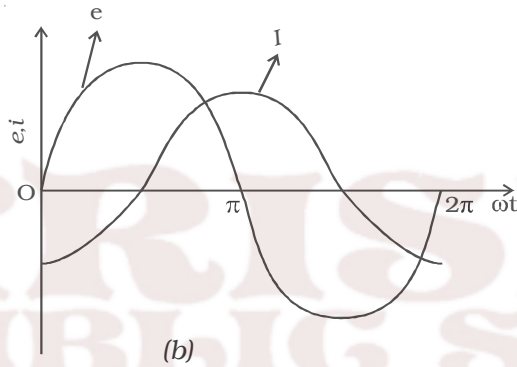


Fig 4.19 Pure inductive circuit

Therefore $e = -e'$

$$E_0 \sin \omega t = -\left(-L \frac{di}{dt}\right)$$

$$\therefore E_0 \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{E_0}{L} \sin \omega t dt$$

Integrating both the sides

$$i = \frac{E_0}{L} \int \sin \omega t dt$$

$$= \frac{E_0}{L} \left[-\frac{\cos \omega t}{\omega} \right] = -\frac{E_0 \cos \omega t}{\omega L}$$

$$i = \frac{E_0}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

$$i = I_0 \cdot \sin \left(\omega t - \frac{\pi}{2} \right) \quad \dots(2)$$

where $I_0 = \frac{E_0}{\omega L}$. Here, ωL is the resistance offered by the coil. It is called inductive reactance. Its unit is ohm .

From equations (1) and (2) it is clear that in an a.c. circuit containing a pure inductor the current i lags behind the voltage e by the phase angle of $\pi/2$. Conversely the voltage across L leads the current by the phase angle of $\pi/2$. This fact is presented graphically in Fig 4.19b.

Fig 4.19c represents the phasor diagram of a.c. circuit containing only L.

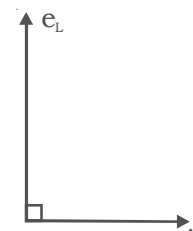


Fig 4.19c
Phasor diagram

Inductive reactance

$X_L = \omega L = 2\pi\nu L$, where ν is the frequency of the a.c. supply

For d.c. $\nu = 0$; $\therefore X_L = 0$

Thus a pure inductor offers zero resistance to d.c. But in an a.c. circuit the reactance of the coil increases with increase in frequency.

4.7.4 AC Circuit with a capacitor

An alternating source of emf is connected across a capacitor of capacitance C (Fig 4.20a). It is charged first in one direction and then in the other direction.

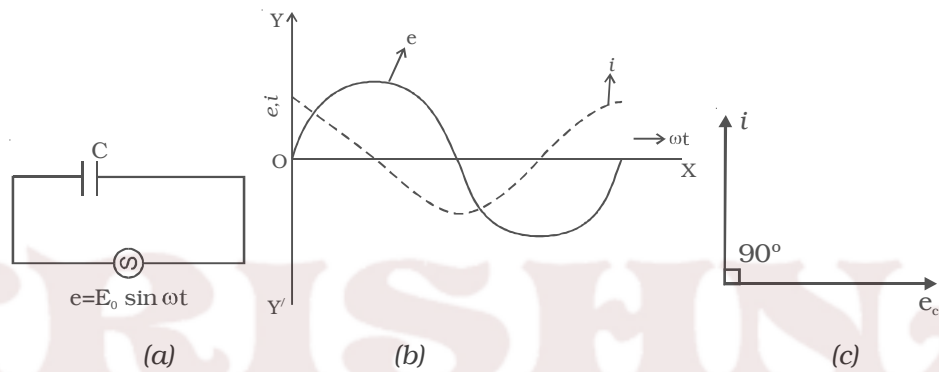


Fig 4.20 Capacitive circuit

The instantaneous value of the applied emf is given by

$$e = E_0 \sin \omega t \quad \dots(1)$$

At any instant the potential difference across the capacitor will be equal to the applied emf

$\therefore e = q/C$, where q is the charge in the capacitor

$$\text{But} \quad i = \frac{dq}{dt} = \frac{d}{dt} (Ce)$$

$$i = \frac{d}{dt} (C E_0 \sin \omega t) = \omega C E_0 \cos \omega t$$

$$i = \frac{E_0}{(1/\omega C)} \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$i = I_0 \sin \left(\omega t + \frac{\pi}{2} \right) \quad \dots(2)$$

where $I_0 = \frac{E_0}{(1/\omega C)}$

$\frac{1}{\omega C} = X_C$ is the resistance offered by the capacitor. It is called capacitive reactance. Its unit is ohm .

From equations (1) and (2), it follows that in an a.c. circuit with a capacitor, the current leads the voltage by a phase angle of $\pi/2$. In other words the emf lags behind the current by a phase angle of $\pi/2$. This is represented graphically in Fig 4.20b.

Fig 4.20c represents the phasor diagram of a.c. circuit containing only C.

$$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

where ν is the frequency of the a.c. supply. In a d.c. circuit $\nu = 0$

$$\therefore X_C = \infty$$

Thus a capacitor offers infinite resistance to d.c. For an a.c. the capacitive reactance varies inversely as the frequency of a.c. and also inversely as the capacitance of the capacitor.

4.7.5 Resistor, inductor and capacitor in series

Let an alternating source of emf e be connected to a series combination of a resistor of resistance R , inductor of inductance L and a capacitor of capacitance C (Fig 4.21a).

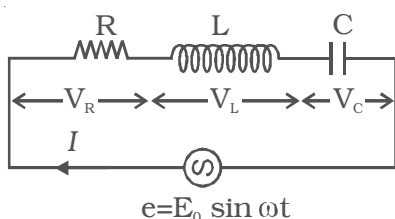
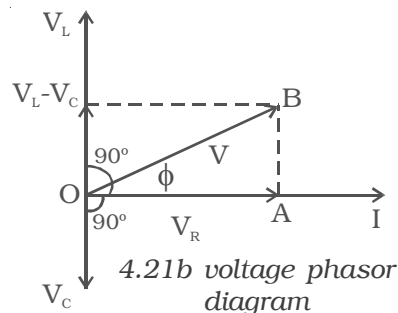


Fig 4.21a RLC series circuit



4.21b voltage phasor diagram

Let the current flowing through the circuit be I .

The voltage drop across the resistor is, $V_R = I R$ (This is in phase with I)

The voltage across the inductor coil is $V_L = I X_L$

(V_L leads I by $\pi/2$)

The voltage across the capacitor is, $V_C = I X_C$

(V_C lags behind I by $\pi/2$)

The voltages across the different components are represented in the voltage phasor diagram (Fig. 4.21b).

V_L and V_C are 180° out of phase with each other and the resultant of V_L and V_C is $(V_L - V_C)$, assuming the circuit to be predominantly inductive. The applied voltage 'V' equals the vector sum of V_R , V_L and V_C .

$$OB^2 = OA^2 + AB^2 ;$$

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 - (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2}$$

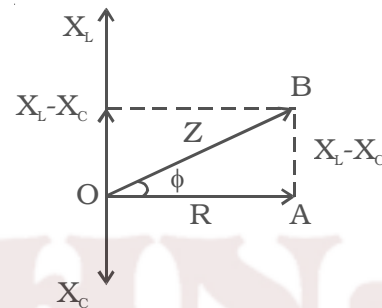


Fig 4.22 Impedance diagram

The expression $\sqrt{R^2 + (X_L - X_C)^2}$ is the net effective opposition offered by the combination of resistor, inductor and capacitor known as the impedance of the circuit and is represented by Z . Its unit is ohm. The values are represented in the impedance diagram (Fig 4.22).

Phase angle ϕ between the voltage and current is given by

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{I X_L - I X_C}{IR}$$

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{\text{net reactance}}{\text{resistance}}$$

$$\therefore \phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$\therefore I_0 \sin (\omega t \pm \phi)$ is the instantaneous current flowing in the circuit.

Series resonance or voltage resonance in RLC circuit

The value of current at any instant in a series RLC circuit is given by

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

At a particular value of the angular frequency, the inductive reactance and the capacitive reactance will be equal to each other (i.e.)

$\omega L = \frac{1}{\omega C}$, so that the impedance becomes minimum and it is given by $Z = R$

i.e. I is in phase with V

The particular frequency ν_0 at which the impedance of the circuit becomes minimum and therefore the current becomes maximum is called Resonant frequency of the circuit. Such a circuit which admits maximum current is called series resonant circuit or acceptor circuit. Thus the maximum current through the circuit at resonance is

$$I_0 = \frac{V}{R}$$

Maximum current flows through the circuit, since the impedance of the circuit is merely equal to the ohmic resistance of the circuit. i.e $Z = R$

$$\omega L = \frac{1}{\omega C}$$

$$\omega = 2\pi \nu_0 = \frac{1}{\sqrt{LC}}$$

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

Acceptor circuit

The series resonant circuit is often called an 'acceptor' circuit. By offering minimum impedance to current at the resonant frequency it is able to select or accept most readily this particular frequency among many frequencies.

In radio receivers the resonant frequency of the circuit is tuned

to the frequency of the signal desired to be detected. This is usually done by varying the capacitance of a capacitor.

Q-factor

The selectivity or sharpness of a resonant circuit is measured by the quality factor or Q factor. In other words it refers to the sharpness of tuning at resonance.

The Q factor of a series resonant circuit is defined as the ratio of the voltage across a coil or capacitor to the applied voltage.

$$Q = \frac{\text{voltage across } L \text{ or } C}{\text{applied voltage}} \quad \dots(1)$$

$$\text{Voltage across } L = I \omega_0 L \quad \dots(2)$$

where ω_0 is the angular frequency of the a.c. at resonance.

The applied voltage at resonance is the potential drop across R, because the potential drop across L is equal to the drop across C and they are 180° out of phase. Therefore they cancel out and only potential drop across R will exist.

$$\text{Applied Voltage} = IR \quad \dots(3)$$

Substituting equations (2) and (3) in equation (1)

$$Q = \frac{I \omega_0 L}{IR} = \frac{\omega_0 L}{R}$$

$$Q = \frac{1}{\sqrt{LC}} \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} \quad \left\{ \because \omega_0 = \frac{1}{\sqrt{LC}} \right\}$$

Q is just a number having values between 10 to 100 for normal frequencies. Circuit with high Q values would respond to a very narrow frequency range and vice versa. Thus a circuit with a high Q value is sharply tuned while one with a low Q has a flat resonance. Q-factor can be increased by having a coil of large inductance but of small ohmic resistance.

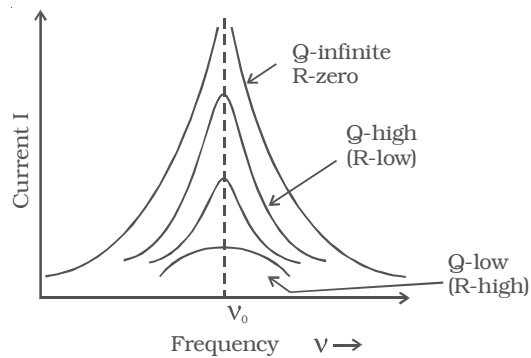


Fig 4.23 variation of current with frequency

Current frequency curve is quite flat for large values of resistance and becomes more sharp as the value of resistance decreases. The curve shown in Fig 4.23 is also called the frequency response curve.

4.7.6 Power in an ac circuit

In an a.c circuit the current and emf vary continuously with time. Therefore power at a given instant of time is calculated and then its mean is taken over a complete cycle. Thus, we define instantaneous power of an a.c. circuit as the product of the instantaneous emf and the instantaneous current flowing through it.

The instantaneous value of emf and current is given by

$$e = E_o \sin \omega t$$

$$i = I_o \sin (\omega t + \phi)$$

where ϕ is the phase difference between the emf and current in an a.c circuit

The average power consumed over one complete cycle is

$$P_{av} = \frac{\int_0^T ie \, dt}{\int_0^T dt} = \frac{\int_0^T [I_o \sin(\omega t + \phi) E_o \sin \omega t] dt}{T}$$

On simplification, we obtain

$$P_{av} = \frac{E_o I_o}{2} \cos \phi$$

$$P_{av} = \frac{E_o}{\sqrt{2}} \cdot \frac{I_o}{\sqrt{2}} \cdot \cos \phi = E_{rms} I_{rms} \cos \phi$$

$$P_{av} = \text{apparent power} \times \text{power factor}$$

where Apparent power = $E_{rms} I_{rms}$ and power factor = $\cos \phi$

The average power of an ac circuit is also called the true power of the circuit.

Choke coil

A choke coil is an inductance coil of very small resistance used for controlling current in an a.c. circuit. If a resistance is used to control current, there is wastage of power due to Joule heating effect in the resistance. On the other hand there is no dissipation of power when a current flows through a pure inductor.

Construction

It consists of a large number of turns of insulated copper wire wound over a soft iron core. A laminated core is used to minimise eddy current loss (Fig. 4.24).

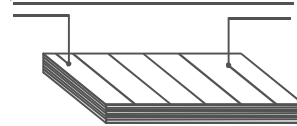


Fig 4.24 Choke coil

Working

The inductive reactance offered by the coil is given by

$$X_L = \omega L$$

In the case of an ideal inductor the current lags behind the emf by a phase angle $\frac{\pi}{2}$.

∴ The average power consumed by the choke coil over a complete cycle is

$$P_{av} = E_{rms} I_{rms} \cos \pi/2 = 0$$

However in practice, a choke coil of inductance L possesses a small resistance r. Hence it may be treated as a series combination of an inductor and small resistance r. In that case the average power consumed by the choke coil over a complete cycle is

$$P_{av} = E_{rms} I_{rms} \cos \phi$$

$$P_{av} = E_{rms} I_{rms} \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \dots(1)$$

where $\frac{r}{\sqrt{r^2 + \omega^2 L^2}}$ is the power factor. From equation (1) the value of average power dissipated works out to be much smaller than the power loss I^2R in a resistance R.

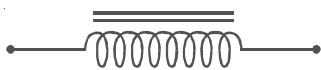


Fig.4.24a A.F Choke



Fig.4.24b R.F. Choke

Chokes used in low frequency a.c. circuit have an iron core so that the inductance may be high. These chokes are known as audio – frequency (A.F) chokes. For radio frequencies, air chokes are used since a low inductance is sufficient. These are called radio frequency (R. F) or high frequency (H.F) chokes and are used in wireless receiver circuits (Fig. 4.24a and Fig. 4.24b).

Choke coils can be commonly seen in fluorescent tubes which work on alternating currents.

Solved problems

- 4.1 Magnetic field through a coil having 200 turns and cross sectional area 0.04 m^2 changes from 0.1 wb m^{-2} to 0.04 wb m^{-2} in 0.02 s Find the induced emf.

Data : $N = 200$, $A = 0.04 \text{ m}^2$, $B_1 = 0.1 \text{ wb m}^{-2}$,
 $B_2 = 0.04 \text{ wb m}^{-2}$, $t = 0.02 \text{ s}$, $e = ?$

Solution : $e = - \frac{d\phi}{dt} = - \frac{d}{dt}(\phi)$

$$e = - \frac{d}{dt}(NBA) = - NA \cdot \frac{dB}{dt} = - NA \cdot \frac{(B_2 - B_1)}{dt}$$

$$e = - 200 \times 4 \times 10^{-2} \frac{(0.04 - 0.1)}{0.02}$$

$$e = 24 \text{ V}$$

- 4.2 An aircraft having a wingspan of 20.48 m flies due north at a speed of 40 ms^{-1} . If the vertical component of earth's magnetic field at the place is $2 \times 10^{-5} \text{ T}$, Calculate the emf induced between the ends of the wings.

Data : $l = 20.48 \text{ m}$; $v = 40 \text{ ms}^{-1}$; $B = 2 \times 10^{-5} \text{ T}$; $e = ?$

Solution : $e = - B l v$

$$= - 2 \times 10^{-5} \times 20.48 \times 40$$

$$e = - 0.0164 \text{ volt}$$

- 4.3 A solenoid of length 1 m and 0.05 m diameter has 500 turns. If a current of 2 A passes through the coil, calculate (i) the coefficient of self induction of the coil and (ii) the magnetic flux linked with a the coil.

Data : $l = 1 \text{ m}$; $d = 0.05 \text{ m}$; $r = 0.025 \text{ m}$; $N = 500$; $I = 2 \text{ A}$;

(i) $L = ?$ (ii) $\phi = ?$

Solution : (i) $L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l}$

$$= \frac{4\pi \times 10^{-7} \times (5 \times 10^2)^2 \times 3.14 (0.025)^2}{1} = 0.616 \times 10^{-3}$$

$$\therefore L = 0.616 \text{ mH}$$

(ii) Magnetic flux $\phi = LI$

$$= 0.616 \times 10^{-3} \times 2 = 1.232 \times 10^{-3}$$

$$\phi = 1.232 \text{ milli weber}$$

- 4.4 Calculate the mutual inductance between two coils when a current of 4 A changing to 8 A in 0.5 s in one coil, induces an emf of 50 mV in the other coil.

Data : $I_1 = 4\text{A}; I_2 = 8\text{A}; dt = 0.5\text{s};$
 $e = 50 \text{ mV} = 50 \times 10^{-3}\text{V}, M = ?$

Solution : $e = - M \cdot \frac{dI}{dt}$

$$\therefore M = -\frac{e}{\left(\frac{dI}{dt}\right)} = -\frac{e}{\left(\frac{I_2 - I_1}{dt}\right)} = -\frac{50 \times 10^{-3}}{\left(\frac{8-4}{0.5}\right)} = -6.25 \times 10^{-3}$$

$\therefore M = 6.25 \text{ mH}$

- 4.5 An a.c. generator consists of a coil of 10,000 turns and of area 100 cm^2 . The coil rotates at an angular speed of 140 rpm in a uniform magnetic field of $3.6 \times 10^{-2} \text{ T}$. Find the maximum value of the emf induced.

Data : $N = 10,000 \quad A = 10^2 \text{ cm}^2 = 10^{-2} \text{ m}^2,$

$v = 140 \text{ rpm} = \frac{140}{60} \text{ rps}, \quad B = 3.6 \times 10^{-2} \text{ T} \quad E_o = ?$

Solution : $E_o = NAB\omega = NAB 2\pi v$

$$= 10^4 \times 10^{-2} \times 3.6 \times 10^{-2} \times 2 \pi \times \frac{7}{3}$$

$E_o = 52.75 \text{ V}$

- 4.6 Write the equation of a 25 cycle current sine wave having rms value of 30 A.

Data : $v = 25 \text{ Hz}, \quad I_{\text{rms}} = 30 \text{ A}$

Solution : $i = I_o \sin \omega t$

$$= I_{\text{rms}} \sqrt{2} \sin 2\pi vt$$

$$i = 30 \sqrt{2} \sin 2\pi \times 25 t$$

$$i = 42.42 \sin 157 t$$

- 4.7 A capacitor of capacitance $2 \mu\text{F}$ is in an a.c. circuit of frequency 1000 Hz. If the rms value of the applied emf is 10 V, find the effective current flowing in the circuit.

Data : $C = 2\mu\text{F}$, $\nu = 1000 \text{ Hz}$, $E_{\text{eff}} = 10\text{V}$

Solution : $X_c = \frac{1}{C\omega} = \frac{1}{C \times 2\pi\nu}$

$$X_c = \frac{1}{2 \times 10^{-6} \times 2\pi \times 10^3} = 79.6 \Omega$$

$$I_{\text{rms}} = \frac{E_{\text{eff}}}{X_c} = \frac{10}{79.6}$$

$$\therefore I_{\text{rms}} = 0.126 \text{ A}$$

4.8 A coil is connected across 250 V, 50 Hz power supply and it draws a current of 2.5 A and consumes power of 400 W. Find the self inductance and power factor.

Data : $E_{\text{rms}} = 250 \text{ V}$, $\nu = 50 \text{ Hz}$; $I_{\text{rms}} = 2.5\text{A}$; $P = 400 \text{ W}$; $L = ?$, $\cos \phi = ?$

Solution : Power $P = E_{\text{rms}} I_{\text{rms}} \cos \phi$

$$\therefore \cos \phi = \frac{P}{E_{\text{rms}} I_{\text{rms}}}$$

$$= \frac{400}{250 \times 2.5}$$

$$\cos \phi = 0.64$$

$$\text{Impedance } Z = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{250}{2.5} = 100 \Omega$$

From the phasor diagram

$$\sin \phi = \frac{X_L}{Z}$$

$$\begin{aligned} \therefore X_L &= Z \cdot \sin \phi = Z \sqrt{1 - \cos^2 \phi} \\ &= 100 \sqrt{1 - (0.64)^2} \end{aligned}$$

$$\therefore X_L = 76.8 \Omega$$

But $X_L = L \omega = L 2 \pi \nu$

$$\therefore L = \frac{X_L}{2\pi\nu} = \frac{76.8}{2\pi \times 50}$$

$$\therefore L = 0.244 \text{ H}$$

- 4.9 A bulb connected to 50 V, DC consumes 20 w power. Then the bulb is connected to a capacitor in an a.c. power supply of 250 V, 50 Hz. Find the value of the capacitor required so that the bulb draws the same amount of current.

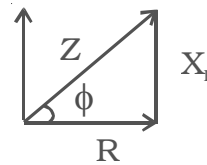
Data : P = 20 W; V = 50 V; ν = 50 Hz; C = ?

Solution : P = VI

$$\therefore I = \frac{P}{V} = \frac{20}{50} = 0.4 \text{ A}$$

$$\therefore \text{Resistance, } R = \frac{V}{I} = \frac{50}{0.4} = 125 \Omega$$

The impedance, $Z = \frac{V}{I} = \frac{250}{0.4} = 625\Omega$



$$\therefore Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{2\pi\nu C}\right)^2}$$

$$Z^2 = R^2 + \frac{1}{4\pi^2\nu^2 C^2}$$

$$C = \frac{1}{2\pi\nu\sqrt{Z^2 - R^2}}$$

$$= \frac{1}{2\pi \times 50 \sqrt{(625)^2 - (125)^2}} = \frac{1}{2\pi \times 50 \times 612.37}$$

$$C = 5.198 \mu\text{F}$$

- 4.10 An AC voltage represented by $e = 310 \sin 314 t$ is connected in series to a 24Ω resistor, 0.1 H inductor and a $25 \mu\text{F}$ capacitor. Find the value of the peak voltage, rms voltage, frequency, reactance of the circuit, impedance of the circuit and phase angle of the current.

Data : R = 24Ω , L = 0.1 H , C = $25 \times 10^{-6}\text{F}$

Solution : $e = 310 \sin 314 t$... (1)

and $e = E_0 \sin \omega t$... (2)

comparing equations (1) & (2)

$$E_0 = 310 \text{ V}$$

$$E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{310}{\sqrt{2}} = 219.2 \text{ V}$$

$$\omega t = 314 \quad t$$

$$2\pi v = 314$$

$$v = \frac{314}{2 \times 3.14} = 50 \text{ Hz}$$

$$\begin{aligned} \text{Reactance} &= X_L - X_C = L \omega - \frac{1}{C\omega} = L.2\pi v - \frac{1}{C.2\pi v} \\ &= 0.1 \times 2 \pi \times 50 - \frac{1}{25 \times 10^{-6} \times 2\pi \times 50} \\ &= 31.4 - 127.4 = -96 \Omega \end{aligned}$$

$$X_L - X_C = -96 \Omega$$

$$\therefore X_C - X_L = 96 \Omega$$

$$\begin{aligned} Z &= \sqrt{R^2 + (X_C - X_L)^2} \\ &= \sqrt{24^2 + 96^2} \\ &= \sqrt{576 + 9216} \\ &= 98.9 \Omega \end{aligned}$$

$$\tan \phi = \frac{X_C - X_L}{R}$$

$$= \left(\frac{127.4 - 31.4}{24} \right)$$

$$\tan \phi = \frac{96}{24} = 4$$

$$\phi = 76^\circ$$

Predominance of capacitive reactance signify that current leads the emf by 76°

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

- 4.1 Electromagnetic induction is not used in
(a) transformer (b) room heater
(c) AC generator (d) choke coil
- 4.2 A coil of area of cross section 0.5 m^2 with 10 turns is in a plane which is perpendicular to an uniform magnetic field of 0.2 Wb/m^2 . The flux through the coil is
(a) 100 Wb (b) 10 Wb
(c) 1 Wb (d) zero
- 4.3 Lenz's law is in accordance with the law of
(a) conservation of charges (b) conservation of flux
(c) conservation of momentum (d) conservation of energy
- 4.4 The self-inductance of a straight conductor is
(a) zero (b) infinity
(c) very large (d) very small
- 4.5 The unit henry can also be written as
(a) Vs A^{-1} (b) Wb A^{-1}
(c) $\Omega \text{ s}$ (d) all
- 4.6 An emf of 12 V is induced when the current in the coil changes at the rate of 40 A S^{-1} . The coefficient of self induction of the coil is
(a) 0.3 H (b) 0.003 H
(c) 30 H (d) 4.8 H
- 4.7 A DC of 5A produces the same heating effect as an AC of
(a) 50 A rms current (b) 5 A peak current
(c) 5A rms current (d) none of these
- 4.8 Transformer works on
(a) AC only (b) DC only
(c) both AC and DC (d) AC more effectively than DC

- 4.9 The part of the AC generator that passes the current from the coil to the external circuit is
- (a) field magnet (b) split rings
(c) slip rings (d) brushes
- 4.10 In an AC circuit the applied emf $e = E_0 \sin(\omega t + \pi/2)$ leads the current $I = I_0 \sin(\omega t - \pi/2)$ by
- (a) $\pi/2$ (b) $\pi/4$
(c) π (d) 0
- 4.11 Which of the following cannot be stepped up in a transformer?
- (a) input current (b) input voltage
(c) input power (d) all
- 4.12 The power loss is less in transmission lines when
- (a) voltage is less but current is more
(b) both voltage and current are more
(c) voltage is more but current is less
(d) both voltage and current are less
- 4.13 Which of the following devices does not allow d.c. to pass through?
- (a) resistor (b) capacitor
(c) inductor (d) all the above
- 4.14 In an ac circuit
- (a) the average value of current is zero.
(b) the average value of square of current is zero.
(c) the average power dissipation is zero.
(d) the rms current is $\sqrt{2}$ time of peak current.
- 4.15 What is electromagnetic induction?
- 4.16 State Faraday's laws of electromagnetic induction.
- 4.17 Define self-inductance. Give its unit
- 4.18 Define the unit of self-inductance.
- 4.19 Define coefficient of mutual induction.
- 4.20 Give the practical application of self-induction.
- 4.21 State Fleming's right hand rule.

- 4.22 Define rms value of a.c.
- 4.23 State the methods of producing induced emf.
- 4.24 What is a poly phase AC generator?
- 4.25 What is inductive reactance?
- 4.26 Define alternating current and give its expression.
- 4.27 What is capacitive reactance?
- 4.28 Mention the difference between a step up and step down transformer.
- 4.29 What is resonant frequency in LCR circuit?
- 4.30 Define power factor.
- 4.31 Why a d.c ammeter cannot read a.c?
- 4.32 Obtain an expression for the rms value of a.c.
- 4.33 Define quality factor.
- 4.34 A capacitor blocks d.c but allows a.c. Explain.
- 4.35 What happens to the value of current in RLC series circuit, if frequency of the source is increased?
- 4.36 State Lenz's law and illustrate through an experiment. Explain how it is in accordance with the law of conservation of energy.
- 4.37 Differentiate between self-inductance and mutual inductance.
- 4.38 Obtain an expression for the self-inductance of a long solenoid.
- 4.39 Explain the mutual induction between two long solenoids. Obtain an expression for the mutual inductance.
- 4.40 Explain how an emf can be induced by changing the area enclosed by the coil.
- 4.41 Discuss with theory the method of inducing emf in a coil by changing its orientation with respect to the direction of the magnetic field.
- 4.42 What are eddy currents? Give their applications. How are they minimised?
- 4.43 Explain how power can be transmitted efficiently to long distance.
- 4.44 Obtain an expression for the current flowing in a circuit containing resistance only to which alternating emf is applied. Find the phase relationship between voltage and current.

- 4.45 Obtain an expression for the current in an ac circuit containing a pure inductance. Find the phase relationship between voltage and current.
- 4.46 Obtain an expression for the current flowing in the circuit containing capacitance only to which an alternating emf is applied. Find the phase relationship between the current and voltage.
- 4.47 Derive an expression for the average power in an ac circuit.
- 4.48 Describe the principle, construction and working of a choke coil.
- 4.49 Discuss the advantages and disadvantages of a.c. over dc.
- 4.50 Describe the principle, construction and working of a single – phase a.c generator.
- 4.51 Describe the principle, construction and working of three–phase a.c generator.
- 4.52 Explain the principle of transformer. Discuss its construction and working.
- 4.53 A source of alternating emf is connected to a series combination of a resistor R an inductor L and a capacitor C. Obtain with the help of a vector diagram and impedance diagram, an expression for (i) the effective voltage (ii) the impedance (iii) the phase relationship between the current and the voltage.

Problems

- 4.54 A coil of 100 turns and resistance 100Ω is connected in series with a galvanometer of resistance 100Ω and the coil is placed in a magnetic field. If the magnetic flux linked with the coil changes from 10^{-3} Wb to $2 \times 10^{-4} \text{ Wb}$ in a time of 0.1 s, calculate the induced emf and current.
- 4.55 Two rails of a railway track insulated from each other and the ground are connected to a millivoltmeter. The train runs at a speed of 180 Km/hr. Vertical component of earth's magnetic field is $0.2 \times 10^{-4} \text{ Wb/m}^2$ and the rails are separated by 1m. Find the reading of the voltmeter.
- 4.56 Air core solenoid having a diameter of 4 cm and length 60 cm is wound with 4000 turns. If a current of 5A flows in the solenoid, calculate the energy stored in the solenoid.

- 4.57 An iron cylinder 5cm in diameter and 100cm long is wound with 3000 turns in a single layer. The second layer of 100 turns of much finer wire is wound over the first layer near its centre. Calculate the mutual inductance between the coils (relative permeability of the core = 500).
- 4.58 A student connects a long air core coil of manganin wire to a 100V DC source and records a current of 1.5A. When the same coil is connected across 100V, 50 Hz a.c. source, the current reduces to 1 A. Calculate the value of reactance and inductance of the coil.
- 4.59 An emf $e = 100 \sin 200 \pi t$ is connected to a circuit containing a capacitance of $0.1 \mu\text{F}$ and resistance of 500Ω in series. Find the power factor of the circuit.
- 4.60 The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1100 V is 12.1 KW, calculate the primary voltage. If the resistance of primary is 0.2Ω and that of secondary is 2Ω and the efficiency of the transformer is 90% calculate
- heat loss in the primary coil
 - heat loss in the secondary coil
- 4.61 A resistance of 50Ω , an inductance of 0.5 H and a capacitance of $5 \mu\text{F}$ are connected in series with an a.c. supply of $e = 311 \sin (314t)$. Find (i) frequency of a.c. supply (ii) maximum voltage (iii) inductive reactance (iv) capacitive reactance (v) impedance.
- 4.62 A radio can tune over the frequency range of a portion of broadcast band (800 KHz to 1200 KHz). If its LC circuit has an effective inductance of $200 \mu\text{H}$, what must be the range of its variable capacitance?
- 4.63 A transformer has an efficiency of 80%. It is connected to a power input of at 4 KW and 100 V. If the secondary voltage is 240 V. Calculate the primary and secondary currents.
- 4.64 An electric lamp which works at 80 volt and 10 A D.C. is connected to 100 V, 50 Hz alternating current. Calculate the inductance of the choke required so that the bulb draws the same current of 10 A.

Answers

- 4.1** (b) **4.2** (c) **4.3** (d) **4.4** (a) **4.5** (d)
4.6 (a) **4.7** (c) **4.8** (a) **4.9** (d) **4.10** (c)
4.11 (c) **4.12**(c) **4.13** (b) **4.14** (a)
4.54 0.8 V and 4 mA **4.55** 1 mV
4.56 0.52575 joule **4.57** 0.37 H
4.58 74.54 Ω and 0.237 H **4.59** 0.0314
4.60 220V, (i) 747 W (ii) 242 W
4.61 (i) 50 Hz (ii) 311 V (iii) 157 Ω (iv) 636.9 Ω (v) 482.5 Ω
4.62 87.9 pF to 198 pF
4.63 40 A, 13.3 A
4.64 0.019 H

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5. Electromagnetic Waves and Wave optics

The phenomenon of Faraday's electromagnetic induction concludes that a changing magnetic field at a point with time produces an electric field at that point. Maxwell in 1865, pointed out that there is a symmetry in nature (i.e) changing electric field with time at a point produces a magnetic field at that point. It means that a change in one field with time (either electric or magnetic) produces another field. This idea led Maxwell to conclude that the variation in electric and magnetic fields perpendicular to each other, produces electromagnetic disturbances in space. These disturbances have the properties of a wave and propagate through space without any material medium. These waves are called electromagnetic waves.

5.1.1 Electromagnetic waves

According to Maxwell, an accelerated charge is a source of electromagnetic radiation.

In an electromagnetic wave, electric and magnetic field vectors are at right angles to each other and both are at right angles to the direction of propagation. They possess the wave character and propagate through free space without any material medium. These waves are transverse in nature.

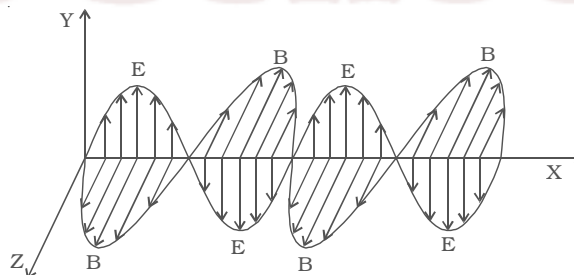


Fig 5.1 Electromagnetic waves.

Fig 5.1 shows the variation of electric field \vec{E} along Y direction and magnetic field \vec{B} along Z direction and wave propagation in + X direction.

5.1.2 Characteristics of electromagnetic waves

- (i) Electromagnetic waves are produced by accelerated charges.
- (ii) They do not require any material medium for propagation.
- (iii) In an electromagnetic wave, the electric (\vec{E}) and magnetic (\vec{B}) field vectors are at right angles to each other and to the direction of propagation. Hence electromagnetic waves are transverse in nature.
- (iv) Variation of maxima and minima in both \vec{E} and \vec{B} occur simultaneously.
- (v) They travel in vacuum or free space with a velocity $3 \times 10^8 \text{ m s}^{-1}$ given by the relation $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.
(μ_0 - permeability of free space and ϵ_0 - permittivity of free space)
- (vi) The energy in an electromagnetic wave is equally divided between electric and magnetic field vectors.
- (vii) The electromagnetic waves being chargeless, are not deflected by electric and magnetic fields.

5.1.3 Hertz experiment

The existence of electromagnetic waves was confirmed experimentally by Hertz in 1888. This experiment is based on the fact that an oscillating electric charge radiates electromagnetic waves. The energy of these waves is due to the kinetic energy of the oscillating charge.

The experimental arrangement is as shown in Fig 5.2. It consists of two metal plates A and B placed at a distance of 60 cm from each other. The metal plates are connected to two polished metal spheres S_1 and S_2 by means of thick copper wires. Using an induction coil a high potential difference is applied across the small gap between the spheres.

Due to high potential difference across S_1 and S_2 , the air in the small gap between the spheres gets ionized and provides a path for the discharge of the plates. A spark is produced between

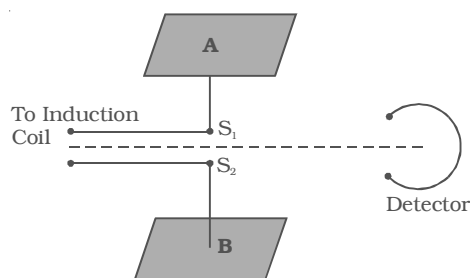


Fig 5.2 Hertz experiment

S_1 and S_2 and electromagnetic waves of high frequency are radiated. Hertz was able to produce electromagnetic waves of frequency about 5×10^7 Hz.

Here the plates A and B act as a capacitor having small capacitance value C and the connecting wires provide low inductance L . The high frequency oscillation of charges between the plates is given by $\nu = \frac{1}{2\pi\sqrt{LC}}$

5.1.4 Electromagnetic Spectrum

After the demonstration of electromagnetic waves by Hertz, electromagnetic waves in different regions of wavelength were produced by different ways of excitation.

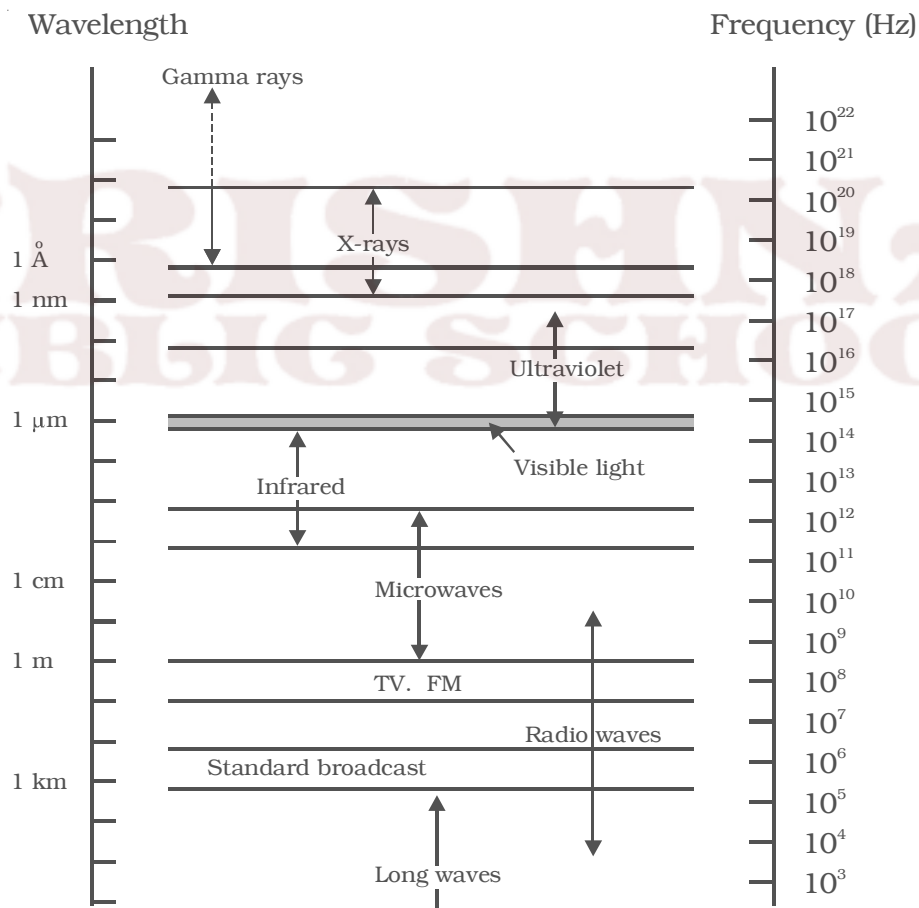


Fig 5.3 Electromagnetic spectrum

The orderly distribution of electromagnetic waves according to their wavelength or frequency is called the electromagnetic spectrum.

Electromagnetic spectrum covers a wide range of wavelengths (or) frequencies. The whole electromagnetic spectrum has been classified into different parts and sub parts, in order of increasing wavelength and type of excitation. All electromagnetic waves travel with the velocity of light. The physical properties of electromagnetic waves are determined by their wavelength and not by their method of excitation. The overlapping in certain parts of the spectrum shows that the particular wave can be produced by different methods.

Table 5.1 shows various regions of electromagnetic spectrum with source, wavelength and frequency ranges of different electromagnetic waves.

Table 5.1
(NOT FOR EXAMINATION)

Sl.No.	Name	Source	Wavelength range (m)	Frequency range (Hz)
1.	γ - rays	Radioactive nuclei, nuclear reactions	$10^{-14} - 10^{-10}$	$3 \times 10^{22} - 3 \times 10^{18}$
2.	x - rays	High energy electrons suddenly stopped by a metal target	$1 \times 10^{-10} - 3 \times 10^{-8}$	$3 \times 10^{18} - 1 \times 10^{16}$
3.	Ultra-violet (UV)	Atoms and molecules in an electrical discharge	$6 \times 10^{-10} - 4 \times 10^{-7}$	$5 \times 10^{17} - 8 \times 10^{14}$
4.	Visible light	incandescent solids Fluorescent lamps	$4 \times 10^{-7} - 8 \times 10^{-7}$	$8 \times 10^{14} - 4 \times 10^{14}$
5.	Infra-red (IR)	molecules of hot bodies	$8 \times 10^{-7} - 3 \times 10^{-5}$	$4 \times 10^{14} - 1 \times 10^{13}$
6.	Microwaves	Electronic device (Vacuum tube)	$10^{-3} - 0.3$	$3 \times 10^{11} - 1 \times 10^9$
7.	Radio frequency waves	charges accelerated through conducting wires	$10 - 10^4$	$3 \times 10^7 - 3 \times 10^4$

5.1.5 Uses of electromagnetic spectrum

The following are some of the uses of electromagnetic waves.

1. Radio waves : These waves are used in radio and television communication systems. AM band is from 530 kHz to 1710 kHz. Higher frequencies upto 54 MHz are used for short waves bands.

Television waves range from 54 MHz to 890 MHz. FM band is from 88 MHz to 108 MHz. Cellular phones use radio waves in ultra high frequency (UHF) band.

2. Microwaves : Due to their short wavelengths, they are used in radar communication system. Microwave ovens are an interesting domestic application of these waves.

3. Infra red waves :

(i) Infrared lamps are used in physiotherapy.

(ii) Infrared photographs are used in weather forecasting.

(iii) As infrared radiations are not absorbed by air, thick fog, mist etc, they are used to take photograph of long distance objects.

(iv) Infra red absorption spectrum is used to study the molecular structure.

4. Visible light : Visible light emitted or reflected from objects around us provides information about the world. The wavelength range of visible light is 4000 Å to 8000 Å.

5. Ultra-violet radiations

(i) They are used to destroy the bacteria and for sterilizing surgical instruments.

(ii) These radiations are used in detection of forged documents, finger prints in forensic laboratories.

(iii) They are used to preserve the food items.

(iv) They help to find the structure of atoms.

6. X rays :

(i) X rays are used as a diagnostic tool in medicine.

(ii) It is used to study the crystal structure in solids.

7. γ -rays : Study of γ rays gives useful information about the nuclear structure and it is used for treatment of cancer.

5.2 Types of spectra

When white light falls on a prism, placed in a spectrometer, the waves of different wavelengths are deviated to different directions by the prism. The image obtained in the field of view of the telescope consists of a number of coloured images of the slit. Such an image is called a spectrum.

If the slit is illuminated with light from sodium vapour lamp, two images of the slit are obtained in the yellow region of the spectrum. These images are the emission lines of sodium having wave lengths 5896\AA and 5890\AA . This is known as spectrum of sodium.

The spectra obtained from different bodies can be classified into two types (i) emission spectra and (ii) absorption spectra.

(i) Emission spectra

When the light emitted directly from a source is examined with a spectrometer, the emission spectrum is obtained. Every source has its own characteristic emission spectrum.

The emission spectrum is of three types.

1. Continuous spectrum 2. Line spectrum and 3. Band spectrum

1. Continuous spectrum

It consists of unbroken luminous bands of all wavelengths containing all the colours from violet to red. These spectra depend only on the temperature of the source and is independent of the characteristic of the source.

Incandescent solids, liquids, Carbon arc, electric filament lamps etc, give continuous spectra.

2. Line spectrum

Line spectra are sharp lines of definite wavelengths. It is the characteristic of the emitting substance. It is used to identify the gas.

Atoms in the gaseous state, i.e. free excited atoms emit line spectrum. The substance in atomic state

	H_α	H_β	H_γ	H_δ
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Fig 5.4 Line spectrum of hydrogen

such as sodium in sodium vapour lamp, mercury in mercury vapour lamp and gases in discharge tube give line spectra (Fig. 5.4).

3. Band Spectrum

It consists of a number of bright bands with a sharp edge at one end but fading out at the other end.

Band spectra are obtained from molecules. It is the characteristic of the molecule. Calcium or Barium salts in a bunsen flame and gases like carbon-di-oxide, ammonia and nitrogen in molecular state in the discharge tube give band spectra. When the bands are examined with high resolving power spectrometer, each band is found to be made of a large number of fine lines, very close to each other at the sharp edge but spaced out at the other end. Using band spectra the molecular structure of the substance can be studied.

(ii) Absorption Spectra

When the light emitted from a source is made to pass through an absorbing material and then examined with a spectrometer, the obtained spectrum is called absorption spectrum. It is the characteristic of the absorbing substance.

Absorption spectra is also of three types

1. continuous absorption spectrum
2. line absorption spectrum and
3. band absorption spectrum

1. Continuous absorption spectrum

A pure green glass plate when placed in the path of white light, absorbs everything except green and gives continuous absorption spectrum.

2. Line absorption spectrum



Fig 5.5 Emission and absorption spectrum of sodium

When light from the carbon arc is made to pass through sodium vapour and then examined by a spectrometer, a continuous spectrum of carbon arc with two dark lines in the yellow region is obtained as shown in Fig.5.5.

3. Band absorption spectrum

If white light is allowed to pass through iodine vapour or dilute solution of blood or chlorophyll or through certain solutions of organic

and inorganic compounds, dark bands on continuous bright background are obtained. The band absorption spectra are used for making dyes.

5.2.1 Fraunhofer lines

If the solar spectrum is closely examined, it is found that it consists of large number of dark lines. These dark lines in the solar spectrum are called Fraunhofer lines. Solar spectrum is an example of line absorption spectrum.

The central core of the sun is called photosphere which is at a very high temperature of the order of 14 million kelvin. It emits continuous spectrum. The sun's outer layer is called chromosphere. This is at a comparatively lower temperature at about 6000 K. It contains various elements in gaseous state.

When light from the central core of the sun passes through sun's atmosphere, certain wavelengths are absorbed by the elements present in the chromosphere and the spectrum is marked by dark lines.

By comparing the absorption spectra of various substances with the Fraunhofer lines in the solar spectrum, the elements present in the sun's atmosphere have been identified.

5.2.2 Fluorescence

When an atomic or molecular system is excited into higher energy state by absorption of energy, it returns back to lower energy state in a time less than 10^{-5} second and the system is found to glow brightly by emitting radiation of longer wavelength.

When ultra violet light is incident on certain substances, they emit visible light.

It may be noted that fluorescence exists as long as the fluorescing substance remain exposed to incident ultraviolet light and re-emission of light stops as soon as incident light is cut off.

5.2.3 Phosphorescence

There are some substances in which the molecules are excited by the absorption of incident ultraviolet light, and they do not return immediately to their original state. The emission of light continues even after the exciting radiation is removed. This type of delayed fluorescence is called phosphorescence.

5.3 Theories of light

Any theory regarding propagation of light must explain the properties of light. Since, light is a form of energy, it is transferred from one place to another. Light does not require a material medium for its propagation.

In general, there are two possible modes of propagation of energy from one place to another (i) by stream of material particles moving with a finite velocity (ii) by wave motion, wherein the matter through which the wave propagates does not move along the direction of the wave. The various theories of light put forward by famous physicists are given below.

5.3.1 Corpuscular theory

According to Newton, a source of light or a luminous body continuously emits tiny, massless (negligibly small mass) and perfectly elastic particles called corpuscles. They travel in straight lines in a homogeneous medium in all directions with the speed of light.

The corpuscles are so small that a luminous body does not suffer any appreciable loss of mass even if it emits light for a long time.

Light energy is the kinetic energy of the corpuscles. The sense of vision is produced, when the corpuscles impinge on the retina of the eye. The sensation of different colours was due to different sizes of the corpuscles. On account of high speed, they are unaffected by the force of gravity and their path is a straight line. When the corpuscles approach a surface between two media, they are either attracted or repelled. Reflection of the particles is due to repulsion and refraction is due to attraction.

According to this theory, the velocity of light in the denser medium is greater than the velocity of light in rarer medium. But the experimental results of Foucault and Michelson showed that velocity of light in a denser medium is lesser than that in a rarer medium. Further, this theory could not explain the phenomena of interference, diffraction and polarisation.

5.3.2 Wave theory

According to Huygens, light is propagated in the form of waves, through a continuous medium. Huygens assumed the existence of an invisible, elastic medium called ether, which pervades all space. The

disturbance from the source is propagated in the form of waves through space and the energy is distributed equally in all directions. Huygens assumed these waves to be longitudinal. Initially rectilinear propagation of light could not be explained. But the difficulty was overcome when Fresnel and Young suggested that light waves are transverse. The wave theory could satisfactorily explain all the basic properties, which were earlier proved by corpuscular theory and in addition, it explains the phenomena of interference, diffraction and polarisation.

According to Huygens, the velocity of light in a denser medium is lesser than that in a rarer medium. This is in accordance with the experimental result of Foucault.

5.3.3 Electromagnetic theory

Maxwell showed that light was an electromagnetic wave, conveying electromagnetic energy and not mechanical energy as believed by Huygens, Fresnel and others. He showed that the variation of electric and magnetic intensities had precisely the same characteristics as a transverse wave motion. He also showed that no medium was necessary for the propagation of electromagnetic waves.

5.3.4 Quantum theory

The electromagnetic theory, however failed to account for the phenomenon of photo electric effect. In 1900, Planck had suggested that energy was emitted and absorbed, not continuously but in multiples of discrete packets of energy called Quantum which could not be subdivided into smaller parts. In 1905, Einstein extended this idea and suggested that light waves consist of small packets of energy called

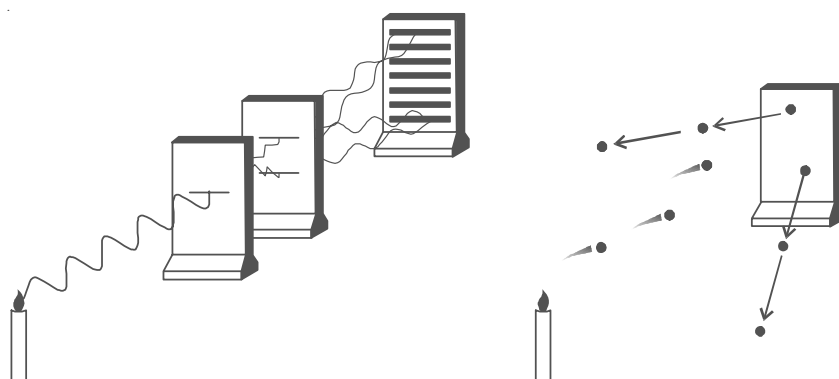


Fig 5.6 Wave and Quantum nature

photons. The energy associated with each photon is $E = h\nu$, where h is Planck's constant ($h = 6.626 \times 10^{-34}$ J s) and ν is the frequency of the electromagnetic radiation.

It is now established that photon seems to have a dual character. It behaves as particles in the region of higher energy and as waves in the region of lower energy (Fig. 5.6).

5.4 Scattering of light

Lord Rayleigh was the first to deal with scattering of light by air molecules. The scattering of sunlight by the molecules of the gases in Earth's atmosphere is called Rayleigh scattering.

The basic process in scattering is absorption of light by the molecules followed by its re-radiation in different directions. The strength of scattering depends on the wavelength of the light and also the size of the particle which cause scattering.

The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as Rayleigh scattering law. Hence, the shorter wavelengths are scattered much more than the longer wavelengths. The blue appearance of sky is due to scattering of sunlight by the atmosphere. According to Rayleigh's scattering law, blue light is scattered to a greater extent than red light. This scattered radiation causes the sky to appear blue.

At sunrise and sunset the rays from the sun have to travel a larger part of the atmosphere than at noon. Therefore most of the blue light is scattered away and only the red light which is least scattered reaches the observer. Hence, sun appears reddish at sunrise and sunset.

5.4.1 Tyndal scattering

When light passes through a colloidal solution its path is visible inside the solution. This is because, the light is scattered by the particles of solution. The scattering of light by the colloidal particles is called Tyndal scattering.

5.4.2 Raman effect

In 1928, Sir C.V. Raman discovered experimentally, that the monochromatic light is scattered when it is allowed to pass through a substance. The scattered light contains some additional frequencies

other than that of incident frequency. This is known as Raman effect.

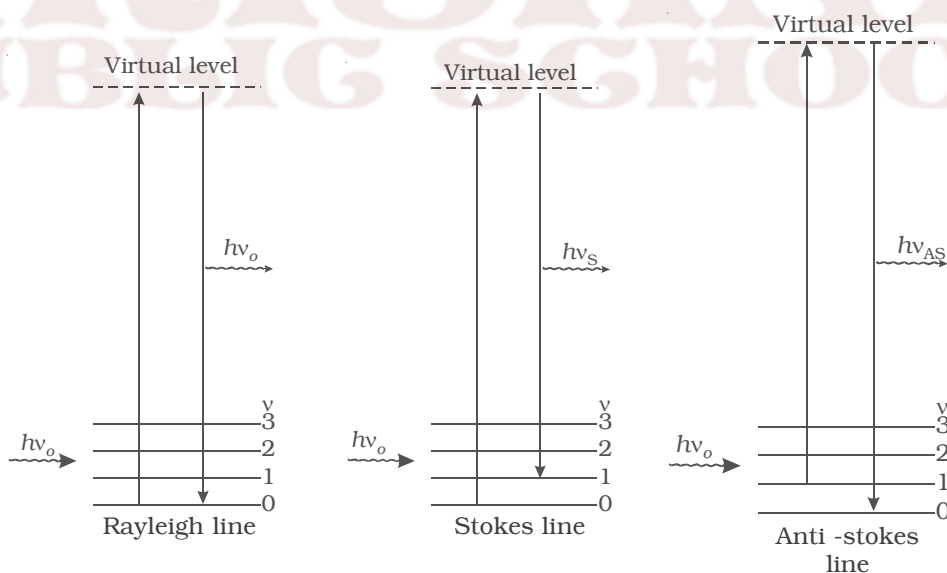
The lines whose frequencies have been modified in Raman effect are called Raman lines. The lines having frequencies lower than the incident frequency are called Stoke's lines and the lines having frequencies higher than the incident frequency are called Anti-stokes lines. This series of lines in the scattering of light by the atoms and molecules is known as Raman Spectrum.

The Raman effect can be easily understood, by considering the scattering of photon of the incident light with the atoms or molecules. Let the incident light consist of photons of energy $h\nu_o$.

1. If a photon strikes an atom or a molecule in a liquid, part of the energy of the incident photon may be used to excite the atom of the liquid and the rest is scattered. The spectral line will have lower frequency and it is called stokes line.

2. If a photon strikes an atom or a molecule in a liquid, which is in an excited state, the scattered photon gains energy. The spectral line will have higher frequency and it is called Anti-stoke's line.

3. In some cases, when a light photon strikes atoms or molecules, photons may be scattered elastically. Then the photons neither gain nor



($v = 0, 1, 2 \dots$ are the vibration levels of the ground electronic state.)

Fig 5.7 Raman Spectrum

lose energy. The spectral line will have unmodified frequency.

If ν_0 is the frequency of incident radiation and ν_s the frequency of scattered radiation of a given molecular sample, then Raman Shift or Raman frequency $\Delta\nu$ is given by the relation $\Delta\nu = \nu_0 - \nu_s$.

The Raman shift does not depend upon the frequency of the incident light but it is the characteristic of the substance producing Raman effect. For Stoke's lines, $\Delta\nu$ is positive and for Anti-stoke's lines $\Delta\nu$ is negative.

The intensity of Stoke's line is always greater than the corresponding Anti-stoke's Line. The different processes giving rise to Rayleigh, Stoke's and Anti-stokes lines are shown in Fig 5.7.

When a system interacts with a radiation of frequency ν_0 , it may make an upward transition to a virtual state. A virtual state is not one of the stationary states of the molecule. Most of the molecules of the system return back to the original state from the virtual state which corresponds to Rayleigh scattering. A small fraction may return to states of higher and lower energy giving rise to Stoke's line and Anti-stoke's line respectively.

5.4.3 Applications of Raman Spectrum

- (i) It is widely used in almost all branches of science.
- (ii) Raman Spectra of different substances enable to classify them according to their molecular structure.
- (iii) In industry, Raman Spectroscopy is being applied to study the properties of materials.
- (iv) It is used to analyse the chemical constitution.

5.5 Wave front

When a stone is dropped in a still water, waves spread out along the surface of water in all directions with same velocity. Every particle on the surface vibrates. At any instant, a photograph of the surface of water would show circular rings on which the disturbance is maximum (Fig. 5.8). It is clear that all the particles on such a circle are vibrating in phase, because these



Fig 5.8 Water waves

particles are at the same distance from the source. Such a surface which envelopes the particles that are in the same state of vibration is

known as a wave front. The wave front at any instant is defined as the locus of all the particles of the medium which are in the same state of vibration.

A point source of light at a finite distance in an isotropic medium* emits a spherical wave front (Fig 5.9a). A point source of light in an isotropic medium at infinite distance will give rise to plane wavefront (Fig. 5.9b). A linear source of light such as a slit illuminated by a lamp, will give rise to cylindrical wavefront (Fig 5.9c).

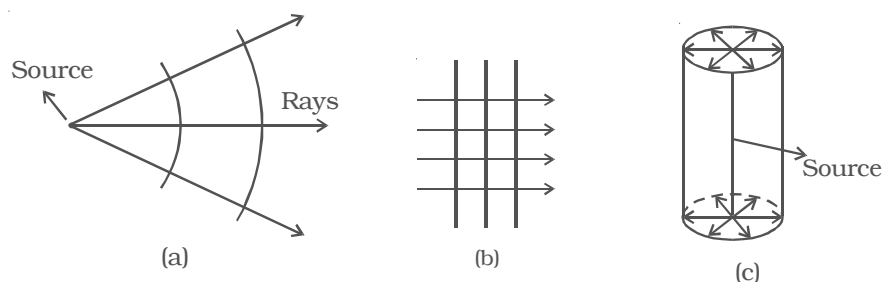


Fig 5.9 Wavefront

5.5.1 Huygen's principle

Huygen's principle helps us to locate the new position and shape of the wavefront at any instant, knowing its position and shape at any previous instant. In other words, it describes the progress of a wave front in a medium.

Huygen's principle states that, (i) every point on a given wave front may be considered as a source of secondary wavelets which spread out with the speed of light in that medium and (ii) the new wavefront is the forward envelope of the secondary wavelets at that instant.

Huygen's construction for a spherical and plane wavefront is shown in Fig. 5.10a. Let AB represent a given wavefront at a time $t = 0$. According to Huygen's principle, every point on AB acts as a source of secondary wavelets which travel with the speed of light c . To find the position of the wave front after a time t , circles are drawn with points P, Q, R ... etc as centres on AB and radii equal to ct . These are the traces of secondary wavelets. The arc A_1B_1 drawn as a forward envelope of the small circles is the new wavefront at that instant. If the source of light is at a large distance, we obtain a plane wave front $A_1 B_1$ as shown in Fig 5.10b.

* Isotropic medium is the medium in which the light travels with same speed in all directions.

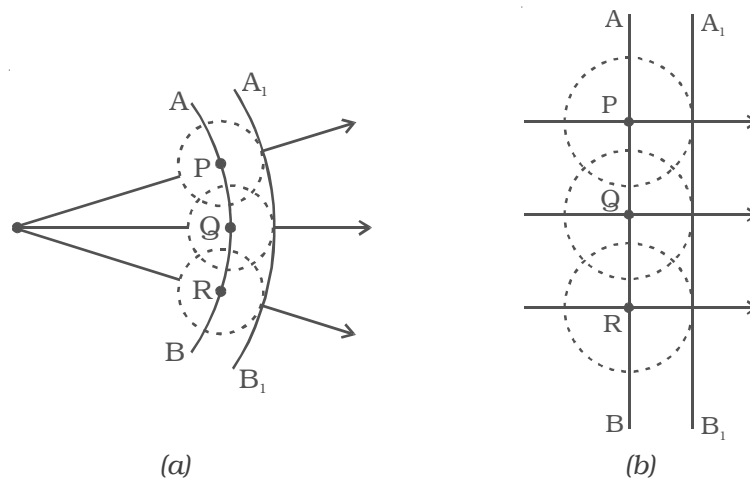


Fig 5.10 Huygen's principle

5.5.2 Reflection of a plane wave front at a plane surface

Let XY be a plane reflecting surface and AB be a plane wavefront incident on the surface at A. PA and QBC are perpendiculars drawn to AB at A and B respectively. Hence they represent incident rays. AN is the normal drawn to the surface. The wave front and the surface are perpendicular to the plane of the paper (Fig. 5.11).

According to Huygen's principle each point on the wavefront acts as the source of secondary wavelet. By the time, the secondary wavelets from B travel a distance BC, the secondary wavelets from A on the reflecting surface would travel the same distance BC after reflection. Taking A as centre and BC as radius an arc is drawn. From C a tangent CD is drawn to this arc. This tangent CD not only envelopes the wavelets from C and A but also the wavelets from all the points between C and A. Therefore CD is the reflected plane wavefront and AD is the reflected ray.

Laws of reflection

- (i) The incident wavefront AB, the reflected wavefront CD and the reflecting surface XY all lie in the same plane.
- (ii) Angle of incidence $i = \angle PAN = 90^\circ - \angle NAB = \angle BAC$
 Angle of reflection $r = \angle NAD = 90^\circ - \angle DAC = \angle DCA$
 In right angled triangles ABC and ADC

$\angle B = \angle D = 90^\circ$
 $BC = AD$ and AC is common
 \therefore The two triangles are congruent
 $\angle BAC = \angle DCA$
 i.e. $i = r$
 Thus the angle of incidence is equal to angle of reflection.

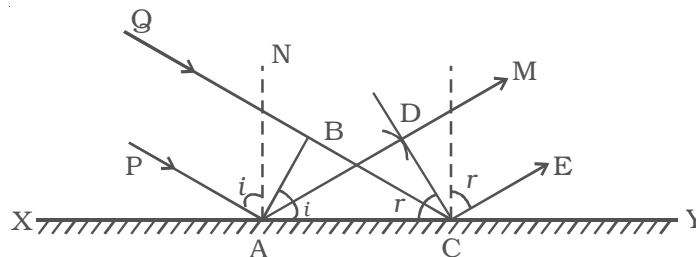


Fig 5.11 Reflection of a plane wavefront at a plane surface.

5.5.3 Refraction of a plane wavefront at a plane surface

Let XY be a plane refracting surface separating two media 1 and 2 of refractive indices μ_1 and μ_2 (Fig 5.12). The velocities of light in these two media are respectively c_1 and c_2 . Consider a plane wavefront AB incident on the refracting surface at A . PA and QBC are perpendiculars drawn to AB at A and B respectively. Hence they represent incident rays. NAN_1 is the normal drawn to the surface. The wave front and the surface are perpendicular to the plane of the paper.

According to Huygen's principle each point on the wave front act as the source of secondary wavelet. By the time, the secondary wavelets from B , reaches C , the secondary wavelets from the point A would travel a distance $AD = c_2 t$, where t is the time taken by the wavelets to travel the distance BC .

$\therefore BC = c_1 t$ and $AD = c_2 t = c_2 \frac{BC}{c_1}$. Taking A as centre and

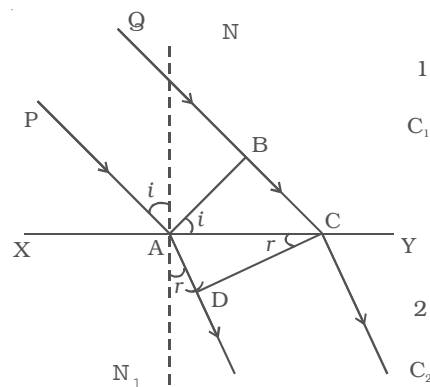


Fig 5.12 Refraction of a plane wavefront at the plane surface.

$C_2 \frac{BC}{C_1}$ as radius an arc is drawn in the second medium. From C a tangent CD is drawn to this arc. This tangent CD not only envelopes the wavelets from C and A but also the wavelets from all the points between C and A. Therefore CD is the refracted plane wavefront and AD is the refracted ray.

Laws of refraction

(i) The incident wave front AB, the refracted wave front CD and the refracting surface XY all lie in the same plane.

(ii) Angle of incidence $i = \angle PAN = 90^\circ - \angle NAB = \angle BAC$

Angle of refraction $r = \angle N_1AD = 90^\circ - \angle DAC = \angle ACD$

$$\frac{\sin i}{\sin r} = \frac{BC/AC}{AD/AC} = \frac{BC}{AD} = \frac{BC}{BC \cdot \frac{C_2}{C_1}} = \frac{C_1}{C_2} = \text{a constant} = {}_1\mu_2$$

${}_1\mu_2$ is called the refractive index of second medium with respect to first medium. This is Snell's law of refraction.

If ${}_1\mu_2 > 1$, the first medium is rarer and the second medium is denser. Then $\frac{C_1}{C_2} > 1$. This means that the velocity of light in rarer medium is greater than that in a denser medium. This conclusion from wave theory is in agreement with the result of Foucault's experiment.

It is clear from above discussions that the refractive index of a medium μ_m is given by

$$\mu_m = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in the medium}} = \frac{C_a}{C_m}$$

The frequency of a wave does not change when a wave is reflected or refracted from a surface, but wavelength changes on refraction.

$$\text{i.e. } \mu_m = \frac{C_a}{C_m} = \frac{v\lambda_a}{v\lambda_m} = \frac{\lambda_a}{\lambda_m}$$

$$\therefore \lambda_m = \frac{\lambda_a}{\mu_m}$$

where λ_a and λ_m are the wavelengths in air and medium respectively.

5.5.4 Total internal reflection by wave theory

Let XY be a plane surface which separates a rarer medium (air) and a denser medium. Let the velocity of the wavefront in these media be C_a and C_m respectively.

A plane wavefront AB passes from denser medium to rarer medium. It is incident on the surface with angle of incidence i . Let r be the angle of refraction.

$$\frac{\sin i}{\sin r} = \frac{(BC/AC)}{(AD/AC)} = \frac{BC}{AD} = \frac{c_m t}{c_a t} = \frac{c_m}{c_a}$$

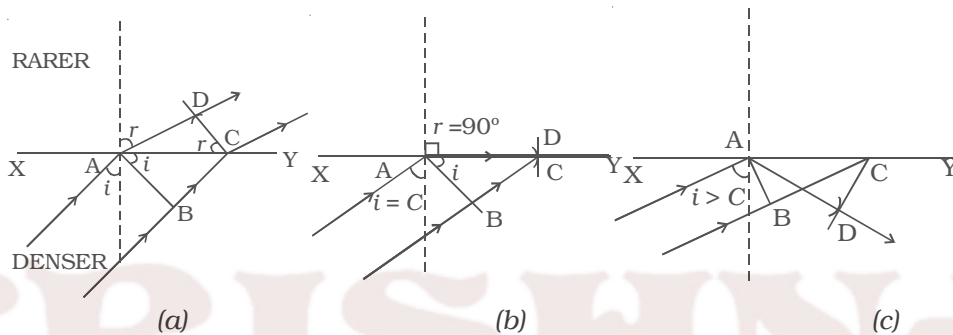


Fig 5.13 Total internal reflection

Since $\frac{c_m}{c_a} < 1$, i is less than r . This means that the refracted wavefront is deflected away from the surface XY.

In right angled triangle ADC, there are three possibilities (i) $AD < AC$ (ii) $AD = AC$ and (iii) $AD > AC$

(i) $AD < AC$: For small values of i , BC will be small and so $AD > BC$ but less than AC (Fig. 5.13a)

$$\sin r = \frac{AD}{AC}, \text{ which is less than unity}$$

$$\text{i.e. } r < 90^\circ$$

For each value of i , for which $r < 90^\circ$, a refracted wavefront is possible

(ii) $AD = AC$: As i increases r also increases. When $AD = AC$, $\sin r = 1$ (or) $r = 90^\circ$. i.e. a refracted wavefront is just possible (Fig. 5.13b). Now the refracted ray grazes the surface of separation of the two media. The angle of incidence at which the angle of refraction is 90° is called the critical angle C.

(iii) $AD > AC$: When $AD > AC$, $\sin r > 1$. This is not possible (Fig 5.13c). Therefore no refracted wave front is possible, when the angle of incidence increases beyond the critical angle. The incident wavefront is totally reflected into the denser medium itself. This is called total internal reflection.

Hence for total internal reflection to take place (i) light must travel from a denser medium to a rarer medium and (ii) the angle of incidence inside the denser medium must be greater than the critical angle. i.e $i > C$.

5.6 Superposition principle

When two or more waves simultaneously pass through the same medium, each wave acts on every particle of the medium, as if the other waves are not present. The resultant displacement of any particle is the vector addition of the displacements due to the individual waves.

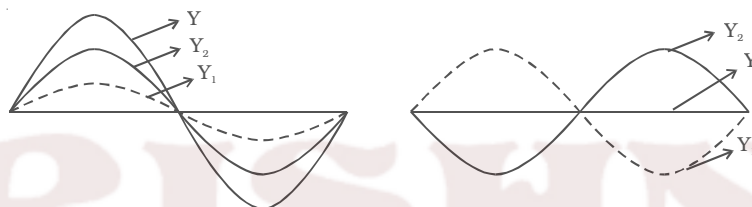


Fig 5.14 Superposition principle

This is known as principle of superposition. If \vec{Y}_1 and \vec{Y}_2 represent the individual displacement then the resultant displacement is given by

$$\vec{Y} = \vec{Y}_1 + \vec{Y}_2$$

5.6.1 Coherent sources

Two sources are said to be coherent if they emit light waves of the same wave length and start with same phase or have a constant phase difference.

Two independent monochromatic sources, emit waves of same wave length. But the waves are not in phase. So they are not coherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.

5.6.2 Phase difference and path difference

A wave of length λ corresponds to a phase of 2π . A distance of δ corresponds to a phase of $\phi = \frac{2\pi}{\lambda} \times \delta$

5.6.3 Interference of light

Two slits A and B illuminated by a single monochromatic source S act as coherent sources. The waves from these two coherent sources travel in the same medium and superpose at various points as shown in Fig. 5.15. The crest of the wavetrains are shown by thick continuous lines and

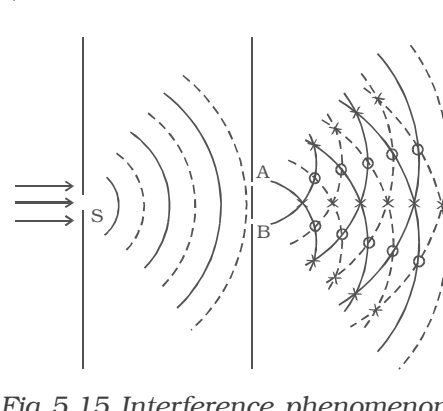


Fig 5.15 Interference phenomenon

troughs are shown by broken lines. At points where the crest of one wave meets the crest of the other wave or the trough of one wave meets the trough of the other wave, the waves are in phase, the displacement is maximum and these points appear bright. These points are marked by crosses (x). This type of interference is said to be constructive interference.

At points where the crest of one wave meets the trough of the other wave, the waves are in opposite phase, the displacement is minimum and these points appear dark. These points are marked by circles (O). This type of interference is said to be destructive interference. Therefore, on a screen XY the intensity of light will be alternatively maximum and minimum i.e. bright and dark bands which are referred as interference fringes. The redistribution of intensity of light on account of the superposition of two waves is called interference.

The intensity of light (I) at a point due to a wave of amplitude (a) is given by $I \propto a^2$.

If a_1 and a_2 are the amplitude of the two interfering waves, then

$$I_1 \propto a_1^2 \text{ and } I_2 \propto a_2^2$$

$$\therefore \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

For constructive interference, $I_{\max} \propto (a_1 + a_2)^2$ and for destructive interference, $I_{\min} \propto (a_1 - a_2)^2$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

5.6.4 Condition for sustained interference

The interference pattern in which the positions of maximum and minimum intensity of light remain fixed with time, is called sustained or permanent interference pattern. The conditions for the formation of sustained interference may be stated as :

- (i) The two sources should be coherent
- (ii) Two sources should be very narrow
- (iii) The sources should lie very close to each other to form distinct and broad fringes.

5.6.5 Young's double slit experiment

The phenomenon of interference was first observed and demonstrated by Thomas Young in 1801. The experimental set up is shown in Fig 5.16.

Light from a narrow slit S, illuminated by a monochromatic source, is allowed to fall on two narrow slits A and B placed very close to each other. The width of each slit is about 0.03 mm and they are about 0.3 mm apart. Since A and B are equidistant from S, light waves from S reach A and B in phase. So A and B act as coherent sources.

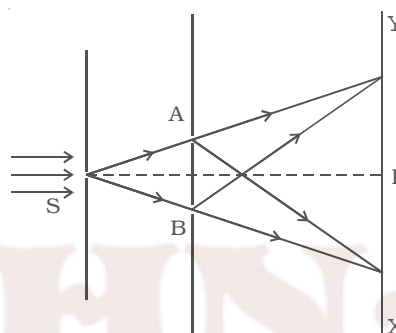


Fig 5.16 Young's double slit experiment

According to Huygen's principle, wavelets from A and B spread out and overlapping takes place to the right side of AB. When a screen XY is placed at a distance of about 1 metre from the slits, equally spaced alternate bright and dark fringes appear on the screen. These are called interference fringes or bands. Using an eyepiece the fringes can be seen directly. At P on the screen, waves from A and B travel equal distances and arrive in phase. These two waves constructively interfere and bright fringe is observed at P. This is called central bright fringe.

When one of the slits is covered, the fringes disappear and there is uniform illumination on the screen. This shows clearly that the bands are due to interference.

5.6.6 Expression for bandwidth

Let d be the distance between two coherent sources A and B of wavelength λ . A screen XY is placed parallel to AB at a distance D from the coherent sources. C is the mid point of AB. O is a point on the screen equidistant from A and B. P is a point at a distance x from O, as shown in Fig 5.17. Waves from A and B meet at P in phase or out of phase depending upon the path difference between two waves.

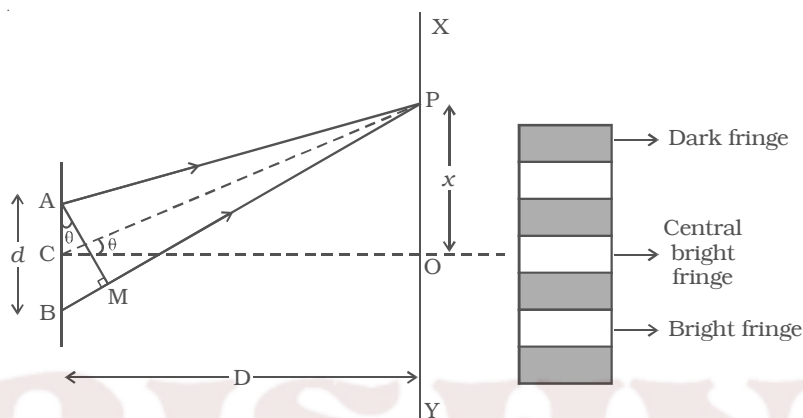


Fig 5.17 Interference band width

Draw AM perpendicular to BP

The path difference $\delta = BP - AP$

$AP = MP$

$$\therefore \delta = BP - AP = BP - MP = BM$$

In right angled ΔABM , $BM = d \sin \theta$

If θ is small, $\sin \theta = \theta$

$$\therefore \text{The path difference } \delta = \theta \cdot d$$

$$\text{In right angled triangle COP, } \tan \theta = \frac{OP}{CO} = \frac{x}{D}$$

For small values of θ , $\tan \theta = \theta$

$$\therefore \text{The path difference } \delta = \frac{xd}{D}$$

Bright fringes

By the principle of interference, condition for constructive interference is the path difference = $n\lambda$

$$\therefore \frac{xd}{D} = n\lambda$$

where $n = 0, 1, 2 \dots$ indicate the order of bright fringes.

$$\therefore x = \frac{D}{d} n\lambda$$

This equation gives the distance of the n^{th} bright fringe from the point O.

Dark fringes

By the principle of interference, condition for destructive interference is the path difference = $(2n-1) \frac{\lambda}{2}$

where $n = 1, 2, 3 \dots$ indicate the order of the dark fringes.

$$\therefore x = \frac{D}{d} (2n-1) \frac{\lambda}{2}$$

This equation gives the distance of the n^{th} dark fringe from the point O. Thus, on the screen alternate dark and bright bands are seen on either side of the central bright band.

Band width (β)

The distance between any two consecutive bright or dark bands is called bandwidth.

The distance between $(n+1)^{\text{th}}$ and n^{th} order consecutive bright fringes from O is given by

$$x_{(n+1)} - x_n = \frac{D}{d} (n+1)\lambda - \frac{D}{d} n\lambda = \frac{D}{d} \lambda$$

$$\text{Bandwidth, } \beta = \frac{D}{d} \lambda$$

Similarly, it can be proved that the distance between two consecutive dark bands is also equal to $\frac{D\lambda}{d}$. Since bright and dark fringes are of same width, they are equi-spaced on either side of central maximum.

Condition for obtaining clear and broad interference bands

- (i) The screen should be as far away from the source as possible.
- (ii) The wavelength of light used must be larger.
- (iii) The two coherent sources must be as close as possible.

5.6.7 Colours of thin films

Everyone is familiar with the brilliant colours exhibited by a thin oil film spread on the surface of water and also by a soap bubble. These colours are due to interference between light waves reflected from the top and the bottom surfaces of thin films. When white light is incident on a thin film, the film appears coloured and the colour depends upon the thickness of the film and also the angle of incidence of the light.

Interference in thin films

Consider a transparent thin film of uniform thickness t and its refractive index μ bounded by two plane surfaces K and K' (Fig 5.18).

A ray of monochromatic light AB incident on the surface K of the film is partly reflected along BC and partly refracted into the film along BD . At the point D on the surface K' , the ray of light is partly reflected along DE and partly transmitted out of the film along DG . The reflected light then emerges into air along EF which is parallel to BC . The ray EH after refraction at H , finally emerges along HJ .

BC and EF are reflected rays parallel to each other and DG and HJ are transmitted rays parallel to each other. Rays BC and EF interfere and similarly the rays DG and HJ interfere.

Interference due to the reflected beam

EM is drawn normal to BC from E . Now the path difference between the waves BC and EF

$$\delta = (BD+DE)_{\text{in film}} - (BM)_{\text{in air}}$$

We know, that a distance in air is numerically equal to μ times the distance in medium

$$\delta = \mu (BD + DE) - BM$$

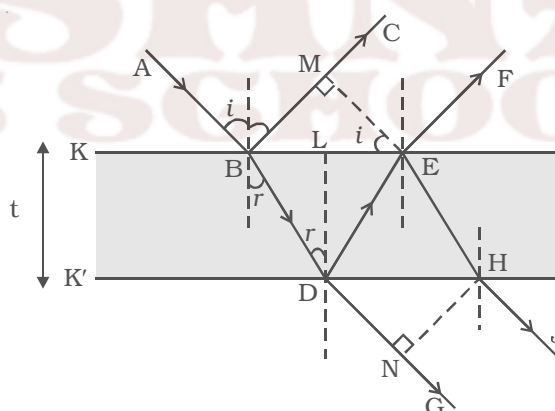


Fig 5.18 Interference in thin films

From the figure, it is clear that $BD = DE$

$$\therefore \delta = (2\mu \cdot BD) - BM$$

$$\text{In the } \Delta BME, \sin i = \frac{BM}{BE} \quad \left[\because \mu = \frac{\sin i}{\sin r} \right]$$

$$BM = BE \sin i = BE \cdot \mu \sin r$$

$$BM = \mu \cdot BE \sin r$$

$$\text{In the } \Delta BDL, \sin r = \frac{BL}{BD} = \frac{\frac{1}{2}BE}{BD}$$

$$BE = 2 (BD) \sin r$$

$$\therefore BM = \mu(2BD) \sin^2 r$$

$$\therefore \delta = 2\mu BD - 2\mu BD \sin^2 r$$

$$\delta = 2\mu BD \cos^2 r$$

$$\text{In the } \Delta BDL, \cos r = \frac{DL}{BD} = \frac{t}{BD}$$

$$\therefore \delta = 2\mu t \cos r$$

A ray of light travelling in air and getting reflected at the surface of a denser medium, undergoes an automatic phase change of π (or) an additional path difference of $\lambda/2$.

Since the reflection at B is at the surface of a denser medium, there is an additional path difference $\frac{\lambda}{2}$.

The effective path difference in this case, $\delta = 2\mu t \cos r + \frac{\lambda}{2}$

(i) For the constructive interference, path difference $\delta = n\lambda$, where $n = 0, 1, 2, 3$ and the film appears bright

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$\therefore 2\mu t \cos r = (2n-1) \frac{\lambda}{2}$$

(ii) For the destructive interference, path difference

$$\delta = (2n+1) \frac{\lambda}{2}$$

where $n = 0, 1, 2, 3 \dots$ and the film appears dark.

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$\therefore 2\mu t \cos r = n\lambda$$

If light is incident normally $i = 0$ and hence $r = 0$. Therefore the condition for bright fringe is $2\mu t = (2n-1) \frac{\lambda}{2}$ and for dark fringe is $2\mu t = n\lambda$.

Interference due to the transmitted light

The path difference between the transmitted rays DG and HJ is, in a similar way, $\delta = 2\mu t \cos r$. In this case there is no additional path difference introduced because both reflections at the point D and E take place backed by rarer medium

Hence, condition for brightness is $2\mu t \cos r = n\lambda$ and condition for darkness is $2\mu t \cos r = (2n - 1) \frac{\lambda}{2}$

5.6.8 Newton's rings

An important application of interference in thin films is the formation of Newton's rings. When a plano convex lens of long focal length is placed over an optically plane glass plate, a thin air film with varying thickness is enclosed between them. The thickness of the air film is zero at the point of contact and gradually increases outwards from the point of contact. When the air film is illuminated by monochromatic light normally, alternate bright and dark concentric circular rings are formed with dark spot at the centre. These rings are known as Newton's rings. When viewed with white light, the fringes are coloured (shown in the wrapper of the text book).

Experiment

Fig 5.19 shows an experimental arrangement for producing and observing Newton's rings. A monochromatic source of light S is kept at the focus of a condensing lens L_1 . The parallel beam of light emerging from L_1 falls on the glass plate G kept at 45° . The glass plate reflects a part of the incident light vertically downwards, normally on the thin air film, enclosed by the plano convex lens L and plane glass plate P. The reflected beam from the air film is viewed with a microscope. Alternate bright and dark circular rings with dark spot as centre is seen.

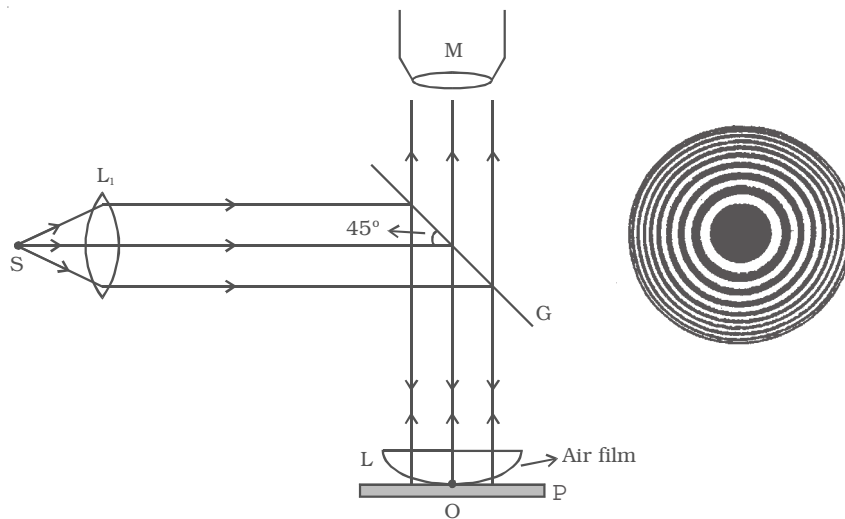


Fig 5.19 Newton's rings

Theory

The formation of Newton's rings can be explained on the basis of interference between waves which are partially reflected from the top and bottom surfaces of the air film. If t is the thickness of the air film at a point on the film, the refracted wavelet from the lens has to travel a distance t into the film and after reflection from the top surface of the glass plate, has to travel the same distance back to reach the point again.

Thus, it travels a total path $2t$. One of the two reflections takes place at the surface of the denser medium and hence it introduces an additional phase change of π or an equivalent path difference

$\frac{\lambda}{2}$ between two wavelets.

\therefore The condition for brightness is,

$$\text{Path difference, } \delta = 2t + \frac{\lambda}{2} = n\lambda$$

$$\therefore 2t = (2n-1) \frac{\lambda}{2}$$

where $n = 1, 2, 3 \dots$ and λ is the wavelength of light used.

The condition for darkness is,

$$\text{path difference } \delta = 2t + \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$\therefore 2t = n\lambda$$

where $n = 0, 1, 2, 3 \dots$

The thickness of the air film at the point of contact of lens L with glass plate P is zero. Hence, there is no path difference between the interfering waves. So, it should appear bright. But the wave reflected from the denser glass plate has suffered a phase change of π while the wave reflected at the spherical surface of the lens has not suffered any phase change. Hence the point O appears dark. Around the point of contact alternate bright and dark rings are formed.

5.6.9 Expression for the radius of the n^{th} dark ring

Let us consider the vertical section SOP of the plano convex lens through its centre of curvature C, as shown in Fig 5.20. Let R be the radius of curvature of the plano convex lens and O be the point of contact of the lens with the plane surface. Let t be the thickness of the air film at S and P. Draw ST and PQ perpendiculars to the plane surface of the glass plate. Then $ST = AO = PQ = t$

Let r_n be the radius of the n^{th} dark ring which passes through the points S and P.

$$\text{Then } SA = AP = r_n$$

If ON is the vertical diameter of the circle, then by the law of segments

$$SA \cdot AP = OA \cdot AN$$

$$r_n^2 = t(2R-t)$$

$$r_n^2 = 2Rt \text{ (neglecting } t^2 \text{ comparing with } 2R)$$

$$2t = \frac{r_n^2}{R}$$

According to the condition for darkness

$$2t = n\lambda$$

$$\therefore \frac{r_n^2}{R} = n\lambda$$

$$r_n^2 = nR\lambda \text{ or } r_n = \sqrt{nR\lambda}$$

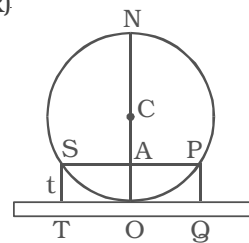


Fig 5.20 Radius of Newton's rings

Since R and λ are constants, we find that the radius of the dark ring is directly proportional to square root of its order. i.e. $r_1 \propto \sqrt{1}$, $r_2 \propto \sqrt{2}$, $r_3 \propto \sqrt{3}$, and so on. It is clear that the rings get closer as n increases.

5.6.10 Applications of Newton's rings

(i) Using the method of Newton's rings, the wavelength of a given monochromatic source of light can be determined. The radius of n^{th} dark ring and $(n+m)^{\text{th}}$ dark ring are given by

$$\begin{aligned} r_n^2 &= nR\lambda & \text{and} & & r_{n+m}^2 &= (n+m)R\lambda \\ r_{n+m}^2 - r_n^2 &= mR\lambda \\ \therefore \lambda &= \frac{r_{n+m}^2 - r_n^2}{mR} \end{aligned}$$

Knowing r_{n+m} , r_n and R , the wavelength can be calculated.

(ii) Using Newton's rings, the refractive index of a liquid can be calculated. Let λ_a and λ_m represent the wavelength of light in air and in medium (liquid). If r_n is the radius of the n^{th} dark ring in air and if r'_n is the radius of the n^{th} dark ring in liquid, then

$$\begin{aligned} r_n^2 &= nR\lambda_a \\ r'_n{}^2 &= nR\lambda_m = \frac{nR\lambda_a}{\mu} \quad \left[\because \mu = \frac{\lambda_a}{\lambda_m} \right] \\ \therefore \mu &= \frac{r_n^2}{r'_n{}^2} \end{aligned}$$

5.7 Diffraction

Sound is propagated in the form of waves. Sound produced in an adjoining room reaches us after bending round the edges of the walls. Similarly, waves on the surface of water also bend round the edges of an obstacle and spread into the region behind it. This bending of waves around the edges of an obstacle is called diffraction. Diffraction is a characteristic property of waves. The waves are diffracted, only when the size of the obstacle is comparable to the wavelength of the wave.

Fresnel showed that the amount of bending produced at an obstacle depends upon the wavelength of the incident wave. Since the sound waves have a greater wavelength, the diffraction effects are pronounced. As the wavelength of light is very small, compared to that of sound wave and even tiny obstacles have large size, compared to the wavelength of light waves, diffraction effects of light are very small.

In practice, diffraction of light can be observed by looking at a source of white light through a fine piece of cloth. A series of coloured images are observed.

5.7.1 Fresnel and Fraunhofer diffraction

Diffraction phenomenon can be classified under two groups (i) Fresnel diffraction and (ii) Fraunhofer diffraction. In the Fresnel diffraction, the source and the screen are at finite distances from the obstacle producing diffraction. In such a case the wave front undergoing diffraction is either spherical or cylindrical. In the Fraunhofer diffraction, the source and the screen are at infinite distances from the obstacle producing diffraction. Hence in this case the wavefront undergoing diffraction is plane. The diffracted rays which are parallel to one another are brought to focus with the help of a convex lens. Fraunhofer pattern is easier to observe practically by a spectrometer.

5.7.2 Diffraction grating

An arrangement consisting of a large number of equidistant parallel narrow slits of equal width separated by equal opaque portions is known as a diffraction grating.

The plane transmission grating is a plane sheet of transparent material on which opaque rulings are made with a fine diamond pointer. The modern commercial form of grating contains about 6000 lines per centimetre.

The rulings act as obstacles having a definite width ' b ' and the transparent space between the rulings act as slit of width ' a '. The combined width of a ruling and a slit is called grating element (e). Points on successive slits separated by a distance equal to the grating element are called corresponding points.

Theory

MN represents the section of a plane transmission grating. AB, CD, EF ... are the successive slits of equal width a and BC, DE ... be the rulings of equal width b (Fig. 5.21). Let $e = a + b$.

Let a plane wave front of monochromatic light of wave length λ be incident normally on the grating. According to Huygen's principle, the points in the slit AB, CD ... etc act as a source of secondary wavelets which spread in all directions on the other side of the grating.

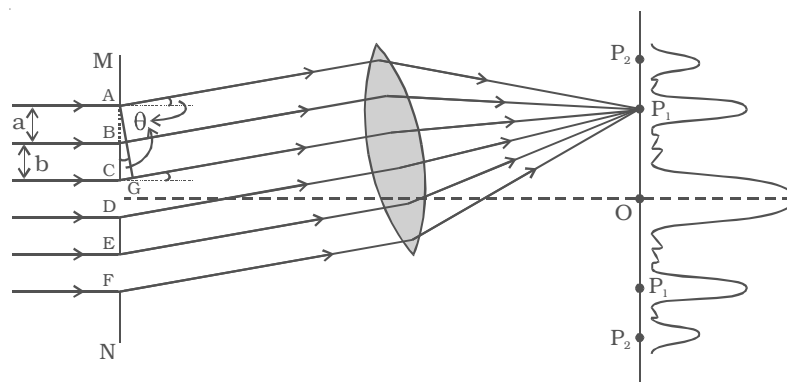


Fig 5.21 Diffraction grating

Let us consider the secondary diffracted wavelets, which makes an angle θ with the normal to the grating.

The path difference between the wavelets from one pair of corresponding points A and C is $CG = (a + b) \sin \theta$. It will be seen that the path difference between waves from any pair of corresponding points is also $(a + b) \sin \theta$

The point P_1 will be bright, when

$$(a + b) \sin \theta = m \lambda \text{ where } m = 0, 1, 2, 3$$

In the undiffracted position $\theta = 0$ and hence $\sin \theta = 0$.

$(a + b) \sin \theta = 0$, satisfies the condition for brightness for $m = 0$. Hence the wavelets proceeding in the direction of the incident rays will produce maximum intensity at the centre O of the screen. This is called zero order maximum or central maximum.

If $(a + b) \sin \theta_1 = \lambda$, the diffracted wavelets inclined at an angle θ_1 to the incident direction, reinforce and the first order maximum is obtained.

Similarly, for second order maximum, $(a + b) \sin \theta_2 = 2\lambda$

On either side of central maxima different orders of secondary maxima are formed at the point P_1, P_2 .

In general, $(a + b) \sin \theta = m \lambda$ is the condition for maximum intensity, where m is an integer, the order of the maximum intensity.

$$\sin \theta = \frac{m\lambda}{a+b} \quad \text{or} \quad \sin \theta = Nm\lambda$$

where $N = \frac{1}{a+b}$, gives the number of grating element or number of lines per unit width of the grating.

When white light is used, the diffraction pattern consists of a white central maximum and on both sides continuous coloured images are formed.

In the undiffracted position, $\theta = 0$ and hence $\sin \theta = 0$. Therefore $\sin \theta = Nm\lambda$ is satisfied for $m=0$ for all values of λ . Hence, at O all the wavelengths reinforce each other producing maximum intensity for all wave lengths. Hence an undispersed white image is obtained.

As θ increases, $(a+b) \sin \theta$ first passes through $\frac{\lambda}{2}$ values for all colours from violet to red and hence darkness results. As θ further increases, $(a+b) \sin \theta$ passes through λ values of all colours resulting in the formation of bright images producing a spectrum from violet to red. These spectra are formed on either side of white, the central maximum.

5.7.3 Experiment to determine the wavelength of monochromatic light using a plane transmission grating.

The wavelength of a spectral line can be very accurately determined with the help of a diffraction grating and spectrometer.

Initially all the preliminary adjustments of the spectrometer are made. The slit of collimator is illuminated by a monochromatic light, whose wavelength is to be determined. The telescope is brought in line with collimator to view the direct image. The given plane transmission grating is then mounted on the prism table with its plane perpendicular to the incident beam of light coming from the collimator. The telescope is slowly turned to one side until the first order diffraction image coincides with the vertical cross wire of the eye piece. The reading of the position of the telescope is noted (Fig. 5.22).

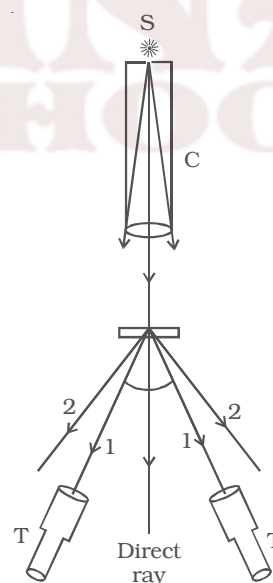


Fig 5.22 Diffraction of monochromatic light

Similarly the first order diffraction image on the other side, is made to coincide with the vertical cross wire and corresponding reading is noted. The difference between two positions gives 2θ . Half of its value gives θ , the diffraction angle for first order maximum. The wavelength of light is calculated from the equation $\lambda = \frac{\sin \theta}{Nm}$. Here N is the number of rulings per metre in the grating.

5.7.4 Determination of wavelengths of spectral lines of white light

Monochromatic light is now replaced by the given source of white light. The source emits radiations of different wavelengths, then the beam gets dispersed by grating and a spectrum of constituent wavelengths is obtained as shown in Fig 5.23.

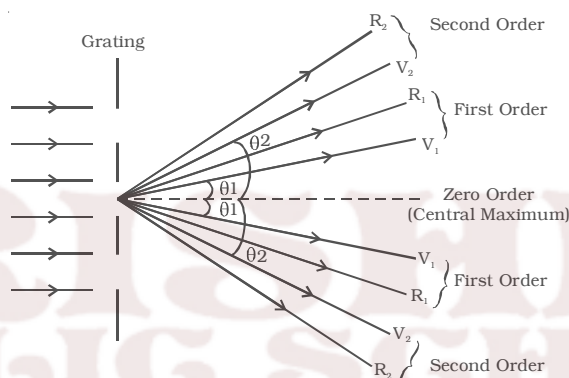


Fig 5.23 Diffraction of white light

knowing N, wave length of any line can be calculated from the relation

$$\lambda = \frac{\sin \theta}{Nm}$$

5.7.5 Difference between interference and diffraction

	Interference	Diffraction
1.	It is due to the superposition of secondary wavelets from two different wavefronts produced by two coherent sources.	It is due to the superposition of secondary wavelets emitted from various points of the same wave front.
2.	Fringes are equally spaced.	Fringes are unequally spaced.
3.	Bright fringes are of same intensity	Intensity falls rapidly
4.	Comparing with diffraction, it has large number of fringes	It has less number of fringes.

5.8. Polarisation

The phenomena of reflection, refraction, interference, diffraction are common to both transverse waves and longitudinal waves. But the transverse nature of light waves is demonstrated only by the phenomenon of polarisation.

5.8.1 Polarisation of transverse waves.

Let a rope AB be passed through two parallel vertical slits S_1 and S_2 placed close to each other. The rope is fixed at the end B. If the free end A of the rope is moved up and down perpendicular to its length, transverse waves are

generated with vibrations parallel to the slit. These waves pass through both S_1 and S_2 without any change in their amplitude. But if S_2 is made horizontal, the two slits are perpendicular to each other. Now, no vibrations will pass through S_2 and amplitude of vibrations will become zero. i.e the portion S_2B is without wave motion as shown in fig 5.24.

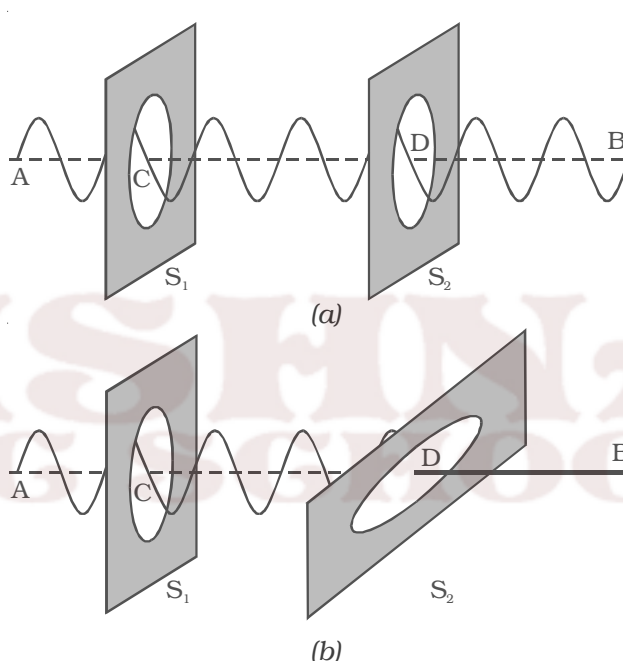


Fig 5.24 Polarisation of transverse waves

On the otherhand, if longitudinal waves are generated in the rope by moving the rope along forward and backward, the vibrations will pass through S_1 and S_2 irrespective of their positions.

This implies that the orientation of the slits has no effect on the propagation of the longitudinal waves, but the propagation of the transverse waves, is affected if the slits are not parallel to each other.

A similar phenomenon has been observed in light, when light passes through a tourmaline crystal.

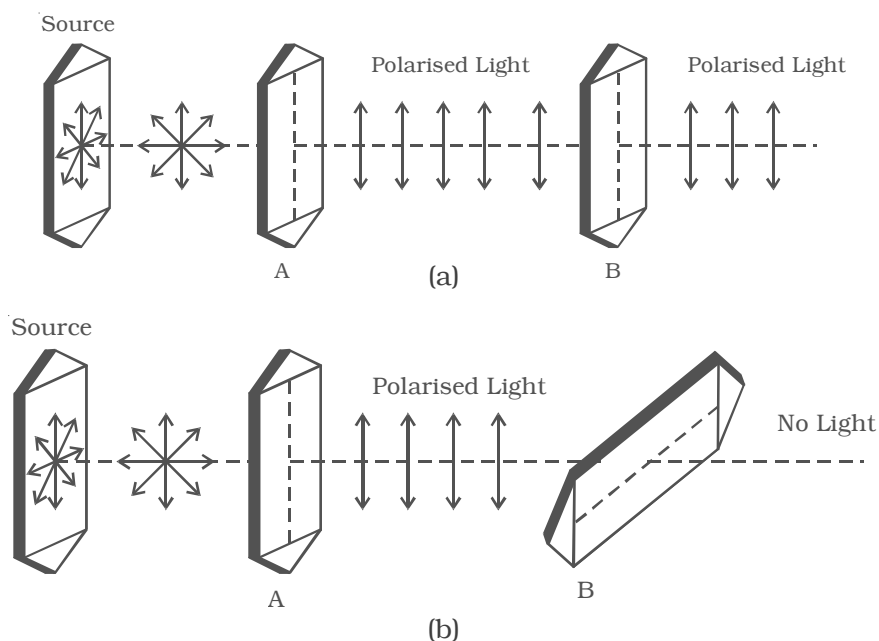


Fig 5.25 Polarisation of transverse waves

Light from the source is allowed to fall on a tourmaline crystal which is cut parallel to its optic axis (Fig. 5.25a).

The emergent light will be slightly coloured due to natural colour of the crystal. When the crystal A is rotated, there is no change in the intensity of the emergent light. Place another crystal B parallel to A in the path of the light. When both the crystals are rotated together, so that their axes are parallel, the intensity of light coming out of B does not change. When the crystal B alone is rotated, the intensity of the emergent light from B gradually decreases. When the axis of B is at right angles to the axis of A, no light emerges from B (Fig. 5.25b).

If the crystal B is further rotated, the intensity of the light coming out of B gradually increases and is maximum again when their axis are parallel.

Comparing these observations with the mechanical analogue discussed earlier, it is concluded that the light waves are transverse in nature.

Light waves coming out of tourmaline crystal A have their vibrations in only one direction, perpendicular to the direction of

propagation. These waves are said to be polarised. Since the vibrations are restricted to only one plane parallel to the axis of the crystal, the light is said to be plane polarised. The phenomenon of restricting the vibrations into a particular plane is known as polarisation.

5.8.2 Plane of vibration and plane of polarisation

The plane containing the optic axis in which the vibrations occur is known as plane of vibration. The plane which is at right angles to the plane of vibration and which contains the direction of propagation of the polarised light is known as the plane of polarisation. Plane of polarisation does not contain vibrations in it.

In the Fig 5.26 PQRS represents the plane of vibration and EFGH represents the plane of polarisation.

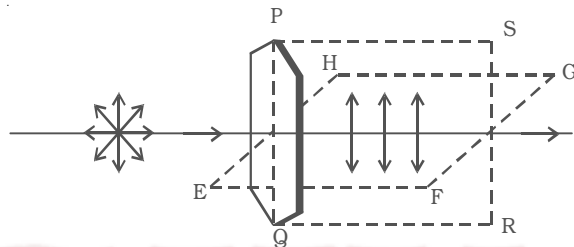


Fig 5.26 Planes of vibration and polarisation

5.8.3 Representation of light vibrations

In an unpolarised light, the vibrations in all directions may be supposed

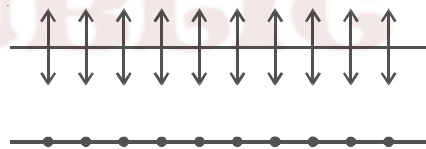


Fig 5.27 Light vibrations

to be made up of two mutually perpendicular vibrations. These are represented by double arrows and dots (Fig 5.27).

The vibrations in the plane of the paper are represented by double arrows, while the vibrations perpendicular to the plane of the paper are represented by dots.

5.8.4 Polariser and Analyser

A device which produces plane polarised light is called a polariser. A device which is used to examine, whether light is plane polarised or not is an analyser. A polariser can serve as an analyser and vice versa.

A ray of light is allowed to pass through an analyser. If the intensity of the emergent light does not vary, when the analyser is rotated, then the incident light is unpolarised; If the intensity of light varies between maximum and zero, when the analyser is rotated

through 90° , then the incident light is plane polarised; If the intensity of light varies between maximum and minimum (not zero), when the analyser is rotated through 90° , then the incident light is partially plane polarised.

5.8.5 Polarisation by reflection

The simplest method of producing plane polarised light is by reflection. Malus, discovered that when a beam of ordinary light is reflected from the surface of transparent medium like glass or water, it gets polarised. The degree of polarisation varies with angle of incidence.

Consider a beam of unpolarised light AB, incident at any angle on the reflecting glass surface XY.

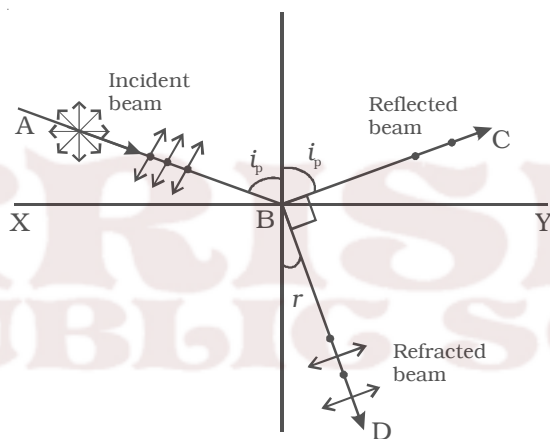


Fig 5.28 Polarisation by reflection

On examining the reflected beam with an analyser, it is found that the ray is partially plane polarised.

When the light is allowed to be incident at a particular angle, (for glass it is 57.5°) the reflected beam is completely plane polarised. The angle of incidence at which the reflected beam is completely plane polarised is called the polarising angle (i_p).

5.8.6 Brewster's law

Sir David Brewster conducted a series of experiments with different reflectors and found a simple relation between the angle of polarisation and the refractive index of the medium. It has been observed experimentally that the reflected and refracted rays are at right angles to

Vibrations in AB which are parallel to the plane of the diagram are shown by arrows. The vibrations which are perpendicular to the plane of the diagram and parallel to the reflecting surface, shown by dots (Fig. 5.28).

A part of the light is reflected along BC, and the rest is refracted along BD.

each other, when the light is incident at polarising angle.

From Fig 5.28, $i_p + 90^\circ + r = 180^\circ$

$$r = 90^\circ - i_p$$

From Snell's law, $\frac{\sin i_p}{\sin r} = \mu$

where μ is the refractive index of the medium (glass)

Substituting for r , we get

$$\frac{\sin i_p}{\sin(90 - i_p)} = \mu \quad ; \quad \frac{\sin i_p}{\cos i_p} = \mu$$

$$\therefore \tan i_p = \mu$$

The tangent of the polarising angle is numerically equal to the refractive index of the medium.

5.8.7 Pile of plates

The phenomenon of polarisation by reflection is used in the construction of pile of plates. It consists of a number of glass plates placed one over the other as shown in Fig 5.29 in a tube of suitable size. The plates are inclined at an angle

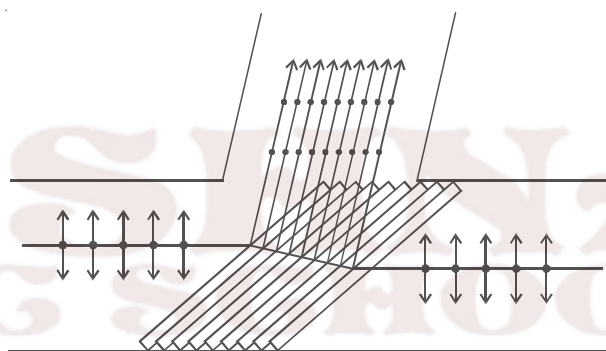


Fig.5.29 Pile of plates

of 32.5° to the axis of the tube. A beam of monochromatic light is allowed to fall on the pile of plates along the axis of the tube. So, the angle of incidence will be 57.5° which is the polarising angle for glass.

The vibrations perpendicular to the plane of incidence are reflected at each surface and those parallel to it are transmitted. The larger the number of surfaces, the greater is the intensity of the reflected plane polarised light. The pile of plates is used as a polariser and an analyser.

5.8.8 Double refraction

Bartholinus discovered that when a ray of unpolarised light is incident on a calcite crystal, two refracted rays are produced. This

phenomenon is called double refraction (Fig. 5.30a). Hence, two images of a single object are formed. This phenomenon is exhibited by several other crystals like quartz, mica etc.

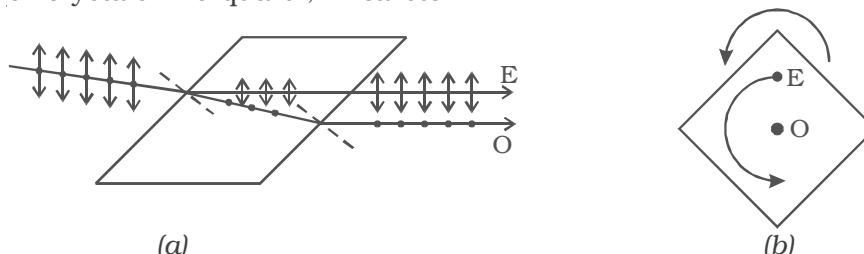


Fig 5.30 Double refraction

When an ink dot on a sheet of paper is viewed through a calcite crystal, two images will be seen (Fig 5.30b). On rotating the crystal, one image remains stationary, while the other rotates around the first. The stationary image is known as the ordinary image (O), produced by the refracted rays which obey the laws of refraction. These rays are known as ordinary rays. The other image is extraordinary image (E), produced by the refracted rays which do not obey the laws of refraction. These rays are known as extraordinary rays.

Inside a double refracting crystal the ordinary ray travels with same velocity in all directions and the extra ordinary ray travels with different velocities along different directions.

A point source inside a refracting crystal produces spherical wavefront corresponding to ordinary ray and elliptical wavefront corresponding to extraordinary ray.

Inside the crystal there is a particular direction in which both the rays travel with same velocity. This direction is called optic axis. The refractive index is same for both rays and there is no double refraction along this direction.

5.8.9 Types of crystals

Crystals like calcite, quartz, ice and tourmaline having only one optic axis are called uniaxial crystals.

Crystals like mica, topaz, selenite and aragonite having two optic axes are called biaxial crystals.

5.8.10 Nicol prism

Nicol prism was designed by William Nicol. One of the most common forms of the Nicol prism is made by taking a calcite crystal whose length is three times its breadth. It is cut into two halves along the diagonal so that their face angles are 72° and 108° . And the two halves are joined together by a layer of Canada balsam, a transparent cement as shown in Fig 5.31. For sodium light, the refractive index for ordinary light is 1.658 and for extra-ordinary light is 1.486. The refractive index for Canada balsam is 1.550 for both rays, hence Canada balsam does not polarise light.

A monochromatic beam of unpolarised light is incident on the face of the nicol prism. It splits up into two rays as ordinary ray (O) and extraordinary ray (E) inside the nicol prism (i.e) double refraction takes place. The ordinary ray is totally internally reflected at the layer of Canada balsam and is prevented from emerging from the other face. The extraordinary ray alone is transmitted through the crystal which is plane polarised. The nicol prism serves as a polariser and also an analyser.

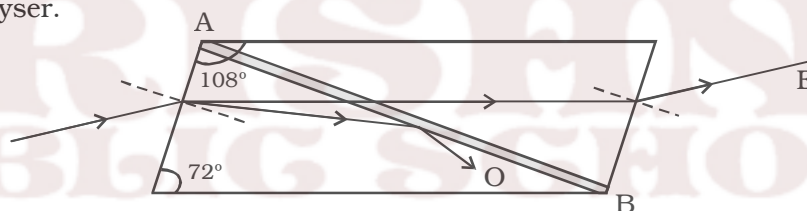


Fig 5.31 Nicol prism

5.8.11 Polaroids

A Polaroid is a material which polarises light. The phenomenon of selective absorption is made use of in the construction of polaroids. There are different types of polaroids.

A Polaroid consists of micro crystals of herapathite (an iodosulphate of quinine). Each crystal is a doubly refracting medium, which absorbs the ordinary ray and transmits only the extra ordinary ray. The modern polaroid consists of a large number of ultra microscopic crystals of herapathite embedded with their optic axes, parallel, in a matrix of nitro -cellulose.

Recently, new types of polaroid are prepared in which thin film of polyvinyl alcohol is used. These are colourless crystals which transmit more light, and give better polarisation.

5.8.12 Uses of Polaroid

1. Polaroids are used in the laboratory to produce and analyse plane polarised light.
2. Polaroids are widely used as polarising sun glasses.
3. They are used to eliminate the head light glare in motor cars.
4. They are used to improve colour contrasts in old oil paintings.
5. Polaroid films are used to produce three – dimensional moving pictures.
6. They are used as glass windows in trains and aeroplanes to control the intensity of light. In aeroplane one polaroid is fixed outside the window while the other is fitted inside which can be rotated. The intensity of light can be adjusted by rotating the inner polaroid.
7. Aerial pictures may be taken from slightly different angles and when viewed through polaroids give a better perception of depth.
8. In calculators and watches, letters and numbers are formed by liquid crystal display (LCD) through polarisation of light.
9. Polarisation is also used to study size and shape of molecules.

5.8.13 Optical activity

When a plane polarised light is made to pass through certain substances, the plane of polarisation of the emergent light is not the same as that of incident light, but it has been rotated through some angle. This phenomenon is known as optical activity. The substances which rotate the plane of polarisation are said to be optically active. Examples : quartz, sugar crystals, turpentine oil, sodium chloride etc.

Optically active substances are of two types, (i) Dextro-rotatory (right handed) which rotate the plane of polarisation in the clock wise direction on looking towards the source. (ii) Laevo – rotatory (left handed) which rotate the plane of polarisation in the anti clockwise direction on looking towards the source.

Light from a monochromatic source S, is made to pass through a polariser P. The plane polarised light is then made to fall on an analyser A, which is in crossed position with P. No light comes out of A. When a quartz plate is inserted between the polariser and analyser some light emerges out of the analyzer A (Fig. 5.32). The emerging light is cut off again, when the analyzer is rotated through a certain angle.

This implies that light emerging from quartz is still plane polarised, but its plane of polarisation has been rotated through certain angle.

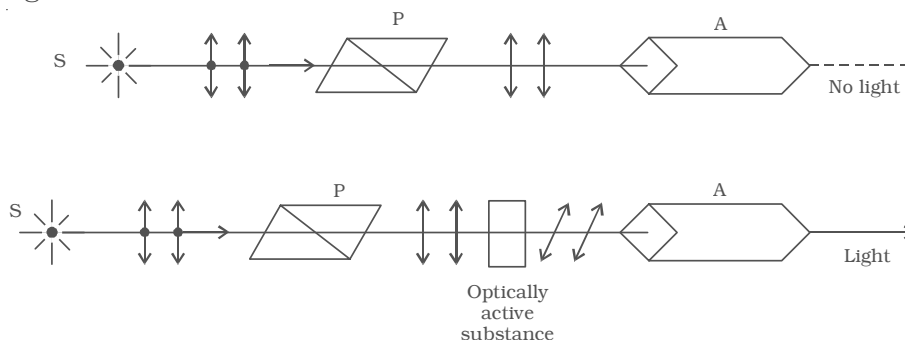


Fig 5.32 Optical activity

The amount of optical rotation depends on :

- (i) thickness of crystal
- (ii) density of the crystal or concentration in the case of solutions.
- (iii) wavelength of light used
- (iv) the temperature of the solutions.

5.8.14 Specific rotation

The term specific rotation is used to compare the rotational effect of all optically active substances.

Specific rotation for a given wavelength of light at a given temperature is defined as the rotation produced by one-decimeter length of the liquid column containing 1 gram of the active material in 1cc of the solution.

If θ is the angle of rotation produced by l decimeter length of a solution of concentration C in gram per cc, then the specific rotation S at a given wavelength λ for a given temperature t is given by

$$S = \frac{\theta}{lc}$$

The instrument used to determine the optical rotation produced by a substance is called polarimeter.

Sugar is the most common optically active substance and this optical activity is used for the estimation of its strength in a solution by measuring the rotation of plane of polarisation.

Solved problems

- 5.1 In Young's double slit experiment two coherent sources of intensity ratio of 64 : 1, produce interference fringes. Calculate the ratio of maximum and minimum intensities.

Data : $I_1 : I_2 :: 64 : 1$ $\frac{I_{max}}{I_{min}} = ?$

Solution : $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{64}{1}$

$$\therefore \frac{a_1}{a_2} = \frac{8}{1}; \quad a_1 = 8a_2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(8a_2 + a_2)^2}{(8a_2 - a_2)^2}$$

$$= \frac{(9a_2)^2}{(7a_2)^2} = \frac{81}{49}$$

$$I_{max} : I_{min} :: 81 : 49$$

- 5.2 In Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2 mm. Calculate the fringe width if the entire apparatus is immersed in a liquid of refractive index 1.33.

Data : $\lambda = 6000 \text{ \AA} = 6 \times 10^{-7} \text{ m}$; $\beta = 2\text{mm} = 2 \times 10^{-3} \text{ m}$
 $\mu = 1.33$; $\beta' = ?$

Solution : $\beta' = \frac{D\lambda'}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}$ $\left[\because \mu = \frac{\lambda}{\lambda'} \right]$

$$\therefore \beta' = \frac{2 \times 10^{-3}}{1.33} = 1.5 \times 10^{-3} \text{ m (or) } 1.5 \text{ mm}$$

- 5.3 A soap film of refractive index 1.33, is illuminated by white light incident at an angle 30°. The reflected light is examined by spectroscope in which dark band corresponding to the wavelength 6000Å is found. Calculate the smallest thickness of the film.

Data : $\mu = 1.33$; $i = 30^\circ$; $\lambda = 6000 \text{ \AA} = 6 \times 10^{-7} \text{ m}$
 $n = 1$ (Smallest thickness); $t = ?$

Solution : $\mu = \frac{\sin i}{\sin r}$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 30^\circ}{1.33} = \frac{0.5}{1.33} = 0.3759$$

$$\therefore \cos r = \sqrt{1 - 0.3759^2} = 0.9267$$

$$2 \mu t \cos r = n\lambda$$

$$t = \frac{\lambda}{2\mu \cos r} = \frac{6 \times 10^{-7}}{2 \times 1.33 \times 0.9267}$$

$$t = \frac{6 \times 10^{-7}}{2.465}$$

$$t = 2.434 \times 10^{-7} \text{ m}$$

- 5.4 A plano – convex lens of radius 3 m is placed on an optically flat glass plate and is illuminated by monochromatic light. The radius of the 8th dark ring is 3.6 mm. Calculate the wavelength of light used.

Data : $R = 3\text{m}$; $n = 8$; $r_8 = 3.6 \text{ mm} = 3.6 \times 10^{-3} \text{ m}$; $\lambda = ?$

Solution : $r_n = \sqrt{nR\lambda}$

$$r_n^2 = nR\lambda$$

$$\lambda = \frac{r_n^2}{nR} = \frac{(3.6 \times 10^{-3})^2}{8 \times 3} = 5400 \times 10^{-10} \text{ m (or) } 5400 \text{ \AA}$$

- 5.5 In Newton's rings experiment the diameter of certain order of dark ring is measured to be double that of second ring. What is the order of the ring?

Data : $d_n = 2d_2$; $n = ?$

Solution : $d_n^2 = 4nR\lambda$... (1)

$$d_2^2 = 8R\lambda$$
 ... (2)

$$\frac{(1)}{(2)} \Rightarrow \frac{d_n^2}{d_2^2} = \frac{n}{2}$$

$$\frac{4d_2^2}{d_2^2} = \frac{n}{2}$$

$$\therefore n = 8.$$

- 5.6 Two slits 0.3 mm apart are illuminated by light of wavelength 4500 Å. The screen is placed at 1 m distance from the slits. Find the separation between the second bright fringe on both sides of the central maximum.

Data : $d = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$; $\lambda = 4500 \text{ Å} = 4.5 \times 10^{-7} \text{ m}$,
 $D = 1 \text{ m}$; $n = 2$; $2x = ?$

Solution : $2x = 2 \frac{D}{d} n\lambda$

$$= \frac{2 \times 1 \times 2 \times 4.5 \times 10^{-7}}{0.3 \times 10^{-3}}$$

$$\therefore 2x = 6 \times 10^{-3} \text{ m (or) 6 mm}$$

- 5.7 A parallel beam of monochromatic light is allowed to incident normally on a plane transmission grating having 5000 lines per centimetre. A second order spectral line is found to be diffracted at an angle 30°. Find the wavelength of the light.

Data : $N = 5000 \text{ lines / cm} = 5000 \times 10^2 \text{ lines / m}$
 $m = 2$; $\theta = 30^\circ$; $\lambda = ?$

Solution : $\sin \theta = Nm \lambda$ $\lambda = \frac{\sin \theta}{Nm}$

$$\lambda = \frac{\sin 30^\circ}{5 \times 10^5 \times 2} = \frac{0.5}{5 \times 10^5 \times 2}$$

$$\lambda = 5 \times 10^{-7} \text{ m} = 5000 \text{ Å}.$$

- 5.8 A 300 mm long tube containing 60 cc of sugar solution produces a rotation of 9° when placed in a polarimeter. If the specific rotation is 60°, calculate the quantity of sugar contained in the solution.

Data : $l = 300 \text{ mm} = 30 \text{ cm} = 3 \text{ decimeter}$

$\theta = 9^\circ$; $S = 60^\circ$; $v = 60 \text{ cc}$

$m = ?$

Solution : $S = \frac{\theta}{l \times c} = \frac{\theta}{l \times (m/v)}$

$$m = \frac{\theta \cdot v}{l \times s}$$

$$= \frac{9 \times 60}{3 \times 60}$$

$$m = 3 \text{ g}$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

5.1 In an electromagnetic wave

- (a) power is equally transferred along the electric and magnetic fields
- (b) power is transmitted in a direction perpendicular to both the fields
- (c) power is transmitted along electric field
- (d) power is transmitted along magnetic field

5.2 Electromagnetic waves are

- (a) transverse
- (b) longitudinal
- (c) may be longitudinal or transverse
- (d) neither longitudinal nor transverse

5.3 Refractive index of glass is 1.5. Time taken for light to pass through a glass plate of thickness 10 cm is

- (a) $2 \times 10^{-8} \text{ s}$
- (b) $2 \times 10^{-10} \text{ s}$
- (c) $5 \times 10^{-8} \text{ s}$
- (d) $5 \times 10^{-10} \text{ s}$

- 5.4 In an electromagnetic wave the phase difference between electric field \vec{E} and magnetic field \vec{B} is
 (a) $\pi/4$ (b) $\pi/2$
 (c) π (d) zero
- 5.5 Atomic spectrum should be
 (a) pure line spectrum (b) emission band spectrum
 (c) absorption line spectrum (d) absorption band spectrum.
- 5.6 When a drop of water is introduced between the glass plate and plano convex lens in Newton's rings system, the ring system
 (a) contracts (b) expands
 (c) remains same (d) first expands, then contracts
- 5.7 A beam of monochromatic light enters from vacuum into a medium of refractive index μ . The ratio of the wavelengths of the incident and refracted waves is
 (a) $\mu : 1$ (b) $1 : \mu$
 (c) $\mu^2 : 1$ (d) $1 : \mu^2$
- 5.8 If the wavelength of the light is reduced to one fourth, then the amount of scattering is
 (a) increased by 16 times (b) decreased by 16 times
 (c) increased by 256 times (d) decreased by 256 times
- 5.9 In Newton's ring experiment the radii of the m^{th} and $(m + 4)^{\text{th}}$ dark rings are respectively $\sqrt{5}$ mm and $\sqrt{7}$ mm. What is the value of m ?
 (a) 2 (b) 4
 (c) 8 (d) 10
- 5.10 The path difference between two monochromatic light waves of wavelength 4000 \AA is $2 \times 10^{-7} \text{ m}$. The phase difference between them is
 (a) π (b) 2π
 (c) $3 \frac{\pi}{2}$ (d) $\pi/2$
- 5.11 In Young's experiment, the third bright band for wavelength of light 6000 \AA coincides with the fourth bright band for another source in the same arrangement. The wave length of the another source is
 (a) 4500 \AA (b) 6000 \AA
 (c) 5000 \AA (d) 4000 \AA

- 5.12 A light of wavelength 6000 \AA is incident normally on a grating 0.005 m wide with 2500 lines. Then the maximum order is
- (a) 3 (b) 2
(c) 1 (d) 4
- 5.13 A diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?
- (a) bands disappear
(b) no change
(c) diffraction pattern becomes narrower and crowded together
(d) diffraction pattern becomes broader and farther apart
- 5.14 The refractive index of the medium, for the polarising angle 60° is
- (a) 1.732 (b) 1.414
(c) 1.5 (d) 1.468
- 5.15 What are electromagnetic waves?
- 5.16 Mention the characteristics of electromagnetic waves.
- 5.17 Give the source and uses of electromagnetic waves.
- 5.18 Explain emission and absorption spectra.
- 5.19 What is fluorescence and phosphorescence?
- 5.20 Distinguish the corpuscle and photon.
- 5.21 What is Tyndal Scattering?
- 5.22 How are Stoke's and Anti-stoke's line formed?
- 5.23 Why the sky appears blue in colour?
- 5.24 Explain the Raman scattering of light.
- 5.25 Explain Huygen's principle.
- 5.26 On the basis of wave theory, explain total internal reflection.
- 5.27 What is principle of superposition of waves?
- 5.28 Give the conditions for sustained interference.
- 5.29 Derive an expression for bandwidth of interference fringes in Young's double slit experiment.
- 5.30 Discuss the theory of interference in thin transparent film due to reflected light and obtain condition for the intensity to be maximum and minimum.

- 5.31 What are Newton's rings? Why the centre of the Newton's rings is dark?
- 5.32 Distinguish between Fresnel and Fraunhofer diffraction.
- 5.33 Discuss the theory of plane transmission grating.
- 5.34 Describe an experiment to demonstrate transverse nature of light.
- 5.35 Differentiate between polarised and unpolarised light.
- 5.36 State and explain Brewster's law.
- 5.37 Bring out the difference's between ordinary and extra ordinary light.
- 5.38 Write a note on : (a) Nicol prism (b) Polaroid
- 5.39 What is meant by optical rotation? On what factors does it depend?

Problems

- 5.40 An LC resonant circuit contains a capacitor 400 pF and an inductor 100 μ H. It is sent into oscillations coupled to an antenna. Calculate the wavelength of the radiated electromagnetic wave.
- 5.41 In Young's double slit experiment, the intensity ratio of two coherent sources are 81 : 1. Calculate the ratio between maximum and minimum intensities.
- 5.42 A monochromatic light of wavelength 589 nm is incident on a water surface having refractive index 1.33. Find the velocity, frequency and wavelength of light in water.
- 5.43 In Young's experiment a light of frequency 6×10^{14} Hz is used. Distance between the centres of adjacent fringes is 0.75 mm. Calculate the distance between the slits, if the screen is 1.5 m away.
- 5.44 The fringe width obtained in Young's double slit experiment while using a light of wavelength 5000 Å is 0.6 cm. If the distance between the slit and the screen is halved, find the new fringe width.
- 5.45 A light of wavelength 6000 Å falls normally on a thin air film, 6 dark fringes are seen between two points. Calculate the thickness of the air film.
- 5.46 A soap film of refractive index 4/3 and of thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle 60° . The reflected light is examined by a spectroscope in which dark band

corresponds to a wavelength of 5000 \AA . Calculate the order of the dark band.

- 5.47 In a Newton's rings experiment the diameter of the 20th dark ring was found to be 5.82 mm and that of the 10th ring 3.36 mm. If the radius of the plano-convex lens is 1 m. Calculate the wavelength of light used.
- 5.48 A plane transmission grating has 5000 lines / cm. Calculate the angular separation in second order spectrum of red line 7070 \AA and blue line 5000 \AA .
- 5.49 The refractive index of the medium is $\sqrt{3}$. Calculate the angle of refraction if the unpolarised light is incident on it at the polarising angle of the medium.
- 5.50 A 20 cm long tube contains sugar solution of unknown strength. When observed through polarimeter, the plane of polarisation is rotated through 10° . Find the strength of sugar solution in g/cc. Specific rotation of sugar is 60° / decimetre / unit concentration.

Answers

- | | | | | |
|---------------------------------------------------------------------------------------------------------|-----------------|-----------------|-------------------------|-----------------|
| 5.1 (b) | 5.2 (a) | 5.3 (d) | 5.4 (d) | 5.5 (a) |
| 5.6 (a) | 5.7 (a) | 5.8 (c) | 5.9 (d) | 5.10 (a) |
| 5.11 (a) | 5.12 (a) | 5.13 (c) | 5.14 (a) | |
| 5.40 377 m | | | 5.41 25 : 16 | |
| 5.42 $2.26 \times 10^8 \text{ m s}^{-1}$, $5.09 \times 10^{14} \text{ Hz}$, 4429 \AA | | | | |
| 5.43 1 mm | | | 5.44 3 mm | |
| 5.45 $18 \times 10^{-7} \text{ m}$ | | | 5.46 6 | |
| 5.47 5645 \AA | | | 5.48 15° | |
| 5.49 30° | | | 5.50 0.0833 g/cc | |

6. Atomic Physics

At atmospheric pressure, air and other gases are poor conductors of electricity. This is because, they do not have free charged particles. However, electric current may be passed through a gas if by some mechanism, charged particles are produced in the gas. This can be done in many ways, such as (i) by applying a large potential difference across a gas column at very low pressure and (ii) by allowing X-rays to pass through the gases.

The study of electric discharge through gases gives valuable information regarding the structure of atoms. This has led to the discovery of electrons by J. J. Thomson and later on, to the discovery of X-rays by Roentgen.

6.1 Discharge of electricity through gases at low pressure - Discovery of electrons

A discharge tube is an arrangement to study the conduction of electricity through gases. It is a closed, strong glass tube of length 50 cm and diameter 4 cm, filled with a gas. Two metal electrodes C and A are fitted inside

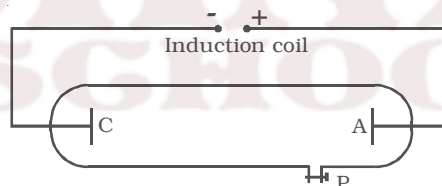


Fig 6.1 Discharge tube

the tube at the ends as shown in Fig 6.1. The side tube P is connected to a high vacuum pump and a low pressure gauge. The electrodes C and A are connected to the secondary of a powerful induction coil, which maintains a potential difference of 50,000 V. The electrode C connected to the negative terminal of the induction coil is called the cathode and the electrode A connected to the positive terminal is called the anode.

When the pressure of the gas inside the discharge tube is reduced by working the vacuum pump, to about 110 mm of Hg, no discharge occurs through the tube. At a pressure of about 100 mm of Hg, the discharge of electricity through the gas begins and irregular streaks of

light appear, accompanied by a crackling sound. As the pressure is reduced to the order of 10 mm of Hg, the irregular streaks broaden out into a luminous column extending from the anode, almost upto the cathode. This column is known as the positive column. With further reduction in pressure to around 0.01 mm of Hg, the positive column disappears and Crooke's dark space fills the whole tube. At this stage, the walls of the glass tube fluoresce with green colour. This greenish glow in the final stage of the gaseous discharge is found to be a fluorescence of the glass produced by some invisible rays emanating from the cathode (shown in the wrapper). These rays are called cathode rays and are found to be electrons.

6.1.1 Properties of Cathode rays

Cathode rays have the following properties:

- (i) They travel in straight lines.
- (ii) Cathode rays possess momentum and kinetic energy.
- (iii) Cathode rays produce heat, when allowed to fall on matter.
- (iv) Cathode rays produce fluorescence when they strike a number of crystals, minerals and salts.
- (v) When cathode rays strike a solid substance of large atomic weight, X-rays are produced.
- (vi) Cathode rays ionize the gas through which they pass.
- (vii) Cathode rays affect the photographic plates.
- (viii) The cathode rays are deflected from their straight line path by both electric and magnetic fields. The direction of deflection shows that they are negatively charged particles.
- (ix) Cathode rays travel with a velocity upto $(1/10)^{\text{th}}$ of the velocity of light.
- (x) Cathode rays comprises of electrons which are fundamental constituents of all atoms.

6.1.2 Positive rays (or) Canal rays

While conducting experiments on the gas discharge, in 1886, German Physicist, E. Goldstein, discovered that if the cathode used is perforated,

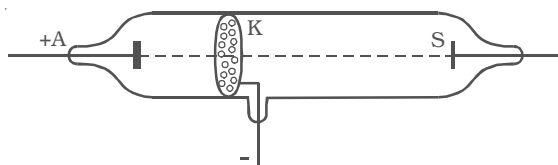


Fig 6.2 Production of canal rays.

luminous streams appeared in the tube behind the cathode. These streams were called as canal rays. The discharge tube designed by Goldstein is shown in Fig 6.2. The tube contains an anode (A), a perforated cathode (K) and a fluorescent screen (S). At a pressure of about 1mm of mercury, a luminous stream of particles were observed behind the cathode proceeding in a direction opposite to that of the cathode rays. Goldstein, called them as canal rays, since they pass through and emerge from the holes, in the cathode in straight lines, opposite to the direction of the cathode rays. From the nature of the deflection produced, by a magnetic field or electric field, these rays were found to be positively charged particles. Hence, canal rays are most commonly known as positive rays.

Properties of Canal rays

- (i) They are the streams of positive ions of the gas enclosed in the discharge tube. The mass of each ion is nearly equal to the mass of the atom.
- (ii) They are deflected by electric and magnetic fields. Their deflection is opposite to that of cathode rays.
- (iii) They travel in straight lines.
- (iv) The velocity of canal rays is much smaller than the velocity of cathode rays.
- (v) They affect photographic plates.
- (vi) These rays can produce fluorescence.
- (vii) They ionize the gas through which they pass.

6.1.3 Determination of specific charge (e/m) of an electron – Thomson's method.

In 1887, J.J. Thomson, measured the specific charge (e/m) of the cathode ray particles. The specific charge is defined as the charge per unit mass of the particle. Thomson discovered that the value of (e/m) was independent of the gas used and also independent of the nature of the electrodes.

Principle

The fact that the cathode rays (electrons) are deflected by electric and magnetic fields is made use of in this method.

Experimental arrangement

A highly evacuated discharge tube used in this experiment is as shown in Fig. 6.3. Cathode rays are produced by the discharge between

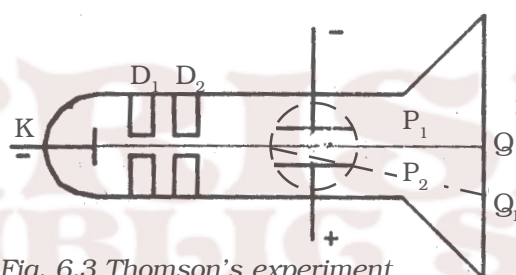


Fig. 6.3 Thomson's experiment

the cathode and the anodes D_1 and D_2 . A thin pencil of cathode ray comes out through fine pin holes in the anode discs. The cathode rays then pass between two parallel metal plates P_1 and P_2 and strike the flat face of the tube. This face is coated with suitable fluorescent material. A spot of

light is produced at Q . But when a potential difference V is applied between P_1 and P_2 , the beam is deflected to point Q_1 . By the use of a pair of coils, uniform magnetic field is produced perpendicular to the plane of the paper and outwards through out the region between P_1 P_2 .

Theory

1. Determination of V

With a given electric intensity between the plates P_1 and P_2 the magnetic induction B is adjusted until the beam strikes the screen at the original position Q . Then the downward force Ee due to the electric field is balanced by the force Bev due to magnetic induction where e is the charge of the cathode ray particle and v is the velocity of the cathode rays.

$$Ee = Bev$$

$$v = \frac{E}{B} \quad \dots(1)$$

2. Determination of e/m

Now the magnetic induction is switched off. The deflection $Q Q_1 = y$ caused by the electric field alone is measured. At the instant when the cathode rays enter the region between the plates P_1 and P_2 , initial velocity in the downward direction $u = 0$

Acceleration along the downward direction $a = \frac{Ee}{m}$, where m is the mass of each cathode ray particle.

Time for which the electron moves in the electric field $t = \frac{l}{v}$ where l is the length of either of the plates and v is the velocity of the electron.

The deflection produced on the cathode rays along the downward direction in the electric field is given by

$$y_1 = \frac{1}{2} \left(\frac{Ee}{m} \right) \left(\frac{l}{v} \right)^2 \quad \dots (2)$$

Substituting for v from equation (1) in equation (2) we get

$$y_1 = \frac{1}{2} \left(\frac{Ee}{m} \right) \left(\frac{l^2}{E^2} \right) B^2 = \frac{1}{2} \frac{e}{m} \frac{l^2 B^2}{E}$$

The displacement of the spot of light on the screen is given by (Fig 6.4)

$y = K y_1$, where K is a constant determined by the geometry of the discharge tube. Substituting for y_1 we get,

$$y = K \frac{1}{2} \frac{e}{m} \frac{l^2 B^2}{E} \quad \dots(3)$$

$$\frac{e}{m} = \frac{2yE}{Kl^2 B^2} \quad \dots(4)$$

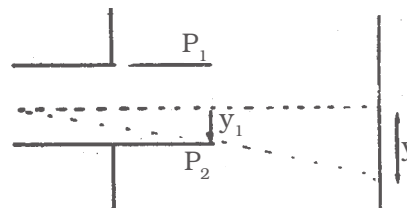


Fig. 6.4 Path of an electron in the electric field

By substituting the known values in the above relation e/m of an electron can be calculated. The value of e/m calculated using this experiment was found to be $1.7592 \times 10^{11} \text{ C kg}^{-1}$

Note: The value of e/m of an electron can also be calculated, by applying uniform magnetic field perpendicular to the path of the electron.

6.1.4 Determination of charge of an electron – Millikan's oil drop experiment

Millikan's experiment is used for the measurement of charge of an electron.

Principle

This method is based on the study of the motion of uncharged oil drop under free fall due to gravity and charged oil drop in a uniform electric field. By adjusting uniform electric field suitably, a charged oil drop can be made to move up or down or even kept balanced in the field of view for sufficiently long time and a series of observations can be made.

Experimental arrangement

The apparatus consists of two horizontal circular metal plates A and B, about 22 cm in diameter and separated by a distance of about 16 mm as shown in Fig 6.5. The upper plate has a hole (H) in the middle. These plates are held together by insulating rods of glass or ebonite, so that they are perfectly parallel to each other.

The plates are surrounded by a constant temperature bath D and the chamber C containing dry air. The plates are connected to a battery which can provide a potential difference of the order of 10000 V.

Theory

A spray of fine droplets of a highly viscous liquid (such as glycerine) is produced by means of an atomiser (AT) near the hole H and enter the space between A and B. The droplets are illuminated by an arc lamp L and are seen through a microscope whose eyepiece is provided with a micrometer scale. One such droplet is viewed through the microscope as it descends under gravity. The viscous force due to air increases and soon it attains a constant terminal velocity and let it be v . The terminal velocity v of the droplet is measured using the microscope.

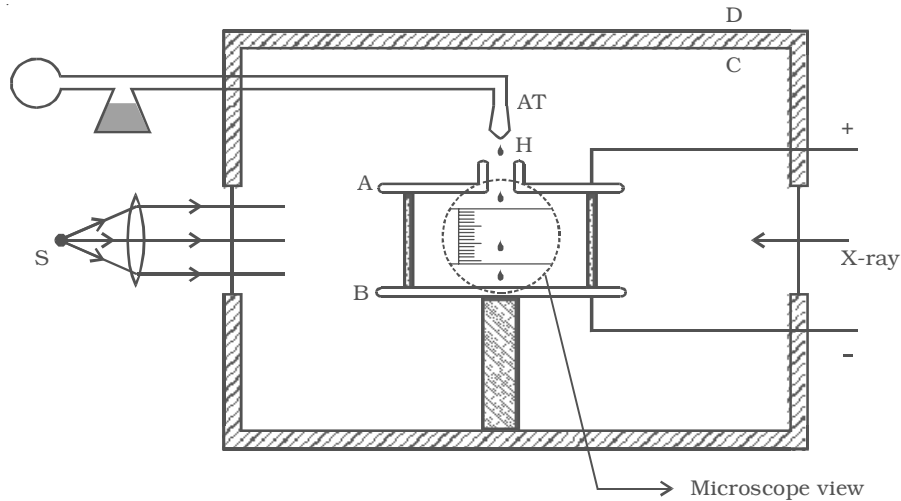


Fig 6.5 Millikan's oil drop experiment

(i) Motion under gravity

The gravitational force acting on the oil drop downwards is equal to $mg = \frac{4}{3} \pi a^3 \rho g$, where a is the radius of the oil drop, ρ is the density of the oil and g , the acceleration due to gravity.

The upthrust experienced by the oil drop due to the displaced air is $\frac{4}{3} \pi a^3 \sigma g$, where σ is the density of air.

\therefore The net downward force acting on the oil drop = weight of the oil drop – upthrust experienced by the oil drop.

$$\begin{aligned}
 &= \frac{4}{3} \pi a^3 \rho g - \frac{4}{3} \pi a^3 \sigma g \\
 &= \frac{4}{3} \pi a^3 (\rho - \sigma) g \quad \dots(1)
 \end{aligned}$$

Since the oil drop attains a terminal velocity v , the net downward force acting on the oil drop is equal to the viscous force acting opposite to the direction of motion of the oil drop.

By Stoke's law, the viscous force on the oil drop is $6\pi a\eta v$, where η is the co-efficient of viscosity of air.

$$\frac{4}{3}\pi a^3(\rho - \sigma)g = 6\pi a\eta v \quad \dots(2)$$

The radius of the oil drop is,

$$a = \left[\frac{9\eta v}{2(\rho - \sigma)g} \right]^{\frac{1}{2}} \quad \dots(3)$$

(ii) Motion under electric field

The air inside the parallel plates is ionized by sending a beam of X-rays. The droplets pickup one or more electrons from the ionized air.

Let q be the charge carried by the droplet under observation. Let E be the electric field applied between the plates A and B, so that the drop moves upwards with a terminal velocity v_1 , which can be determined using the microscope.

The force on the droplet due to electric field is Eq . Since the velocity of the droplet is uniform, we have

$$Eq = \frac{4}{3}\pi a^3(\rho - \sigma)g + 6\pi a\eta v_1$$

$$\therefore Eq - \frac{4}{3}\pi a^3(\rho - \sigma)g = 6\pi a\eta v_1 \quad \dots(4)$$

Adding equations (2) and (4),

$$Eq = 6\pi a\eta(v + v_1) \quad \dots(5)$$

Substituting the value of a in equation (5) from equation (3),

$$Eq = 6\pi\eta^{3/2}(v + v_1) \left[\frac{9v}{2(\rho - \sigma)g} \right]^{\frac{1}{2}} \quad \dots(6)$$

If V is the potential difference between A and B, d is the distance between them, then $E = \frac{V}{d}$

Millikan determined the value q for a large number of oil drops using equation (6) and found that they are an integral multiple of a

least value. The greatest common factor gives the charge e of the electron.

The charge of an electron was found to be 1.602×10^{-19} coulomb.

6.2 Atom models

As far back as 1803, Dalton, an English teacher, showed that the matter is made up of extremely small particles called atoms. Prout (1815), suggested that all elements are made up of atoms of hydrogen. Since many of the elements were found to have atomic weights that were not exact multiples of that of hydrogen, the suggestion was not accepted. After the discovery of radioactivity and electron, interest in the study of atomic structure increased and many models of atom were suggested at different times.

6.2.1 Thomson atom model

From the study of discharge of electricity through gases, it became clear that an atom consists of positive and negative charges.

J.J. Thomson tried to explain the arrangement of positive charge and the electrons inside the atom. According to him, an atom is a sphere of positive charge having a radius of the order of 10^{-10} m. The positive charge is uniformly

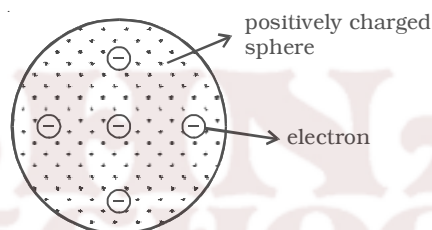


Fig 6.6 Thomson atom model

distributed over the entire sphere and the electrons are embedded in the sphere of positive charge as shown in Fig 6.6. The total positive charge inside the atom is equal to the total negative charge carried by the electrons, so that every atom is electrically neutral.

According to Thomson, if there is a single electron in the atom (like a hydrogen atom), the electron must be situated at the centre of the positive sphere. For an atom with two electrons (helium atom), the electrons should be situated symmetrically with respect to the centre of

the sphere i.e., opposite sides of the centre at a distance of $\frac{r}{2}$, where

r is the radius of the positive sphere. In a three electron system of the atom, the electrons should be at the corners of a symmetrically placed equilateral triangle, the side of which was equal to the radius of the

sphere. In general, the electrons of an atom are located in a symmetrical pattern with respect to the centre of the sphere.

It was suggested that spectral radiations are due to the simple harmonic motion of these electrons on both sides of their mean positions. Moreover, the stability of the atom was very well explained on the basis of this model.

Drawbacks

(i) According to electromagnetic theory, the vibrating electron should radiate energy and the frequency of the emitted spectral line should be the same as the electron. In the case of hydrogen atom, Thomson's model gives only one spectral line of about 1300 \AA . But the experimental observations reveal that hydrogen spectrum consists of five different series with several lines in each series.

(ii) It could not account for the scattering of α -particles through large angles.

6.2.2 Rutherford's α - particle scattering experiment

Rutherford and his associates studied the scattering of the α - particles by a thin gold foil in order to investigate the structure of the atom. An α -particle is a positively charged particle having a mass equal to that of helium atom and positive charge in magnitude equal to twice the charge of an electron. They are emitted by many radioactive elements. The scattering of α -particles provide useful information about the structure of the atom.

Experimental arrangement

A fine pencil of α -particles was obtained from a radioactive material like radium or radon by placing it in a lead box with narrow opening as shown in Fig 6.7.

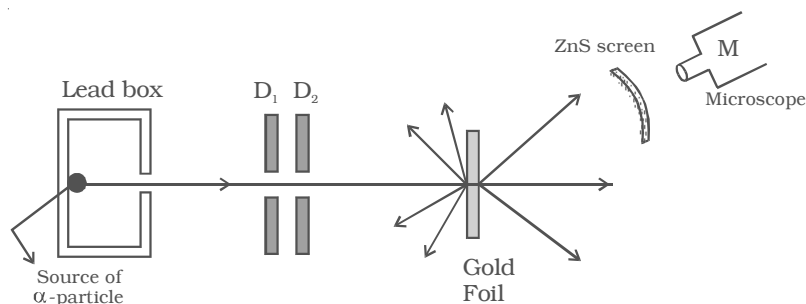


Fig 6.7 Rutherford's α - particle scattering experiment

The α -particles are emitted from the source in all possible directions, but only a narrow beam emerges from the lead box. The remaining α -particles are absorbed in the lead box itself. After passing through the diaphragms D_1 and D_2 , a narrow beam of α -particles incident on a thin gold foil, are scattered through different angles. The scattered α -particles strike a fluorescent screen coated with zinc sulphide. When the α -particles strike the screen, tiny flashes of light are produced. The observations can be made with the help of a low power microscope.

Observations and conclusions

(i) Most of the α particles either passed straight through the gold foil or were scattered by only small angles of the order of a few degrees. This observation led to the conclusion that an atom has a lot of empty space as shown in Fig 6.8.

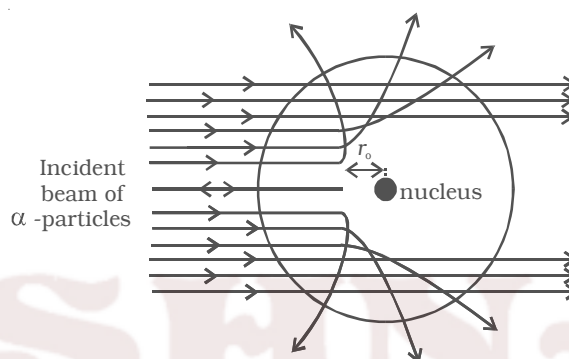


Fig 6.8 Scattering of α -rays

(ii) A few α particles were scattered in the backward direction, which led Rutherford to conclude that the whole of the positive charge was concentrated in a tiny space of about 10^{-14}m . This region of the atom was named as nucleus. Only a small number of particles approaches the nucleus of the atom and they were deflected at large angles.

Distance of closest approach

An α particle directed towards the centre of the nucleus will move close upto a distance r_0 as shown in Fig 6.8, where its kinetic energy will appear as electrostatic potential energy. After this, the α particle begins to retrace its path. This distance r_0 is known as the distance of the closest approach.

Let m and v be the mass and velocity of the α particle directed towards the centre of the nucleus. Then, the kinetic energy of the particle,

$$E_k = \frac{1}{2}mv^2 \quad \dots(1)$$

Since, charge of an α -particle is $2e$ and that of the nucleus of the atom is Ze , the electrostatic potential energy of the α particle, when at a distance r_o from the centre of the nucleus is given by,

$$E_p = \frac{1}{4\pi\epsilon_o} \cdot \frac{(2e)(Ze)}{r_o} \quad \dots(2)$$

where Z is the atomic number of the atom and ϵ_o , the permittivity of free space.

On reaching the distance of the closest approach r_o , the kinetic energy of the α particle appears as its potential energy.

$$\therefore E_p = E_k$$

$$\frac{1}{4\pi\epsilon_o} \cdot \frac{(2e)(Ze)}{r_o} = \frac{1}{2}mv^2$$

$$\text{(or)} \quad r_o = \frac{1}{4\pi\epsilon_o} \cdot \frac{4Ze^2}{mv^2} \quad \dots(3)$$

6.2.3 Rutherford atom model

Based on the results of α -particle scattering experiment, Rutherford suggested the following picture of the atom.

(i) Atom may be regarded as a sphere of diameter 10^{-10}m , but whole of the positive charge and almost the entire mass of the atom is concentrated in a small central core called nucleus having diameter of about 10^{-14}m as shown in Fig 6.9a.

(ii) The electrons in the atom were considered to be distributed around the nucleus in the empty space of the atom. If the electrons were at rest, they would be attracted and neutralized by the nucleus. To overcome this, Rutherford suggested that the electrons are revolving around the nucleus in circular orbits, so that the centripetal force is provided by the electrostatic force of attraction between the electron and the nucleus.

(iii) As the atom is electrically neutral, the total positive charge of the nucleus is equal to the total negative charge of the electrons in it.

Drawbacks

Rutherford atom model offered serious difficulties as regards the stability of the atom. Following are the two drawbacks of Rutherford's model: (i) The electron in the circular orbit experiences a centripetal acceleration. According to electromagnetic theory, an accelerated

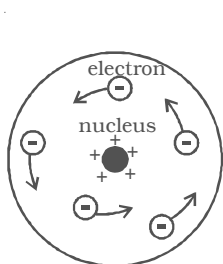


Fig 6.9a Rutherford atom model

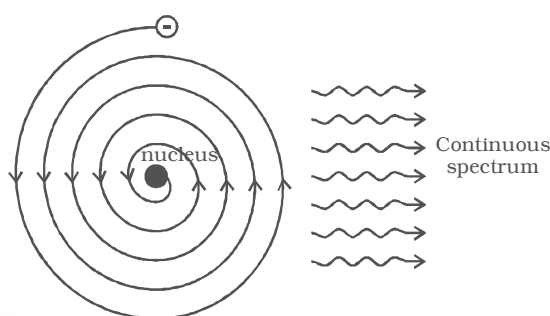


Fig 6.9b Motion of electrons in spiral path into the nucleus.

electric charge must radiate energy in the form of electromagnetic waves. Therefore, if the accelerated electron lose energy by radiation, the energy of the electron continuously decreases and it must spiral down into the nucleus, as shown in Fig 6.9b. Thus, the atom cannot be stable. But, it is well known that most of the atoms are stable. (ii) According to classical electromagnetic theory, the accelerating electron must radiate energy at a frequency proportional to the angular velocity of the electron. Therefore, as the electron spiral towards the nucleus, the angular velocity tends to become infinity and hence the frequency of the emitted energy will tend to infinity. This will result in a continuous spectrum with all possible wavelengths. But experiments reveal only line spectra of fixed wavelength from atoms.

6.2.4 Bohr atom model

Neils Bohr in 1913, modified Rutherford's atom model in order to explain the stability of the atom and the emission of sharp spectral lines. He proposed the following postulates :

(i) An electron cannot revolve round the nucleus in all possible orbits. The electrons can revolve round the nucleus only in those allowed or permissible orbits for which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$ (where h is Planck's constant = 6.626×10^{-34} Js). These orbits are called stationary orbits or non-radiating orbits and an electron revolving in these orbits does not radiate any energy.

If m and v are the mass and velocity of the electron in a permitted orbit of radius r then angular momentum of electron = $mvr = \frac{nh}{2\pi}$, where n is called principal quantum number and has the integral values 1,2,3 ... This is called Bohr's quantization condition.

(ii) An atom radiates energy, only when an electron jumps from a stationary orbit of higher energy to an orbit of lower energy. If the electron jumps from an orbit of energy E_2 to an orbit of energy E_1 , a photon of energy $h\nu = E_2 - E_1$ is emitted. This condition is called Bohr's frequency condition.

Radius of the n^{th} orbit (r_n)

Consider an atom whose nucleus has a positive charge Ze , where Z is the atomic number that gives the number of protons in the nucleus and e , the charge of the electron which is numerically equal to that of proton. Let an electron revolve around the nucleus in the n^{th} orbit of radius r_n .

By Coulomb's law, the electrostatic force of attraction between the nucleus and the electron = $\frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze)(e)}{r_n^2}$... (1)

where ϵ_0 is the permittivity of the free space.

Since, the electron revolves in a circular orbit, it experiences a centripetal force, $\frac{mv_n^2}{r_n} = mr_n\omega_n^2$... (2)

where m is the mass of the electron, v_n and ω_n are the linear velocity and angular velocity of the electron in the n^{th} orbit respectively.

The necessary centripetal force is provided by the electrostatic force of attraction.

For equilibrium, from equations (1) and (2),

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2} = \frac{mv_n^2}{r_n} \quad \dots(3)$$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2} = mr_n\omega_n^2 \quad \dots(4)$$

From equation (4),

$$\omega_n^2 = \frac{Ze^2}{4\pi\epsilon_0mr_n^3} \quad \dots(5)$$

The angular momentum of an electron in n^{th} orbit is,

$$L = mv_nr_n = mr_n^2\omega_n \quad \dots(6)$$

By Bohr's first postulate, the angular momentum of the electron

$$L = \frac{nh}{2\pi} \quad \dots(7)$$

where n is an integer and is called as the principal quantum number.

From equations. (6) and (7),

$$mr_n^2\omega_n = \frac{nh}{2\pi}$$

$$\text{(or)} \quad \omega_n = \frac{nh}{2\pi mr_n^2}$$

squaring both sides,

$$\omega_n^2 = \frac{n^2h^2}{4\pi^2m^2r_n^4} \quad \dots(8)$$

From equations (5) and (8),

$$\frac{Ze^2}{4\pi\epsilon_0mr_n^3} = \frac{n^2h^2}{4\pi^2m^2r_n^4}$$

$$(or) \quad r_n = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2} \quad \dots(9)$$

From equation (9), it is seen that the radius of the n^{th} orbit is proportional to the square of the principal quantum number. Therefore, the radii of the orbits are in the ratio 1 : 4 : 9....

For hydrogen atom, $Z = 1$

\therefore From equation (9)

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2} \quad \dots(10)$$

Substituting the known values in the above equation we get,

$$r_n = n^2 \times 0.53 \text{ \AA}$$

$$\text{If } n = 1, r_1 = 0.53 \text{ \AA}$$

This is called Bohr radius.

Energy of an electron in the n^{th} orbit (E_n)

The total energy of the electron is the sum of its potential energy and kinetic energy in its orbit (Fig 6.10).

The potential energy of the electron in the n^{th} orbit is given by,

$$E_p = \frac{(Ze)(-e)}{4\pi\epsilon_0 r_n} = \frac{-Ze^2}{4\pi\epsilon_0 r_n} \quad \dots(11)$$

The kinetic energy of the electron in the n^{th} orbit is,

$$E_k = \frac{1}{2} m v_n^2 \quad \dots(12)$$

From equation (3),

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n} = m v_n^2 \quad \dots(13)$$

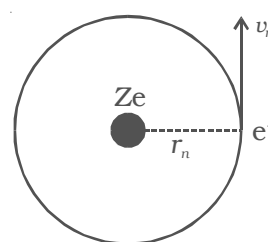


Fig 6.10 Energy of the electron

Substituting equation (13) in (12)

$$E_k = \frac{1}{2} \left[\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n} \right] = \frac{Ze^2}{8\pi\epsilon_0 r_n} \quad \dots(14)$$

The total energy of an electron in its n^{th} orbit is,

$$\begin{aligned} E_n &= E_p + E_k = \frac{-Ze^2}{4\pi\epsilon_0 r_n} + \frac{Ze^2}{8\pi\epsilon_0 r_n} \\ E_n &= \frac{-Ze^2}{8\pi\epsilon_0 r_n} \quad \dots(15) \end{aligned}$$

Substituting the value of r_n , from equation (10) in equation (15),

$$E_n = \frac{-Z^2 m e^4}{8\epsilon_0^2 n^2 h^2} \quad \dots(16)$$

For hydrogen atom, $Z = 1$

$$\therefore E_n = \frac{-m e^4}{8\epsilon_0^2 n^2 h^2}$$

Substituting the known values and calculating in electron-volt,

$$E_n = \frac{-13.6}{n^2} \text{eV} \quad \dots(17) \quad [\because 1\text{eV} = 1.602 \times 10^{-19}\text{J}]$$

As there is a negative sign in equation (17), it is seen that the energy of the electron in its orbit increases as n increases.

Frequency of spectral line

According to Bohr's second postulate, when an electron jumps from an outer orbit of quantum number n_2 to an inner orbit of quantum number n_1 , the frequency of the photon emitted is given by,

$$\nu = \frac{E_{n_2} - E_{n_1}}{h}$$

Using equation (16)

$$\nu = \frac{Z^2 m e^4}{8\epsilon_0^2 h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \dots(18)$$

The wave number $\bar{\nu}$ of a radiation is defined as number of waves per unit length. It is equal to reciprocal of the wavelength

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} \quad [\text{since } c = \nu\lambda]$$

where c is the velocity of light

\therefore From equation (18),

$$\bar{\nu} = \frac{Z^2 m e^4}{8 \epsilon_0^2 c h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \dots(19)$$

For hydrogen atom, $Z=1$

$$\bar{\nu} = \frac{m e^4}{8 \epsilon_0^2 c h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \dots(20)$$

where R is a constant called Rydberg's constant

$$\therefore R = \frac{m e^4}{8 \epsilon_0^2 c h^3} \quad \dots(21)$$

Substituting the known values, we get

$$R = 1.094 \times 10^7 \text{ m}^{-1}$$

6.2.5 Spectral series of hydrogen atom

Whenever an electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference in energies of the two levels is emitted as a radiation of particular wavelength. It is called a spectral line. As the wavelength of the spectral line depends upon the two orbits (energy levels) between which the transition of electron takes place, various spectral lines are obtained. The different wavelengths constitute spectral series which are the characteristic of the atoms emitting them. The following are the spectral series of hydrogen atom.

(i) Lyman series

When the electron jumps from any of the outer orbits to the first orbit, the spectral lines emitted are in the ultraviolet region of the spectrum and they are said to form a series called Lyman series (Fig 6.11).

Here, $n_1 = 1$, $n_2 = 2,3,4 \dots$

The wave number of the Lyman series is given by,

$$\bar{\nu} = R \left(1 - \frac{1}{n_2^2} \right)$$

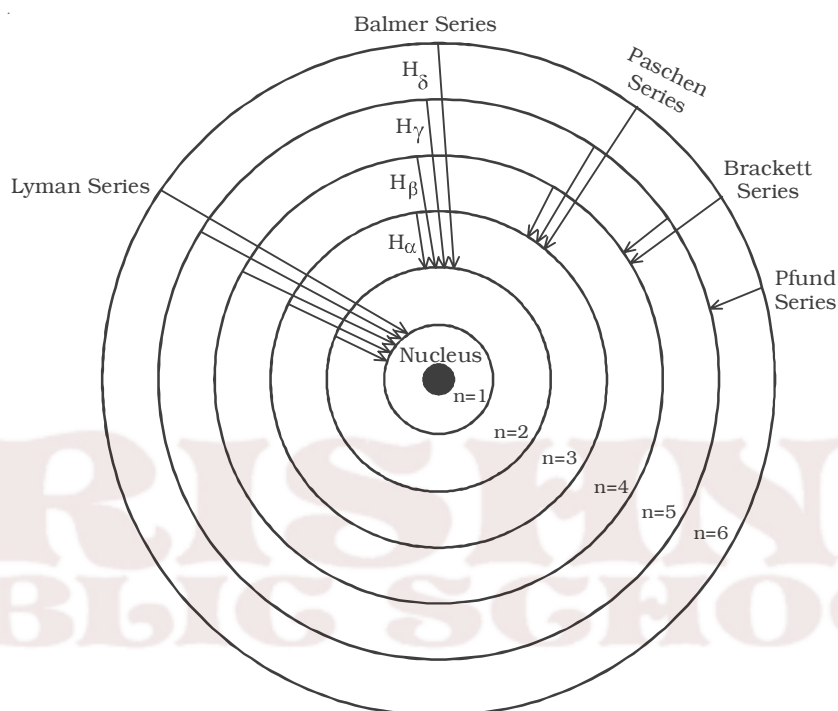


Fig 6.11 Spectral series of hydrogen atom

(ii) Balmer series

When the electron jumps from any of the outer orbits to the second orbit, we get a spectral series called the Balmer series. All the lines of this series in hydrogen have their wavelength in the visible region. Here $n_1=2$, $n_2 = 3,4,5 \dots$

The wave number of the Balmer series is,

$$\bar{\nu} = R \left(\frac{1}{2^2} - \frac{1}{n_2^2} \right) = R \left(\frac{1}{4} - \frac{1}{n_2^2} \right)$$

The first line in this series ($n_2 = 3$), is called the H_α -line, the second ($n_2=4$), the H_β -line and so on.

(iii) Paschen series

This series consists of all wavelengths which are emitted when the electron jumps from outer most orbits to the third orbit. Here $n_2 = 4, 5, 6 \dots$ and $n_1 = 3$. This series is in the infrared region with the wave number given by

$$\bar{\nu} = R \left(\frac{1}{3^2} - \frac{1}{n_2^2} \right) = R \left(\frac{1}{9} - \frac{1}{n_2^2} \right)$$

(iv) Brackett series

The series obtained by the transition of the electron from $n_2 = 5, 6, \dots$ to $n_1 = 4$ is called Brackett series. The wavelengths of these lines are in the infrared region. The wave number is,

$$\bar{\nu} = R \left(\frac{1}{4^2} - \frac{1}{n_2^2} \right) = R \left(\frac{1}{16} - \frac{1}{n_2^2} \right)$$

(v) Pfund series

The lines of the series are obtained when the electron jumps from any state $n_2 = 6, 7, \dots$ to $n_1 = 5$. This series also lies in the infrared region. The wave number is,

$$\bar{\nu} = R \left(\frac{1}{5^2} - \frac{1}{n_2^2} \right) = R \left(\frac{1}{25} - \frac{1}{n_2^2} \right)$$

6.2.6 Energy level diagram

The energy of the electron in the n^{th} orbit of the hydrogen atom is given by,

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

Energy associated with the first orbit of the hydrogen atom is,

$$E_1 = \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

It is called ground state energy of the hydrogen atom.

Energy associated with the second orbit is given by,

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

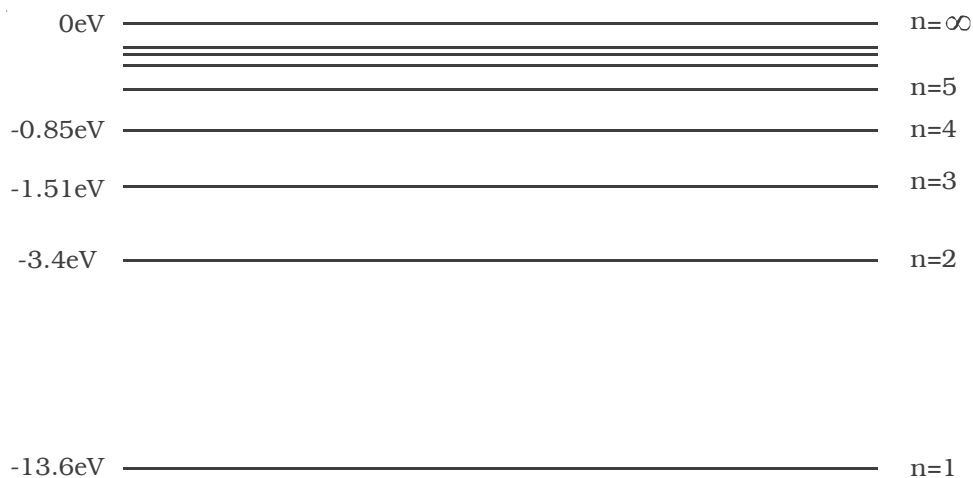


Fig 6.12 Energy level diagram for hydrogen atom

It is called energy of first excited state of the hydrogen atom. The energy of second, third, fourth, ... excited states of the hydrogen atom are, $E_3 = -1.51$ eV, $E_4 = -0.85$ eV, $E_5 = -0.54$ eV ...

$$\text{when } n = \infty, E_{\infty} = \frac{-13.6}{\infty^2} = 0$$

Therefore, it is seen from the above values, that, the energy associated with a state becomes less negative and approaches closer and closer to the maximum value zero corresponding to $n = \infty$.

Taking these energies on a linear scale, horizontal lines are drawn which represent energy levels of the hydrogen atom (Fig 6.12). This diagram is known as energy level diagram.

6.2.7 Sodium and mercury spectra

The sodium vapour lamp is commonly used in the laboratory as a source of monochromatic (single colour) light. Sodium vapour lamps and mercury lamps have been used for street lighting, as the two lamps give a more intense light at comparatively low cost. The mercury light is a composite light consisting of all colours in the visible spectrum. The wavelength of prominent lines emitted by the mercury source is presented in Table 6.1. The two lamps work on the principle of hot cathode positive column. Since, sodium and mercury atoms are in the

vapour state, they emit line spectra. The sodium vapour lamp emits yellow light of wavelength 5896Å and 5890Å. These lines are called sodium D₁ and D₂ lines.

**Table 6.1 Wavelength of spectral lines emitted by mercury
(NOT FOR EXAMINATION)**

Colour	Wavelength (nm)
Violet I	398
Violet II	403
Blue	436
Bluish Green I	492
Bluish Green II	496
Green	546
Yellow I	577
Yellow II	579
Orange	609
Red	625

6.2.8 Excitation and ionization potential of an atom

According to Bohr's theory, there are certain discrete orbits permitted for the motion of the electron. Electron can revolve in these orbits without radiating energy. An atom is said to be in the ground state, when its energy is least. Before an atom can emit spectral radiation, the electron in it has to be raised to a higher orbit. This process is known as the excitation of the atom. The energy required to raise an atom from its normal state into an excited state is called *excitation potential energy* of the atom. For example, the energy required to transfer the electron in hydrogen atom from the ground state to the first excited state = $(13.6 - 3.4) = 10.2\text{eV}$. The energy required to raise it to the second excited state = $(13.6 - 1.51) = 12.09\text{ eV}$. The potentials corresponding to these energies are called as the excitation potentials.

The ionisation potential is that accelerating potential which makes the impinging electron acquire sufficient energy to knock out an electron from the atom and thereby ionise the atom. For hydrogen atom, the energy required to remove an electron from first orbit to its outermost orbit($n=\infty$) $13.6 - 0 = 13.6\text{eV}$. This energy is known as the ionization potential energy for hydrogen atom. 13.6 V is the ionisation potential of hydrogen atom.

The excitation potential and ionization potential are called as the critical potentials of the atom. The *critical potential* of an atom, is defined as the minimum potential required to excite a free neutral atom from its ground state to higher state.

6.2.9 Shortcomings of Bohr's theory

Bohr's theory was able to explain successfully a number of experimental observations and has correctly predicted the spectral lines of hydrogen atom. However, the theory fails in the following aspects.

(i) The theory could not account for the spectra of atoms more complex than hydrogen.

(ii) The theory does not give any information regarding the distribution and arrangement of electrons in an atom.

(iii) It does not explain, the experimentally observed variations in intensity of the spectral lines of the element.

(iv) When the spectral line of hydrogen atom is examined by spectrometers having high resolving power, it is found that a single line is composed of two or more close components. This is known as the fine structure of spectral lines. Bohr's theory could not account for the fine structure of spectral lines.

(v) It is found that when electric or magnetic field is applied to the atom, each of the spectral line split into several lines. The former effect is called as Stark effect, while the latter is known as Zeeman effect. Bohr's theory could not explain the Stark effect and Zeeman effect.

6.2.10 Sommerfeld atom model

In order to explain the observed fine structure of spectral lines, Sommerfeld introduced two main modifications in Bohr's theory.

(i) According to Sommerfeld, the path of an electron around the nucleus, in general, is an ellipse with the nucleus at one of its foci.

(ii) The velocity of the electron moving in an elliptical orbit varies at different parts of the orbit. This causes the relativistic variation in the mass of the moving electron.

Now, when elliptical orbits are permitted, one has to deal with two variable quantities.

(i) The varying distance of the electron from the nucleus (r).

(ii) The varying angular position of the electron with respect to the nucleus i.e the azimuthal angle ϕ (Fig 6.13).

To deal with these two variables, two quantum numbers are introduced

(i) The principal quantum number n of Bohr's theory, which determines the energy of the electrons, and (ii) a new quantum number called orbital (or azimuthal) quantum number (l) which has been introduced to characterize the angular momentum in an orbit i.e., it determines the orbital angular momentum of the electron. Its values vary from zero to $(n-1)$ in steps of unity.

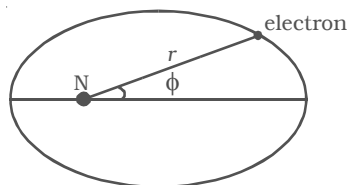


Fig 6.13 Sommerfeld atom model

This orbital quantum number (l) is useful in finding the possible elliptical orbits. The possible elliptical orbits are such that

$$\frac{b}{a} = \frac{l+1}{n}$$

where a and b are semi-major and semi-minor axes respectively of the ellipse.

According to Sommerfeld's model, for any principal quantum number n , there are n possible orbits of varying eccentricities called sub-orbits or sub-shells. Out of n subshells, one is circular and the remaining (i.e., $n-1$) are elliptical in shape.

These possible sub-orbits possess slightly different energies because of the relativistic variation of the electron mass.

Consider the first energy level ($n=1$). When $n = 1$, $l = 0$ i.e., in this energy level, there is only one orbit or sub-shell for the electron. Also, when $a = b$, the two axes of the ellipse are equal. As a result of this, the orbit corresponding to $n=1$ is circular. This subshell is designated as s sub-shell. Since, this sub-shell belongs to $n=1$, it is designated as $1s$ (Fig 6.14a).

Similarly, for the second energy level $n=2$, there are two permissible sub-shells for the electrons. For $n=2$, l can take two values, 0 and 1.

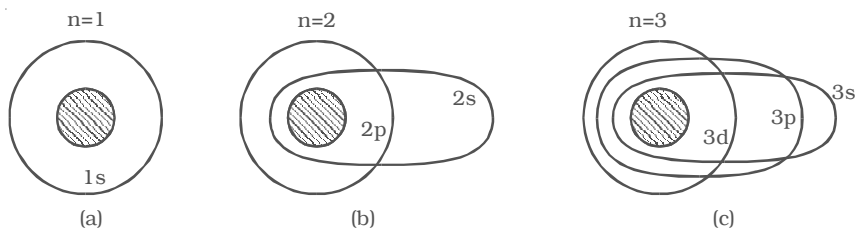


Fig 6.14 Various sub-shells for the electrons

When $n = 2$, $l = 0$.

$$\frac{b}{a} = \frac{0+1}{2} = \frac{1}{2} \quad (\text{or}) \quad b = \frac{a}{2}$$

This subshell corresponding to $l = 0$ is elliptical in shape and is designated as $2s$.

when $n = 2$, $l = 1$.

$$\frac{b}{a} = \frac{1+1}{2} = \frac{2}{2} = 1 \quad (\text{or}) \quad b = a$$

This sub-shell corresponding to $l = 1$ is circular in shape and is designated as $2p$ (Fig 6.14b).

For $n = 3$, l has three values 0, 1 and 2, i.e. there are three permissible sub-shells for the electrons.

when $n = 3$, $l = 0$.

$$\frac{b}{a} = \frac{0+1}{3} = \frac{1}{3} \quad (\text{or}) \quad b = \frac{a}{3}$$

when $n = 3$, $l = 1$.

$$\frac{b}{a} = \frac{1+1}{3} = \frac{2}{3} \quad (\text{or}) \quad b = \frac{2a}{3}$$

and when $n = 3$, $l = 2$.

$$\frac{b}{a} = \frac{2+1}{3} = \frac{3}{3} \quad (\text{or}) \quad b = a$$

The sub-shells corresponding to $l = 0, 1$ and 2 are designated as $3s, 3p$ and $3d$ respectively. The circular shell is designated as $3d$ and the other two are elliptical in shape (Fig 6.14c).

It is common practice to assign letters to l -values as given below:

Orbital quantum number l	:	0	1	2	3	4
electron state	:	s	p	d	f	g

Hence, electrons in the $l = 0, 1, 2, 3 \dots$ states are said to be in the $s, p, d, f \dots$ states.

Fine structure of spectral line

Based on Sommerfeld atom model, the total energy of an electron in the elliptical orbit can be shown as,

$$E_n = \frac{-me^4Z^2}{8\epsilon_0^2h^2n^2}$$

This expression is the same as that obtained by Bohr. Thus the introduction of elliptical orbits gives no new energy levels and hence no new transition. In this way, the attempt of Sommerfeld to explain the fine structure of spectral lines failed. But soon, on the basis of variation of mass of electron with velocity, Sommerfeld could find the solution for the problem of the fine structure of the spectral lines.

According to Sommerfeld, the velocity of the electron is maximum when the electron is nearest to the nucleus and minimum when it is farthest from the nucleus, since the orbit of the electron is

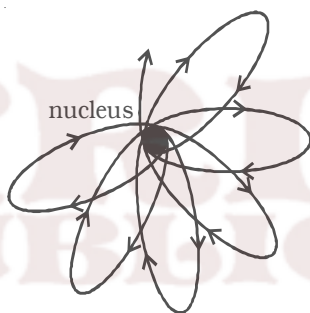


Fig 6.15 Rosette path of an electron

elliptical. This implies that the effective mass of the electron will be different at different parts of its orbit. Taking into account the relativistic variation of the mass of the electron, Sommerfeld modified his theory and showed that the path of electron is not a simple ellipse but a precessing ellipse called a rosette (Fig 6.15).

Based on this idea, Sommerfeld successfully explained the fine structure of spectral lines of hydrogen atom.

Drawbacks

(i) Though Sommerfeld's modification gave a theoretical background of the fine structure of spectral lines of hydrogen, it could not predict the correct number of observed fine structure of these lines.

(ii) It could not explain the distribution and arrangement of electrons in atoms.

(iii) Sommerfeld's model was unable to explain the spectra of alkali metals such as sodium, potassium etc.

(iv) It could not explain Zeeman and Stark effect.

(v) This model does not give any explanation for the intensities of the spectral lines.

6.3. X-rays

A German scientist, Wilhelm Roentgen, in 1895, discovered X-rays when he was studying the phenomenon of discharge of electricity through gases.

After performing a series of experiments, Roentgen concluded that when a beam of fast moving electrons strike a solid target, an invisible penetrating radiation is produced. Due to the unknown nature of the radiation, Roentgen called these radiations as X - rays.

X-rays are electromagnetic waves of short wavelength in the range of 0.5 \AA to 10 \AA . Roentgen was awarded Nobel prize in 1901 for the discovery of X-rays.

6.3.1 Production of X-rays - Modern Coolidge tube

X-rays are produced, when fast moving electrons strike a metal target of suitable material. The basic requirement for the production of X-rays are: (i) a source of electrons, (ii) effective means of accelerating the electrons and (iii) a target of suitable material of high atomic weight.

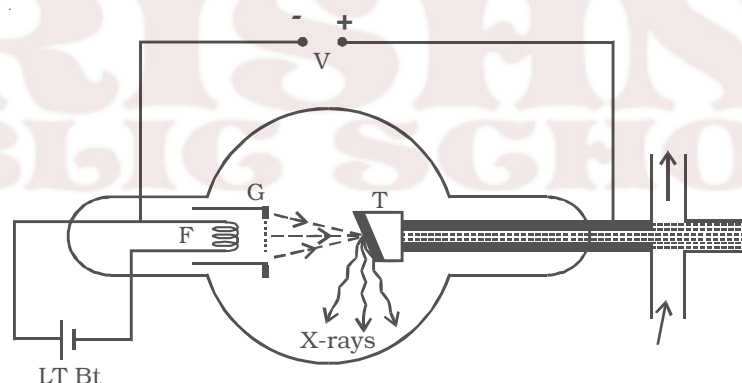


Fig 6.16 Coolidge tube

The modern type of X-ray tube designed by Coolidge is shown in Fig 6.16. It consists of a highly evacuated hard glass bulb containing a cathode and an anode target. The pressure inside the tube is 10^{-6} mm of mercury. The cathode is a tungsten filament F and is heated by passing a current through it from a low tension battery. The electrons are emitted by the process of thermionic emission from the cathode. The filament is surrounded by a molybdenum cylinder G kept at a

negative potential to the filament. Hence, the electrons emitted from the filament are collimated into a fine pencil of electron beam.

The target T consists of a copper block in which a piece of tungsten or molybdenum is fixed. The anode should have the following characteristics :

(i) high atomic weight – to produce hard X-rays

(ii) high melting point – so that it is not melted due to the bombardment of fast moving electrons, which cause lot of heat generation.

(iii) high thermal conductivity – to carry away the heat generated.

The face of the copper anode is sloped at about 45° to the electron beam. Being good conductor of heat, copper helps to conduct the heat efficiently to the water cooling system. A high potential of about 20 kV is applied between filament F and the target T. Due to this high potential difference, the electrons emitted from the filament are accelerated. When these accelerated electrons strike the target, they give up their kinetic energy and thereby produce X-rays.

The intensity of X-rays depends upon the number of electrons striking the target. i.e. the rate of emission of electrons from the filament. This can be controlled by varying the filament current.

6.3.2 Soft X-rays and Hard X-rays

X-rays are of two types : (i) Soft X-rays and (ii) Hard X-rays

(i) Soft X-rays

X-rays having wavelength of 4\AA or above, have lesser frequency and hence lesser energy. They are called soft X – rays due to their low penetrating power. They are produced at comparatively low potential difference.

(ii) Hard X-rays

X-rays having low wavelength of the order of 1\AA have high frequency and hence high energy. Their penetrating power is high, therefore they are called hard X-rays. They are produced at comparatively high potential difference.

The wavelength of X-rays depends upon the kinetic energy of the electrons producing them and this kinetic energy depends upon the potential difference between the filament and the target.

6.3.3 Properties of X-rays

(i) X-rays are electromagnetic waves of very short wave length. They travel in straight lines with the velocity of light. They are invisible to eyes.

(ii) They undergo reflection, refraction, interference, diffraction and polarisation.

(iii) They are not deflected by electric and magnetic fields. This indicates that X-rays do not have charged particles.

(iv) They ionize the gas through which they pass.

(v) They affect photographic plates.

(vi) X-rays can penetrate through the substances which are opaque to ordinary light e.g. wood, flesh, thick paper, thin sheets of metals.

(vii) When X-rays fall on certain metals, they liberate photo electrons (Photo electric effect).

(viii) X-rays have destructive effect on living tissue. When the human body is exposed to X-rays, it causes redness of the skin, sores and serious injuries to the tissues and glands. They destroy the white corpuscles of the blood.

(ix) X-rays do not pass through heavy metals such as lead and bones. If such objects are placed in their path, they cast their shadow.

6.3.4 Detection of X-rays

The basic properties which are generally used for the detection of X-rays are : (i) blackening of a photographic plate and (ii) the ionization produced by X-rays in a gas or vapour. An ionization chamber, which utilizes the property of ionization, is generally used to detect and measure the intensity of X-rays.

6.3.5 Absorption of X-rays

When X-rays pass through any material, a part of the X-ray energy is absorbed by the material. If a beam of X-rays of intensity I passes through a length dx of any material, its intensity is decreased by dI . For any given material, the amount of absorbed intensity is μdx where μ is called the absorption co-efficient. i.e., $dI = -\mu I dx$

6.3.6 Diffraction of X-rays

Soon after the discovery of X-rays, Schuster pointed out that X-rays behave as electromagnetic waves of wavelength much shorter than that of visible light. Attempts were made to measure the wavelength of X-rays by means of diffraction gratings which proved unsuccessful, as the grating failed to disperse X-rays on account of their very small wavelength. Obviously, diffraction effects can only be observed if the spacing between the lines ruled on the grating is of the order of magnitude of wavelength of the wave used. Thus, in order to diffract X-rays, grating with much finer rulings, having distance between rulings comparable to the wavelength of X-rays are required. It is impossible to construct a grating of such fine dimensions artificially.

In a crystal, the atoms or molecules are arranged symmetrically in a three dimensional space. Any plane containing an arrangement of atoms is known as lattice plane or cleavage plane. The spacing between the atoms is of the order of 10^{-10} m, comparable to the wavelength of X-rays. It was suggested that the regular arrangement of atoms or molecules in the cleavage planes of a crystal might provide a grating element suitable to diffract X-rays. The crystal might serve as a three dimensional grating, whereas optical grating is a two dimensional one.

6.3.7 Laue experiment

Von Laue, in 1913, suggested that a crystal can act as a three dimensional grating for an X-ray beam. The experimental arrangement used to produce diffraction in X-rays by Laue is shown in Fig 6.17(a). X-rays from the X-ray tube is collimated into a fine beam by two slits

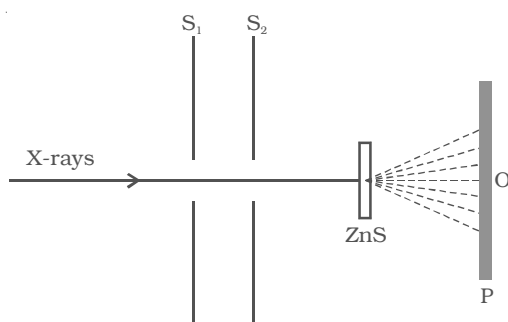


Fig 6.17 (a) Laue experimental set up

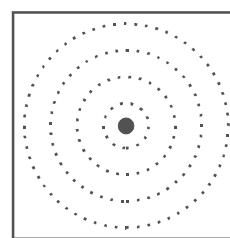


Fig 6.17 (b) Laue spot

S_1 and S_2 . The beam is now allowed to pass through a zinc sulphide (ZnS) crystal. The emergent rays are made to fall on a photographic plate P . The diffraction pattern so obtained consists of a central spot at O and a series of spots arranged in a definite pattern about O as shown in Fig 6.17b. The central spot is due to the direct beam, whereas the regularly arranged spots are due to the diffraction pattern from the atoms of the various crystal planes. These spots are known as Laue spots. The Laue experiment has established following two important facts :

- (i) X-rays are electro magnetic waves of extremely short wave length.
- (ii) The atoms in a crystal are arranged in a regular three dimensional lattice.

6.3.8 Bragg's law for X-ray diffraction

W.L. Bragg and W.H. Bragg studied the diffraction of X-rays in detail and used a crystal of rock salt to diffract X-rays and succeeded in measuring the wavelength of X-rays.

Consider homogeneous X-rays of wave length λ incident on a crystal at a glancing angle θ . The incident rays AB and DE after reflection from the lattice planes Y and Z travel along BC and EF respectively as shown in Fig 6.18.

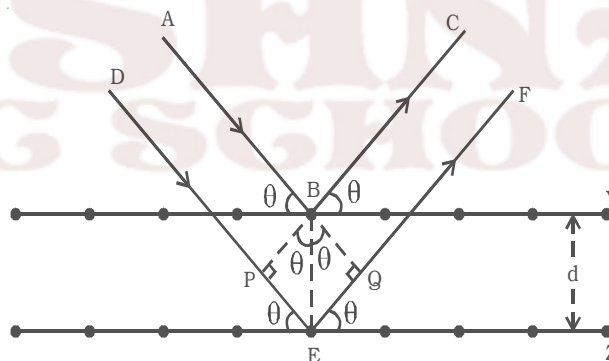


Fig 6.18 Bragg's law

Let the crystal lattice spacing between the planes be d . BP and BQ are perpendiculars drawn from B on DE and EF respectively. Therefore, the path difference between the two waves ABC and DEF is equal to $PE + EQ$.

$$\text{In the } \triangle PBE, \sin \theta = \frac{PE}{BE} \text{ (or) } PE = BE \sin \theta = d \sin \theta$$

In the $\triangle QBE$, $\sin \theta = \frac{EQ}{BE}$ (or) $EQ = BE \sin \theta = d \sin \theta$

\therefore Path difference = $PE + EQ = d \sin \theta + d \sin \theta = 2d \sin \theta$

If this path difference $2d \sin \theta$ is equal to integral multiple of wavelength of X-ray i.e. $n\lambda$, then constructive interference will occur between the reflected beams and they will reinforce with each other. Therefore the intensity of the reflected beam is maximum.

$\therefore 2d \sin \theta = n\lambda$

where, $n = 1, 2, 3 \dots$ etc.

This is known as Bragg's law.

6.3.9 Bragg's X-ray spectrometer

Bragg's spectrometer used to determine the wavelength of X-rays is shown in Fig 6.19. Bragg's spectrometer is similar in construction to an ordinary optical spectrometer.

X-rays from an X-ray tube are made to pass through two fine slits S_1 and S_2 which collimate it into a fine pencil. This fine X-ray beam is then made to fall upon the crystal 'C' (usually sodium chloride crystal) mounted on the spectrometer table. This table is capable of rotation about a vertical axis and its rotation can be read on a circular graduated scale S. The reflected beam after passing through the slits S_3 and S_4

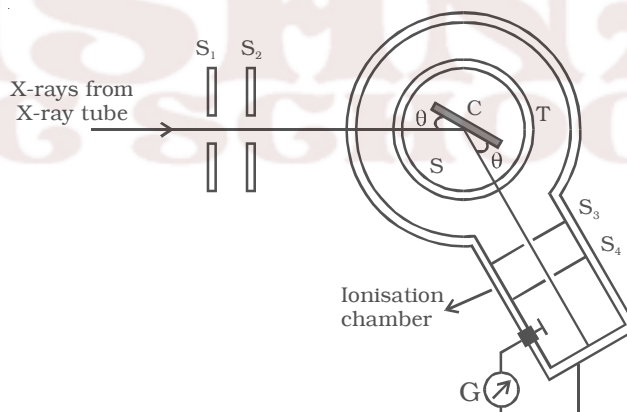


Fig 6.19 Bragg's spectrometer

enters the ionization chamber. The X-rays entering the ionization chamber ionize the gas which causes a current to flow between the electrodes and the current can be measured by galvanometer G . The ionization current is a measure of the intensity of X-rays reflected by the crystal.

The ionization current is measured for different values of glancing angle θ . A graph is drawn between the glancing angle θ and ionization current (Fig 6.20). For certain values of glancing angle, the ionization current increases abruptly. The

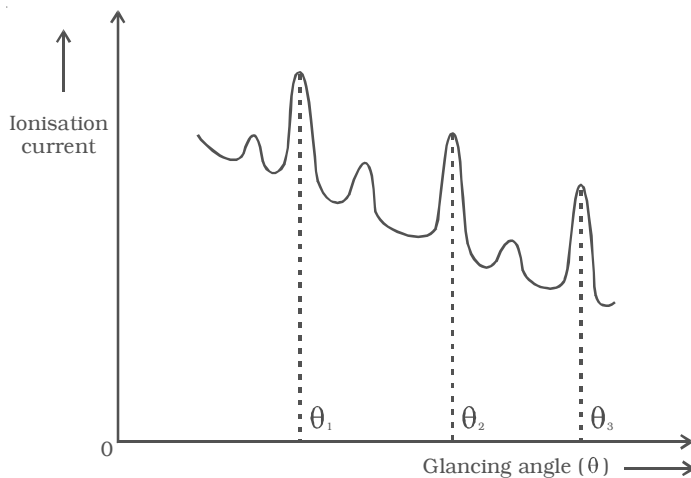


Fig 6.20 Plot of glancing angle and ionisation current

The first peak corresponds to first order, the second peak to second order and so on. From the graph, the glancing angles for different orders of reflection can be measured. Knowing the angle θ and the spacing d for the crystal, wavelength of X-rays can be determined.

6.3.10 X-ray spectra - continuous and characteristic X-ray spectra.

The spectrum from an X-ray tube contains two distinct parts :

(i) Continuous X-ray spectra

It consists of radiations of all possible wavelengths, from a certain lower limit to higher values continuously, as in the case of visible light.

Origin - Continuous X-ray spectra

X-rays are produced, when high velocity electrons strike the target material of high atomic number. It has also been mentioned in the production of X-rays, that most of the energy of the electrons goes into the heating of the target material.

A few fast moving electrons penetrate deep into the interior of the atoms of the target material and are attracted towards the nuclei by the attractive forces of their nuclei. Due to these forces, the electrons get deflected from their original path. As a result of this, the electrons are decelerated, and hence energy of the electron decreases continuously. This loss of energy during retardation is given off in the form of X-rays

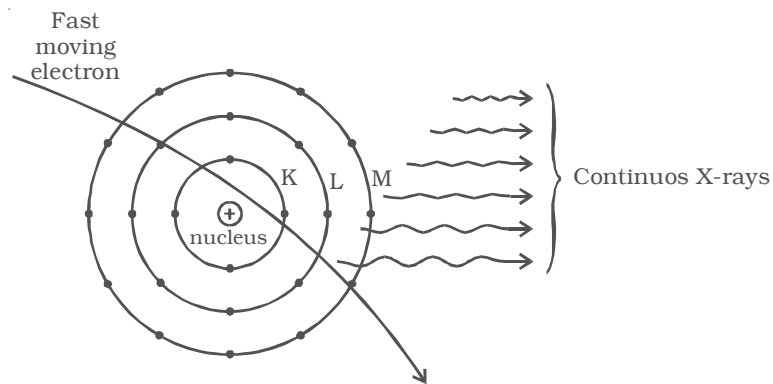


Fig 6.21 Origin of continuous X – rays

of continuously varying wavelength. The X – rays consist of continuous range of frequencies upto a maximum frequency ν_{\max} or minimum wave length λ_{\min} . This is called as continuous X – rays. The minimum wave length depends on the anode voltage. If V is the potential difference between the anode and the cathode

$$eV = h\nu_{\max} = \frac{hc}{\lambda_{\min}} \quad [\because \nu = \frac{c}{\lambda}]$$

The minimum wavelength of the given radiation is,

$$\lambda_{\min} = \frac{hc}{eV}$$

where h is Planck's constant, c is the velocity of light and e , the charge of the electron. Substituting the known values in the above equation.

$$\lambda_{\min} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times V} = \frac{12400}{V} \text{ \AA}$$

For the given operating voltage, the minimum wave length is same for all metals.

(ii) Characteristic X-ray spectra

It consists of definite, well defined wavelengths superimposed on the continuous spectrum. These spectral lines generally occur in the form of small groups and are characteristic of the material of the target.

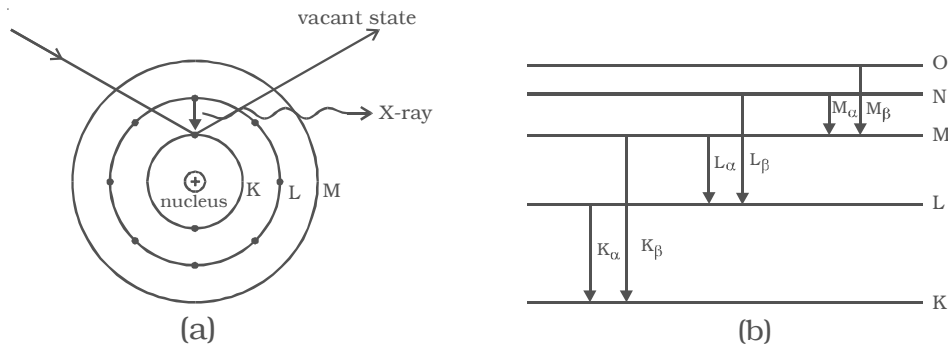


Fig 6.22 Characteristic X-ray spectra

Origin - Characteristic X-ray spectra

Few of the fast moving electrons having velocity of about $(1/10)^{\text{th}}$ of the velocity of light may penetrate the surface atoms of the target materials and knock out the tightly bound electrons even from the inner most shells (like K, L shells) of the atom. Fig 6.22a shows the case, when the fast moving electrons knock off one electron from K-Shell and the vacancy is filled by the nearby electron from the L shell. During this transition, the energy difference is radiated in the form of X-rays of very small wave length. This corresponds to K_{α} - line of the series. The frequency ν_1 of this line is given by the relation $(E_K - E_L) = h\nu_1$. Suppose, the electron from M shell jumps to the K shell, it gives out K_{β} line and so on. If an electron jumps from the M-Shell to the vacant state in L-Shell, it contributes L_{α} line and if the vacancy in L-Shell is filled up by an electron of N shell, it contributes L_{β} and so on (Fig 6.22b). The frequency of radiation depends upon the target material. The X-ray spectra consists of sharp lines and is the characteristic of target material. Hence this spectra is known as characteristic spectra.

6.3.11 Moseley's law

Moseley investigated systematically, the characteristic radiations emitted by different targets. Based on his experiments, he has concluded that the frequency of the spectral line in the characteristic X-ray spectrum is directly proportional to the square of the atomic number (Z) of the element considered. This is known as Moseley's law.

$$\text{i.e } \nu \propto Z^2 \quad \text{or} \quad \sqrt{\nu} = a(Z - b)$$

where a and b are constants depending upon the particular spectral line.

Applications of Moseley's law

(i) Any discrepancy in the order of the elements in the periodic table can be removed by Moseley's law by arranging the elements according to the atomic numbers and not according to the atomic weights.

(ii) Moseley's law has led to the discovery of new elements like hafnium (72), technetium (43), rhenium (75) etc.

(iii) This law has been helpful in determining the atomic number of rare earths, thereby fixing their position in the periodic table.

6.3.12 Applications of X-rays

X-rays have a number of applications. Some of them are listed below:

Medical applications

(i) X-rays are being widely used for detecting fractures, tumours, the presence of foreign matter like bullet etc., in the human body.

(ii) X-rays are also used for the diagnosis of tuberculosis, stones in kidneys, gall bladder etc.

(iii) Many types of skin diseases, malignant sores, cancer and tumours have been cured by controlled exposure of X-rays of suitable quality.

(iv) Hard X-rays are used to destroy tumours very deep inside the body.

Industrial applications

(i) X-rays are used to detect the defects or flaws within a material

(ii) X-rays can be used for testing the homogeneity of welded joints, insulating materials etc.

(iii) X-rays are used to analyse the structure of alloys and the other composite bodies.

(iv) X-rays are also used to study the structure of materials like rubber, cellulose, plastic fibres etc.

Scientific research

(i) X-rays are used for studying the structure of crystalline solids and alloys.

(ii) X-rays are used for the identification of chemical elements including determination of their atomic numbers.

(iii) X-rays can be used for analyzing the structure of complex molecules by examining their X-ray diffraction pattern.

6.4 Laser

The light emitted from an ordinary light source is incoherent, because the radiation emitted from different atoms has no definite phase relationship with each other. For interference of light, coherent sources are essential. Two independent sources cannot act as coherent sources. For experimental purposes, from a single source, two coherent sources are obtained. In recent years, some sources have been developed, which are highly coherent known as LASER. The word 'Laser' is an acronym for Light Amplification by Stimulated Emission of Radiation. The difference between ordinary light and laser beam is pictorially depicted in Fig 6.23.

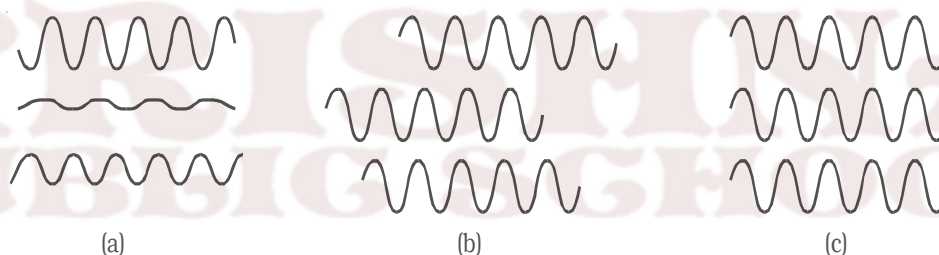


Fig 6.23 Wave forms from (a) ordinary light source
(b) monochromatic source and (c) laser source

6.4.1 Characteristics of laser

The laser beam (i) is monochromatic. (ii) is coherent, with the waves, all exactly in phase with one another, (iii) does not diverge at all and (iv) is extremely intense

6.4.2 Spontaneous and stimulated emission

An atom may undergo transition between two energy states E_1 and E_2 , if it emits or absorbs a photon of the appropriate energy $E_2 - E_1 = h\nu$.

In a system of thermal equilibrium, the number of atoms in the ground state (N_1) is greater than the number of atoms in the excited state (N_2). This is called normal population (Fig 6.24). Consider a sample of free atoms, some of which are

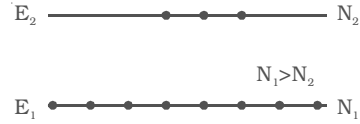


Fig 6.24 Normal population

in the ground state with energy E_1 and some in the excited energy state with energy E_2 . If photons of energy $h\nu = E_2 - E_1$ are incident on the sample, the photons can interact with the atoms in the ground state and are taken to excited state. This is called stimulated or induced absorption (Fig 6.25). The process by which the atoms in the ground

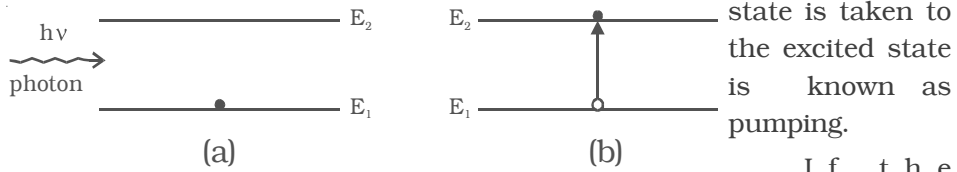


Fig 6.25 Stimulated or induced absorption

to the higher energy levels with the help of light, it is called optical pumping. If the atoms in the ground state are pumped to the excited state by means of external agency, the number of atoms in the excited state (N_2) becomes greater than the number of atoms in the ground state (N_1). This is called population inversion (Fig 6.26).



Fig 6.26 Population inversion

The life time of atoms in the excited state is normally 10^{-8} second. Some of the excited energy levels have greater life times for atoms (10^{-3} s). Such energy levels are called as the metastable states. If the excited energy level is an ordinary level, the excited atoms return to the lower (or) ground energy state immediately without the help of any external agency. During this transition (Fig 6.27), a photon of energy $E_2 - E_1 = h\nu$ is emitted. This is called spontaneous emission.

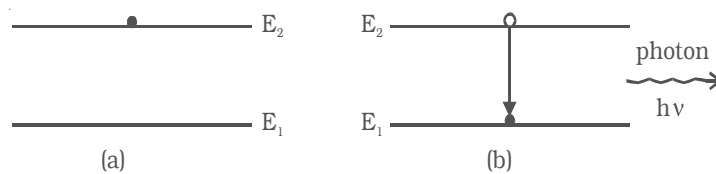


Fig 6.27 Spontaneous emission

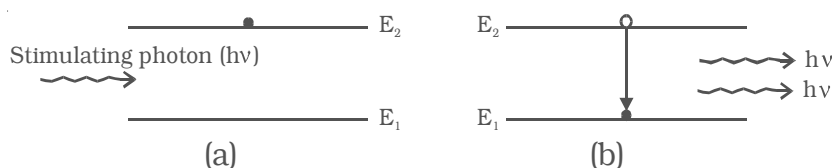


Fig 6.28 Stimulated emission

If the excited state is a metastable state, the atoms stay for some time in these levels. The atoms in such metastable state can be brought to the lower energy levels with the help of photons of energy $h\nu = E_2 - E_1$. During this process, a photon of energy $E_2 - E_1 = h\nu$ is emitted. This is known as stimulated emission (or) induced emission (Fig 6.28). A photon produced by stimulated emission is called secondary photon (or) stimulated photon. The secondary photon is always in phase with the stimulating photon. These photons in turn stimulate the emission further and the process continues to give a chain - reaction. This is called laser action and by this action all the emitted photons having same energy and same frequency are in phase with each other. Hence, a highly monochromatic, perfectly coherent, intense radiation is obtained in laser.

Conditions to achieve laser action

- (i) There must be an inverted population i.e. more atoms in the excited state than in the ground state.
- (ii) The excited state must be a metastable state.
- (iii) The emitted photons must stimulate further emission. This is achieved by the use of the reflecting mirrors at the ends of the system.

6.4.3 Ruby laser

The Ruby laser was first developed by T.Maiman in 1960. It consists of a single crystal of ruby rod of length 10 cm and 0.8 cm in diameter. A ruby is a crystal of aluminium oxide Al_2O_3 , in which some of aluminium ions (Al^{3+}) are replaced by the chromium ions (Cr^{3+}). The opposite ends of ruby rod are flat and parallel; one end is fully silvered and the other is partially silvered (i.e.) semi transparent. The ruby rod is surrounded by a helical xenon flash tube which provides the pumping light to raise the chromium ions to upper energy level (Fig 6.29). In the xenon flash tube, each flash lasts several milliseconds and in each flash a few thousand joules of energy is consumed.

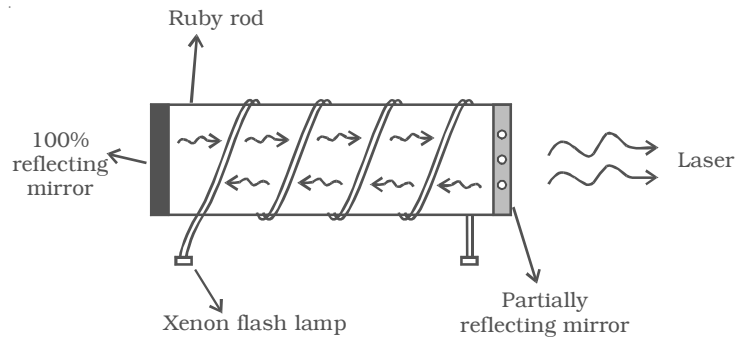


Fig 6.29 Ruby laser

The simplified energy level diagram of chromium ions in a ruby laser, indicating appropriate excitation and decay is shown in Fig 6.30. In normal state, most of the chromium ions are in the ground state E_1 . When the ruby rod is irradiated by a flash of light, the 5500 \AA radiation (green colour) photons are absorbed by the chromium ions which are

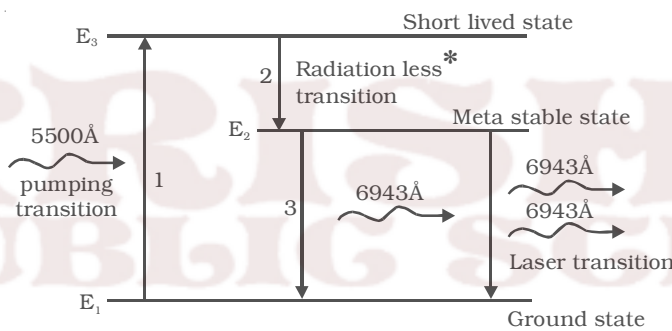


Fig 6.30 Energy level diagram for ruby laser

pumped to the excited state E_3 . The excited ion gives up part of its energy to the crystal lattice and decays without giving any radiation to the meta stable state E_2 . Since, the state E_2 has a much longer lifetime (10^{-3} s), the number of ions in this state goes on increasing. Thus population inversion is achieved between the states E_2 and E_1 . When the excited ion from the metastable state E_2 drops down spontaneously to the ground state E_1 , it emits a photon of wavelength 6943 \AA . This photon travels through the ruby rod and is reflected back and forth by the silvered ends until it stimulates other excited ion and causes it to emit a fresh photon in phase with stimulating photon. Thus the reflections will amount to the additional stimulated emission – the so called amplification by stimulated

* In this process, radiation emitted, is used to heat up the system, hence no radiation comes out of the system.

emission. This stimulated emission is the laser transition. Finally, a pulse of red light of wave length 6943 \AA emerges through the partially silvered end of the crystal.

6.4.4. Helium - neon laser

A continuous and intense laser beam can be produced with the help of gas lasers. A simplified diagram showing basic features of a He - Ne gas laser is shown in Fig 6.31.

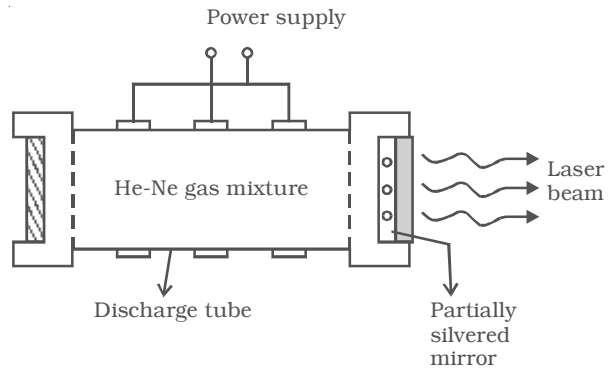


Fig 6.31 He - Ne laser

He - Ne laser system consists of a quartz discharge tube containing helium and neon in the ratio of 1 : 4 at a total pressure of about 1 mm of Hg. One end of the tube is fitted with a perfectly reflecting mirror and the other end with partially reflecting mirror.

A powerful radio frequency generator is used to produce a discharge in the gas, so that the helium atoms are excited to a higher energy level.

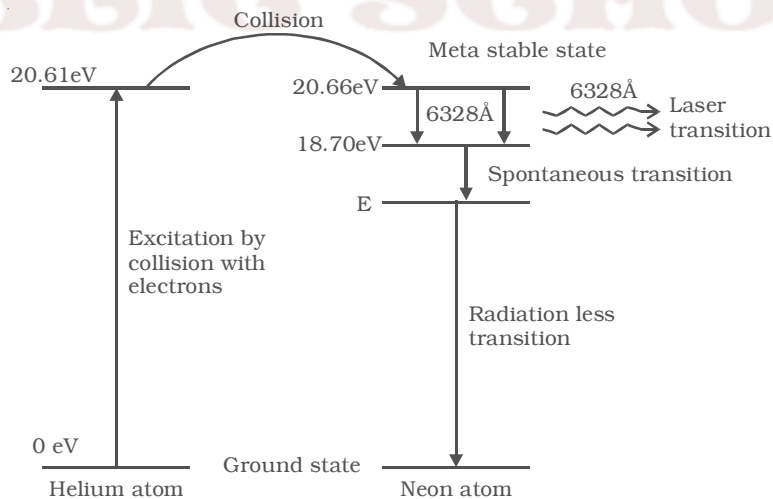


Fig 6.32 Energy level diagram for He-Ne laser system

The energy level diagram for *He* and *Ne* atoms are shown in Fig 6.32. When an electric discharge passes through the gas, the electron in the discharge tube collide with the *He* and *Ne* atoms and excite them to metastable states of energy 20.61 eV and 20.66 eV respectively above the ground state. Some of the excited helium atoms transfer their energy to unexcited Ne atoms by collision. Thus, He atom help in achieving a population inversion in Ne atoms. When an excited Ne atom drops down spontaneously from the metastable state at 20.66 eV to lower energy state at 18.70 eV, it emits a 6328 Å photon in the visible region. This photon travelling through the mixture of the gas, is reflected back and forth by the reflector ends, until it stimulates an excited neon atom and causes it to emit a fresh 6328 Å photon in phase with the stimulating photon. This stimulated transition from 20.66 eV level to 18.70 eV level is the laser transition. The output radiations escape from the partially reflecting mirror. The neon atoms drop down from the 18.70 eV level to lower state E, through spontaneous emission emitting incoherent light. From this level E, the Ne atoms are brought to the ground state through collision with the walls of the tube. Hence the final transition is radiationless.

6.4.5 Applications of laser

Due to high coherence, high intensity, laser beams have wide applications in various branches of science and engineering.

Industrial applications

- (i) The laser beam is used to drill extremely fine holes in diamonds, hard sheets etc.,
- (ii) They are also used for cutting thick sheets of hard metals and welding.
- (iii) The laser beam is used to vapourize the unwanted material during the manufacture of electronic circuit on semiconductor chips.
- (iv) They can be used to test the quality of the materials.

Medical applications

- (i) In medicine, micro surgery has become possible due to narrow angular spread of the laser beam.
- (ii) It can be used in the treatment of kidney stone, tumour, in cutting and sealing the small blood vessels in brain surgery and retina detachment.
- (iii) The laser beams are used in endoscopy.
- (iv) It can also be used for the treatment of human and animal cancer.

Scientific and Engineering applications

(i) Since the laser beam can stay on at a single frequency, it can be modulated to transmit large number of messages at a time in radio, television and telephone.

(ii) The semiconductor laser is the best light source for optical fiber communication.

(iii) Narrow angular spread of the laser beam makes it a very useful tool for microwave communication. Communication with earth satellites and in rocketry. Laser is also used in accurate range finders for detecting the targets.

(iv) The earth-moon distance has been measured with the help of lasers.

(v) It is used in laser Raman Spectroscopy.

(vi) Laser is also used in holography (three dimensional lensless photography)

(vii) Laser beam can determine precisely the distance, velocity and direction as well as the size and form of the objects by means of the reflected signal as in radar.

Holography

When an object is photographed by a camera, a two dimensional image of three dimensional object is obtained. A three dimensional image of an object can be formed by holography. In ordinary photography, the amplitude of the light wave is recorded on the photographic film. In holography, both the phase and amplitude of the light waves are recorded on the film. The resulting photograph is called hologram.

6.4.6 MASER

The term MASER stands for Microwave Amplification by Stimulated Emission of Radiation. The working of maser is similar to that of laser. The maser action is based on the principle of population inversion followed by stimulated emission. In maser, the emitted photon, during the transition from the metastable state belongs to the microwave frequencies. The paramagnetic ions are used as maser materials. Practical maser materials are often chromium or gadolinium ions doped as impurities in ionic crystals. Ammonia gas is also a maser material. Maser provides a very strong tool for analysis in molecular spectroscopy.

Solved Problems

- 6.1 A beam of electrons moving with a uniform speed of $4 \times 10^7 \text{ ms}^{-1}$ is projected normal to the uniform magnetic field where $B = 1 \times 10^{-3} \text{ Wb/m}^2$. What is the path of the beam in magnetic field?

Data : $v = 4 \times 10^7 \text{ ms}^{-1}$
 $B = 1 \times 10^{-3} \text{ Wb/m}^2$
path of the electron, $r = ?$

Solution : Since, the electrons are released normally to the magnetic field, the electrons travel in a circular path.

$$\therefore Bev = \frac{mv^2}{r} \quad (\text{or}) \quad r = \frac{mv}{Be}$$
$$r = \frac{9.1 \times 10^{-31} \times 4 \times 10^7}{1 \times 10^{-3} \times 1.6 \times 10^{-19}}$$
$$r = 0.2275 \text{ m.}$$

- 6.2 In Thomson's experiment for the measurement of $\left(\frac{e}{m}\right)$ of an electron, the beam remains undeflected, when the electric field is $E = 10^5 \text{ V/m}$ and the magnetic field is $B = 10^{-2} \text{ tesla}$. The beam was originally accelerated through a potential difference of 285 volt. Calculate the value of the specific charge $\left(\frac{e}{m}\right)$ of the electron.

Data : $E = 10^5 \text{ Vm}^{-1}$, $B = 10^{-2} \text{ tesla}$, $V = 285 \text{ volt}$, $\frac{e}{m} = ?$

Solution : The energy of the electron = eV

$$\text{The kinetic energy of the electron} = \frac{1}{2}mv^2$$

$$\therefore eV = \frac{1}{2}mv^2 \quad (\text{or}) \quad \frac{e}{m} = \frac{v^2}{2V}$$

But, under the action of crossed electric and magnetic fields,

$$Bev = Ee \text{ (or) } v = \frac{E}{B}$$

$$\therefore \frac{e}{m} = \frac{E^2}{2B^2V} = \frac{(10^5)^2}{2 \times (10^{-2})^2 \times 285}$$

$$\frac{e}{m} = 1.754 \times 10^{11} \text{ C kg}^{-1}$$

- 6.3 In Millikan's experiment, an oil drop of mass 4.9×10^{-14} kg is balanced by applying a potential difference of 2 kV between the two plates which are 8 mm apart. Calculate the number of elementary charges on the drop. Take $g = 10 \text{ ms}^{-2}$.

Data : $m = 4.9 \times 10^{-14}$ kg; $V = 2 \text{ kV} = 2 \times 10^3$ V;

$d = 8 \text{ mm} = 8 \times 10^{-3}$ m; $g = 10 \text{ ms}^{-2}$;

$n =$ number of elementary charges on the drop = ?

Solution : $Eq = mg$ (or) $q = \frac{mg}{E}$

But $E = \frac{V}{d}$

$$\therefore q = \frac{mg}{(V/d)} = \frac{mgd}{V} = \frac{4.9 \times 10^{-14} \times 10 \times 8 \times 10^{-3}}{2 \times 10^3}$$

$$q = 1.96 \times 10^{-18} \text{ C}$$

But $q = ne$

$$\therefore n = \frac{q}{e} = \frac{1.96 \times 10^{-18}}{1.6 \times 10^{-19}} = 12.25 \approx 12$$

- 6.4 An α - particle is projected with an energy of 4 MeV directly towards a gold nucleus. Calculate the distance of its closest approach. Given : atomic number of gold = 79 and atomic number of α particle = 2.

Data : energy of the α - particle = 4 MeV = 4×10^6 eV

$$= 4 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$E_K = 6.4 \times 10^{-13} \text{ J.}$$

$$Z = 79, r_o = ?$$

$$\text{Solution : } r_o = \frac{2Ze^2}{4\pi\epsilon_o E_K} = \frac{2 \times 79 \times (1.6 \times 10^{-19})^2 \times 9 \times 10^9}{6.4 \times 10^{-13}}$$

$$r_o = 5.688 \times 10^{-14} \text{ m.}$$

6.5 Wavelength of Balmer first line is 6563Å. Calculate the wavelength of second line.

Data : For Balmer I line, $n_1 = 2, n_2 = 3, \lambda_1 = 6563\text{Å}$

For Balmer II line, $n_1 = 2, n_2 = 4, \lambda_2 = ?$

Solution : For Balmer I line,

$$\frac{1}{\lambda_1} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\frac{1}{\lambda_1} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = R \left[\frac{5}{36} \right] \quad \dots(1)$$

For Balmer II line,

$$\frac{1}{\lambda_2} = R \left[\frac{1}{2^2} - \frac{1}{4^2} \right] = \frac{3}{16} R \quad \dots(2)$$

Dividing equation (1) by equation (2),

$$\frac{\lambda_2}{\lambda_1} = \frac{R(5/36)}{R(3/16)} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$$

$$\text{(or) } \lambda_2 = \lambda_1 \cdot \left(\frac{20}{27} \right) = 6563 \times \frac{20}{27} = 4861 \text{ Å}$$

6.6 On the basis of Bohr's theory, calculate the velocity and time period of revolution of the electron in the innermost orbit ($n=1$) of the hydrogen atom. Given : Bohr's Radius (r_1) = 0.53Å.

Data : $n = 1, r_1 = 0.53\text{Å} = 0.53 \times 10^{-10} \text{ m}$

$v = ?, T = ?$

$$\text{Solution : Kinetic energy of the electron} = \frac{1}{2} m v_n^2 = \frac{Z e^2}{8\pi\epsilon_o r_n}$$

$$\frac{1}{2} m v_n^2 = \frac{Z e^2}{8 \pi \epsilon_0 \left[\frac{n^2 h^2 \epsilon_0}{\pi m Z e^2} \right]} \quad \left[\because r_n = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2} \right]$$

$$\text{(or)} \quad v_n = \frac{Z e^2}{2 \epsilon_0 n h}$$

For hydrogen atom $Z = 1$ and for $n = 1$,

$$\begin{aligned} v_1 &= \frac{e^2}{2 \epsilon_0 h} = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.854 \times 10^{-12} \times 6.626 \times 10^{-34}} \\ &= 2.1818 \times 10^6 \text{ ms}^{-1} \end{aligned}$$

(Note: Velocity can also be calculated using Bohr's first postulate)

$$\text{Period} = \text{Time taken for one revolution} = T = \frac{2 \pi r_1}{v_1}$$

$$\begin{aligned} \therefore T &= \frac{2 \times 3.142 \times 0.53 \times 10^{-10}}{2.1818 \times 10^6} \quad \left[\because \text{time} = \frac{\text{distance}}{\text{velocity}} \right] \\ T &= 1.5265 \times 10^{-16} \text{ s} \end{aligned}$$

6.7 The Rydberg constant for hydrogen is $1.097 \times 10^7 \text{ ms}^{-1}$. Calculate the short and long wavelength limits of Lyman series.

Data : $R = 1.097 \times 10^7 \text{ m}^{-1}$

For short wavelength limit of Lyman Series,

$$n_1 = 1, n_2 = \infty, \lambda_s = ?$$

For long wavelength limit of Lyman series,

$$n_1 = 1, n_2 = 2, \lambda_l = ?$$

Solution: The wave number for Lyman series is,

$$\bar{\nu} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

For short wavelength limit,

$$\bar{\nu}_s = \frac{1}{\lambda_s} = R \left[\frac{1}{1^2} - \frac{1}{(\infty)^2} \right] = R$$

$$\text{(or)} \quad \lambda_s = \frac{1}{R} = \frac{1}{1.097 \times 10^7} = 911.6 \text{ \AA}$$

For long wavelength limit,

$$\frac{1}{\lambda_l} = \frac{1}{\lambda_1} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = \frac{3}{4} R$$

$$\lambda_l = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7}$$

$$\lambda_l = 1215 \text{ \AA}$$

- 6.8 In Bragg's spectrometer, the glancing angle for first order spectrum was observed to be 8° . Calculate the wavelength of X-ray, if $d = 2.82 \times 10^{-10}$ m. At what angle will the second maximum occur?

Data : For $n = 1$, $\theta_1 = 8^\circ$; $d = 2.82 \times 10^{-10}$ m; $\lambda = ?$

For $n = 2$, $\theta_2 = ?$

Solution : When $n = 1$, $2d \sin \theta_1 = (1) \lambda$

$$\text{(or)} \quad \lambda = 2 \times 2.82 \times 10^{-10} \times \sin 8^\circ = 0.7849 \text{ \AA}$$

When $n = 2$, $2d \sin \theta_2 = (2) \lambda$

$$\sin \theta_2 = \frac{\lambda}{d} = \frac{0.7849 \times 10^{-10}}{2.82 \times 10^{-10}} = 0.2783$$

$$\theta_2 = \sin^{-1} (0.2783) = 16^\circ 9'$$

- 6.9 Calculate the longest wavelength that can be analysed by a rock salt crystal of spacing $d = 2.82 \text{ \AA}$ in the first order.

Data : $d = 2.82 \text{ \AA} = 2.82 \times 10^{-10}$ m ; $n = 1$; $\lambda_{\max} = ?$

Solution : For longest wavelength, $(\sin \theta)_{\max} = 1$

$$\therefore 2d (\sin \theta)_{\max} = \lambda_{\max}$$

$$\text{(or)} \quad \lambda_{\max} = \frac{2 \times 2.82 \times 10^{-10} \times 1}{1}$$

$$\lambda_{\max} = 5.64 \times 10^{-10} \text{ m}$$

6.10 Monochromatic X-ray of wavelength 1\AA , when falls on a crystal, successive reflections take place at angles 30° and 45° respectively. Find the lattice constant of the crystal.

Data : $\theta_1 = 30^\circ$; $\theta_2 = 45^\circ$; $\lambda = 1\text{\AA}$; $d = ?$

Solution :

$$n\lambda = 2d \sin \theta_1 \quad \dots(1)$$

$$(n+1)\lambda = 2d \sin \theta_2 \quad \dots(2)$$

equation (2) - equation (1)

$$\lambda = 2d (\sin \theta_2 - \sin \theta_1)$$

$$\begin{aligned} \therefore d &= \frac{\lambda}{2(\sin \theta_2 - \sin \theta_1)} = \frac{1 \times 10^{-10}}{2(\sin 45^\circ - \sin 30^\circ)} \\ &= \frac{1}{2\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)} = \frac{1}{\sqrt{2} - 1} = 0.414 \end{aligned}$$

$$\therefore d = 2.415 \text{\AA}$$

6.11 Hydrogen atom in its ground state is excited by means of a monochromatic radiation of wavelength 970.6\AA . How many different transitions are possible in the resulting emission spectrum? Find the longest wavelength amongst these. (Ionisation energy of hydrogen atom in its ground state is 13.6 eV and take $h = 6.6 \times 10^{-34} \text{ Js}$)

Data : Wavelength of incident radiation = $970.6 \text{\AA} = 970.6 \times 10^{-10} \text{ m}$

Ionisation energy of hydrogen atom in its ground state = 13.6 eV

(i) number of possible transitions = ?

(ii) longest wavelength emitted = ?

Solution : Energy of the excited state,

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{970.6 \times 10^{-10} \times 1.6 \times 10^{-19}} = 12.75 \text{ eV}$$

$$E_n = -13.6 + 12.75 = -0.85 \text{ eV}$$

$$E_n = \frac{-13.6}{n^2}$$

$$\text{or } n^2 = \frac{-13.6}{E_n} = \frac{-13.6}{-0.85} = 16$$

$$\text{or } n = 4$$

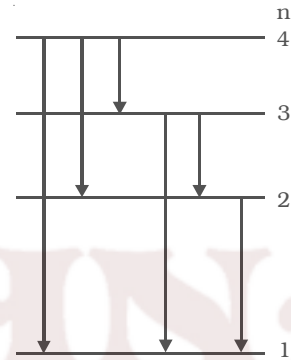
The number of possible transitions in going to the lower state and hence the number of different wavelengths in the spectrum will be six (shown in the figure)

The longest wavelength corresponds to minimum energy difference, (ie for transition $4 \rightarrow 3$)

$$E_3 = \frac{-13.6}{3^2} = -1.51 \text{ eV}$$

$$\frac{hc}{\lambda_{\max}} = E_4 - E_3$$

$$\begin{aligned} \text{or } \lambda_{\max} &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{(1.51 - 0.85) \times 1.6 \times 10^{-19}} \\ &= 18.75 \times 10^{-7} \text{ m} \\ \lambda_{\max} &= 18750 \text{ \AA} \end{aligned}$$



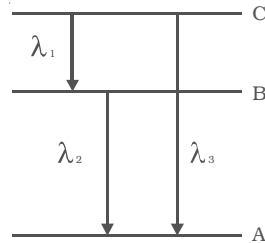
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PUBLIC SCHOOL

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

- 6.1 The cathode rays are
(a) a stream of electrons (b) a stream of positive ions
(c) a stream of uncharged particles (d) the same as canal rays
- 6.2 A narrow electron beam passes undeviated through an electric field $E = 3 \times 10^4$ V/m and an overlapping magnetic field $B = 2 \times 10^{-3}$ Wb/m². The electron motion, electric field and magnetic field are mutually perpendicular. The speed of the electron is
(a) 60 ms^{-1} (b) $10.3 \times 10^7 \text{ ms}^{-1}$
(c) $1.5 \times 10^7 \text{ ms}^{-1}$ (d) $0.67 \times 10^{-7} \text{ ms}^{-1}$
- 6.3 According to Bohr's postulates, which of the following quantities take discrete values?
(a) kinetic energy (b) potential energy
(c) angular momentum (d) momentum
- 6.4 The ratio of the radii of the first three Bohr orbit is,
(a) $1 : 1/2 : 1/3$ (b) $1 : 2 : 3$
(c) $1 : 4 : 9$ (d) $1 : 8 : 27$
- 6.5 The first excitation potential energy or the minimum energy required to excite the atom from ground state of hydrogen atom is,
(a) 13.6 eV (b) 10.2eV
(c) 3.4 eV (d) 1.89 eV
- 6.6 According to Rutherford atom model, the spectral lines emitted by an atom is,
(a) line spectrum (b) continuous spectrum
(c) continuous absorption spectrum (d) band spectrum
- 6.7 Energy levels A, B, C of a certain atom correspond to increasing values of energy (i.e.,) $E_A < E_B < E_C$. If $\lambda_1, \lambda_2, \lambda_3$ are the wavelengths of radiations corresponding to the transitions C to B, B to A and C to A respectively, which of the following statements is correct.

- (a) $\lambda_3 = \lambda_1 + \lambda_2$
 (b) $\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$
 (c) $\lambda_1 = \lambda_2 + \lambda_3 = 0$
 (d) $\lambda^2_3 = \lambda^2_1 + \lambda^2_2$



- 6.8 The elliptical orbits of electron in the atom were proposed by
 (a) J.J.Thomson (b) Bohr
 (c) Sommerfeld (d) de Broglie
- 6.9 X-ray is
 (a) phenomenon of conversion of kinetic energy into radiation.
 (b) conversion of momentum
 (c) conversion of energy into mass
 (d) principle of conservation of charge
- 6.10 In an X-ray tube, the intensity of the emitted X-ray beam is increased by
 (a) increasing the filament current
 (b) decreasing the filament current
 (c) increasing the target potential
 (d) decreasing the target potential
- 6.11 The energy of a photon of characteristic X-ray from a Coolidge tube comes from
 (a) the kinetic energy of the free electrons of the target
 (b) the kinetic energy of ions of the target
 (c) the kinetic energy of the striking electron
 (d) an atomic transition in the target.
- 6.12 A Coolidge tube operates at 24800 V. The maximum frequency of X-radiation emitted from Coolidge tube is
 (a) 6×10^{18} Hz (b) 3×10^{18} Hz
 (c) 6×10^8 Hz (d) 3×10^8 Hz

- 6.13 In hydrogen atom, which of the following transitions produce a spectral line of maximum wavelength
- (a) $2 \rightarrow 1$ (b) $4 \rightarrow 1$
(c) $6 \rightarrow 5$ (d) $5 \rightarrow 2$
- 6.14 In hydrogen atom, which of the following transitions produce a spectral line of maximum frequency
- (a) $2 \rightarrow 1$ (b) $6 \rightarrow 2$
(c) $4 \rightarrow 3$ (d) $5 \rightarrow 2$
- 6.15 After pumping process in laser,
- (a) the number of atoms in the ground state is greater than the number of atoms in the excited state.
(b) the number of atoms in the excited state is greater than the number of atoms in the ground state.
(c) the number of atoms in the ground state is equal to the number atoms in the excited state.
(d) No atoms are available in the excited state.
- 6.16 The chromium ions doped in the ruby rod
- (a) absorbs red light (b) absorbs green light
(c) absorbs blue light (d) emits green light
- 6.17 What are cathode rays?
- 6.18 Write the properties of cathode rays.
- 6.19 Describe the J.J. Thomson method for determining the specific charge of electron.
- 6.20 Describe Millikan's oil drop experiment to determine the charge of an electron.
- 6.21 Explain the results of Rutherford α -particle scattering experiment.
- 6.22 What are the drawbacks of Rutherford atom model?
- 6.23 State the postulates of Bohr atom model.
- 6.24 Obtain the expression for the radius of the n^{th} orbit of an electron based on Bohr's theory.
- 6.25 Prove that the energy of an electron for hydrogen atom in n^{th} orbit

$$\text{is, } E_n = \frac{-me^4}{8\epsilon_0^2 n^2 h^2} .$$

- 6.26 Explain the spectral series of hydrogen atom.
- 6.27 What is meant by energy level diagram?
- 6.28 What are the drawbacks of Sommerfeld atom model?
- 6.29 Define : excitation potential energy and ionization potential energy.
- 6.30 What are X-rays?
- 6.31 What are hard X-rays and soft x-rays?
- 6.32 Write the properties of X-rays?
- 6.33 Why ordinary plane transmission gratings cannot be used to produce diffraction effects in X-rays?
- 6.34 State and obtain Bragg's law.
- 6.35 Explain how a Bragg's spectrometer can be used to determine the wavelength of X-rays.
- 6.36 Explain the origin of characteristic x-rays.
- 6.37 State : Moseley's law.
- 6.38 Write the differences between spontaneous emission and stimulated emission.
- 6.39 What is meant by normal population?
- 6.40 What are the important characteristics of laser?
- 6.41 How does the laser light differ from ordinary light?
- 6.42 Explain the working of Ruby laser with neat sketch.
- 6.43 With the help of energy level diagram, explain the working of He-Ne laser.
- 6.44 What are various applications of laser in medical field?

Problems

- 6.45 An electron beam passes through a transverse magnetic field of 2×10^{-3} tesla and an electric field E of 3.4×10^4 V/m acting simultaneously. If the path of the electrons remain undeviated, calculate the speed of electrons. If the electric field is removed, what will be the radius of the electron path?
- 6.46 In Millikan's experiment, a drop of diameter 10^{-4} cm with a density 0.9 gm/cm^3 is observed. The plates are 2 cm apart. A potential of 72 V applied across the plates keeps the drops just in balance. How many electronic charges are there in the oil drop?

- 6.47 Calculate the distance of closest approach of α -particles to the copper nucleus when α -particles of 5 MeV are scattered back by a thin sheet of copper (Z for copper = 29).
- 6.48 For the Paschen series, calculate the wavelength of the first member and the series limit. Given R , Rydberg constant = $1.094 \times 10^7 \text{m}^{-1}$.
- 6.49 An X-ray diffraction of a crystal gave the closest line at an angle of $6^\circ 27'$. If the wavelength of X-ray is 0.58\AA , find the distance between the two cleavage planes.
- 6.50 How much should be the voltage of an X-ray tube so that the electrons emitted from the cathode may give an X-ray of wavelength 1\AA after striking the target.
- 6.51 Calculate the mass of an electron from the known values of specific charge and charge of electron.
- 6.52 Find the minimum wavelength of X-rays produced by an X-ray tube at 1000 kV.

Answers

- 6.1** (a) **6.2** (c) **6.3** (c) **6.4** (c)
- 6.5** (b) **6.6** (b) **6.7** (b) **6.8** (c)
- 6.9** (a) **6.10** (a) **6.11** (d) **6.12** (a)
- 6.13** (c) **6.14** (a) **6.15** (b) **6.16** (b)
- 6.45** $v = 1.7 \times 10^7 \text{ms}^{-1}$; $r = 4.834 \times 10^{-2} \text{m}$
- 6.46** $n = 8$ **6.47** $r_o = 1.670 \times 10^{-14} \text{m}$
- 6.48** $\lambda_1 = 18752 \text{\AA}$; $\lambda_\alpha = 8204 \text{\AA}$ **6.49** $d = 2.581 \text{\AA}$
- 6.50** $V = 12.4 \text{kV}$ **6.51** $m = 9.11 \times 10^{-31} \text{kg}$
- 6.52** $\lambda_{\min} = 0.0124 \text{\AA}$

7. Dual Nature of Radiation and Matter and Relativity

The true nature of light is difficult to assess. Experiments showed that light exhibited wavelike properties of diffraction and interference. On the other hand, photoelectric effect indicates that light has the aspects of a particle photon, with both energy and momentum. Thus light exhibits a wave-particle duality.

The wave-particle duality was extended to particles as matter waves by Louis de Broglie. His theoretical study on the nature of particles and waves led to the invention of a new mechanics of particles called quantum mechanics.

7.1 Photoelectric effect

Photoelectric emission is the phenomena by which a good number of substances, chiefly metals, emit electrons under the influence of radiation such as γ rays, X-rays, ultraviolet and even visible light. This effect was discovered by Heinrich Hertz in 1887 while working with resonance electrical circuits. A year later, Hallwachs, Elster and Geitel investigated the phenomenon with a simple experimental arrangement.

7.1.1 Hallwachs Experiment

Hallwachs experimental set-up to study the photo electric effect is shown in Fig 7.1. It consists of an evacuated quartz bulb with two zinc plates cathode C and anode A. The plates are connected to a battery and a sensitive galvanometer. In the absence of any radiation incident on the plates, there is no flow of current and hence there is no deflection in the galvanometer. But, when an electro magnetic radiation like ultraviolet radiation is allowed to fall on the plate C which is connected to the negative terminal of the battery, a current begins to flow, indicated by the deflection in the galvanometer (G). But, when ultraviolet radiation is

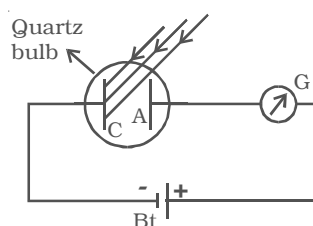


Fig 7.1 Photo-electric effect

made to fall on A, there is no deflection in the galvanometer. These observations reveal that the particles emitted by the plate C due to the photoelectric effect are negatively charged. These particles were found to be electrons. The observed current known as the photoelectric current is due to the flow of electrons.

After the study of photoelectric effect by Hallwachs, scientists J.J.Thomson, Lenard, Richardson, Compton did a series of experiments to study the relationship between photoelectric current, intensity of incident radiation, velocity and the kinetic energy of the photo electrons, and their dependence on the wave length of incident radiation used.

7.1.2 Effect of intensity of incident radiation on photo electric current

Keeping the frequency of the incident radiation and the potential difference between the cathode and the anode at constant values, the intensity of incident radiation is varied. The corresponding photoelectric current is measured in the microammeter.

It is found that the photo electric current increases linearly with the intensity of incident radiation (Fig 7.2).

Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second, it implies that the number of photoelectrons emitted per second is proportional to the intensity of incident radiation.

7.1.3 Effect of potential difference on the photoelectric current

For a given metallic surface C, keeping the intensity (I_1) and frequency of the incident radiation constant, the effect of potential difference between the plates on the photoelectric current can be studied. Fig 7.3 shows the variation of photo electric current with the potential difference V between the two plates. When the positive potential of A is increased, the photoelectric current is also increased. However, if the positive potential is further increased such that it is large enough to collect all the photo electrons emitted from the plate C, the photoelectric current reaches a certain maximum value and this

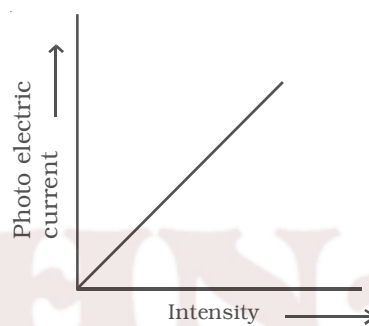


Fig 7.2 Variation of photoelectric current with intensity of incident radiation.

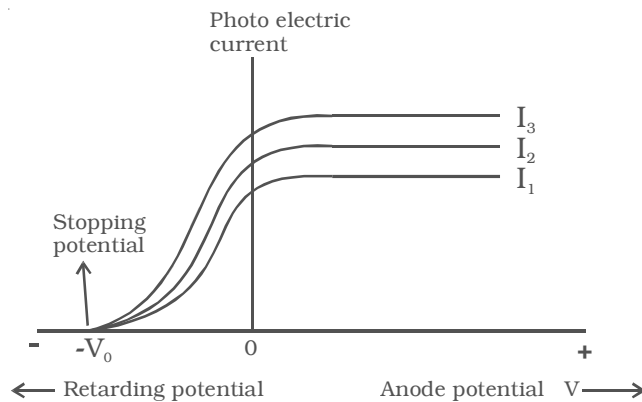


Fig 7.3 Variation of photo electric current with potential difference.

current is known as saturation current. If the potential of the plate A is made negative, the photocurrent does not immediately drop to zero but flows in the same direction as for positive potential. This shows that the photo electrons are emitted from the plate C with finite velocity. If the negative or retarding potential is further increased, the photo current decreases and finally becomes zero at a particular value. Thus, the minimum negative (retarding) potential given to the anode for which the photo electric current becomes zero is called the cut-off or stopping potential.

If m is the mass of the photo electron emitted with a velocity v_{max} , then the kinetic energy associated with it is $\frac{1}{2}mv_{max}^2$.

Since at the stopping potential V_0 , the fastest electron is just prevented from reaching the plate A, workdone in bringing the fastest electron to rest = kinetic energy of the fastest electron.

$$eV_0 = \frac{1}{2}mv_{max}^2$$

The above equation indicates that the stopping potential depends upon the velocity of the fastest electron.

The experiment is repeated with the incident radiation of same frequency, but of higher intensities I_2 and I_3 . It is found from the graph, the saturation currents are proportional to the intensities of the radiation. But, the stopping potential remains the same for all the intensities. Thus, for a given frequency of incident radiation, the stopping potential is independent of its intensity.

7.1.4 Effect of frequency of incident radiation on stopping potential

Keeping the photosensitive plate (C) and intensity of incident radiation a constant, the effect of frequency of the incident radiations

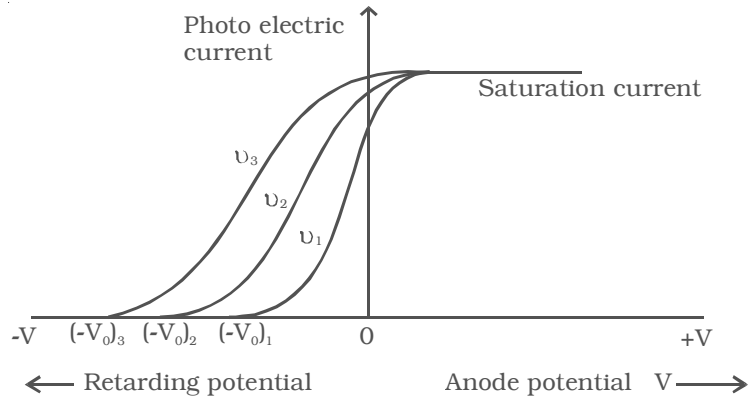


Fig 7.4 Variation of photoelectric current with applied potential difference for different frequency of incident radiation.

on stopping potential is studied. Fig 7.4 shows the variation of the photo electric current with the applied potential difference V for three different frequencies. From the graph, it is found that higher the frequency of the incident radiation, higher is the value of stopping potential V_o . For frequencies $\nu_3 > \nu_2 > \nu_1$, the corresponding stopping potentials are in the same order $(V_o)_3 > (V_o)_2 > (V_o)_1$.

It is concluded from the graph that, the maximum kinetic energy of the photoelectrons varies linearly with the frequency of incident radiation but is independent of its intensity.

If the frequency of the incident radiation is plotted against the corresponding stopping potential, a straight line is obtained as shown in Fig 7.5. From this graph, it is found that at a frequency ν_o , the value of the

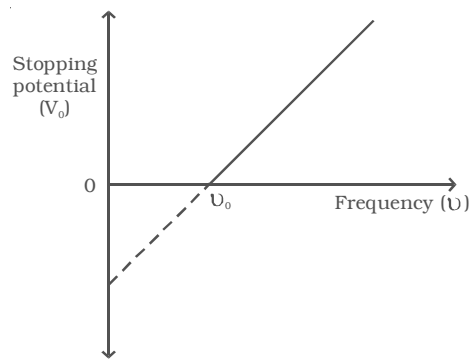


Fig 7.5 Variation of stopping potential with frequency of incident radiation.

stopping potential is zero. This frequency is known as the threshold frequency for the photo metal used. The photoelectric effect occurs above this frequency and ceases below it. Therefore, threshold frequency is defined as the minimum frequency of incident radiation below which the photoelectric emission is not possible completely, however high the intensity of incident radiation may be. The threshold frequency is different for different metals.

7.1.5 Laws of photoelectric emission

The experimental observations on photoelectric effect may be summarized as follows, which are known as the fundamental laws of photoelectric emission.

(i) For a given photo sensitive material, there is a minimum frequency called the threshold frequency, below which emission of photoelectrons stops completely, however great the intensity may be.

(ii) For a given photosensitive material, the photo electric current is directly proportional to the intensity of the incident radiation, provided the frequency is greater than the threshold frequency.

(iii) The photoelectric emission is an instantaneous process. i.e. there is no time lag between the incidence of radiation and the emission of photo electrons.

(iv) The maximum kinetic energy of the photo electrons is directly proportional to the frequency of incident radiation, but is independent of its intensity.

7.1.6 Light waves and photons

The electromagnetic theory of light proposed by Maxwell could not explain photoelectric effect. But, Max Planck's quantum theory successfully explains photoelectric effect. According to Planck's quantum theory, light is emitted in the form of discrete packets of energy called 'quanta' or photon. The energy of each photon is $E = h\nu$, where h is Planck's constant.

Photon is neither a particle nor a wave. In the phenomena like interference, diffraction, polarisation, the photon behaves like a wave. In the phenomena like emission, absorption and interaction with matter (photo electric effect) photon behaves as a particle. Hence light photon has a dual nature.

7.1.7 Einstein's photoelectric equation

In 1905, Albert Einstein, successfully applied quantum theory of radiation to photoelectric effect.

According to Einstein, the emission of photo electron is the result of the interaction between a single photon of the incident radiation and an electron in the metal. When a photon of energy $h\nu$ is incident on a metal surface, its energy is used up in two ways :

(i) A part of the energy of the photon is used in extracting the electron from the surface of metal, since the electrons in the metal are bound to the nucleus. This energy W spent in releasing the photo electron is known as photoelectric work function of the metal. The work function of a photo metal is defined as the minimum amount of energy required to liberate an electron from the metal surface.

(ii) The remaining energy of the photon is used to impart kinetic energy to the liberated electron.

If m is the mass of an electron and v , its velocity then

Energy of the incident photon =
Work function + Kinetic energy of the electron

$$h\nu = W + \frac{1}{2} mv^2 \quad \dots(1)$$

If the electron does not lose energy by internal collisions, as it escapes from the metal, the entire energy $(h\nu - W)$ will be exhibited as the kinetic energy of the electron. Thus, $(h\nu - W)$ represents the maximum kinetic energy of the ejected photo electron. If v_{max} is the maximum velocity with which the photoelectron can be ejected, then

$$h\nu = W + \frac{1}{2} mv_{max}^2 \quad \dots(2)$$

This equation is known as Einstein's photoelectric equation.

When the frequency (ν) of the incident radiation is equal to the threshold frequency (ν_0) of the metal surface, kinetic energy of the electron is zero. Then equation (2) becomes,

$$h\nu_0 = W \quad \dots(3)$$

Substituting the value of W in equation (2) we get,

$$h\nu - h\nu_0 = \frac{1}{2} mv_{max}^2 \quad \text{(or)} \quad h(\nu - \nu_0) = \frac{1}{2} mv_{max}^2$$

This is another form of Einstein's photoelectric equation.

7.1.8 Experimental verification of Einstein's photoelectric equation

Einstein's photoelectric equation is,

$$\frac{1}{2} mv^2 = h(\nu - \nu_0) \quad \dots(1)$$

If V_0 is the stopping potential and e , the electronic charge, then

$$\frac{1}{2} mv^2 = eV_0 \quad \dots(2)$$

From equations (1) and (2),

$$eV_0 = h(\nu - \nu_0)$$

$$\text{(or)} \quad V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{h}{e}\right)\nu_0 \quad \dots(3)$$

This is an equation of a straight line. Millikan verified equation (3) experimentally and found that it is in harmony with the observed facts.

7.2 Photoelectric cells and their applications

The photoelectric cell is a device which converts light energy into electrical energy. The photo electric cells are of three types:

- (i) Photo emissive cell
- (ii) Photo voltaic cell and
- (iii) Photo conductive cell

In this section, a simple photo emissive cell is discussed. It consists of a highly evacuated bulb B made of glass or quartz. A semi cylindrical metal plate C connected to the negative terminal of a battery, acts as cathode. This plate is coated with a low work function material such as caesium oxide, in order to get large number of photo electrons. A thin platinum wire A is connected to the positive terminal of the battery and kept along the axis of the metal plate C and this serves as the anode as shown in Fig 7.6.

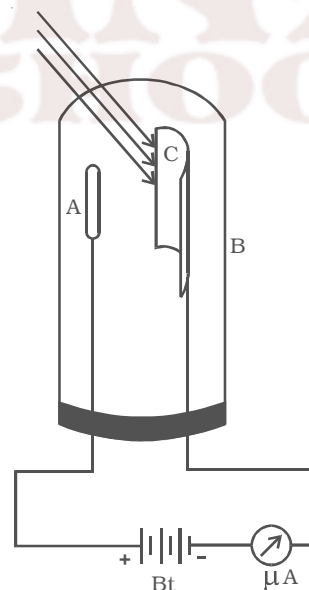


Fig 7.6 Photo emissive cell

When a light of suitable wave length falls on the cathode, photo electrons are emitted, which are attracted by the anode A. The resulting current is measured by a micro ammeter. The current produced by this type of cell is proportional to the intensity of the incident light for a given frequency.

Applications of photo electric cells

(i) Photoelectric cells are used for reproducing sound in cinematography.

(ii) They are used for controlling the temperature of furnaces.

(iii) Photoelectric cells are used for automatic switching on and off the street lights.

(iv) Photoelectric cells are used in the study of temperature and spectra of stars.

(v) Photoelectric cells are also used in obtaining electrical energy from sunlight during space travel.

(vi) These cells are used in instruments measuring light illumination.

(vii) These cells are used in opening and closing of door automatically.

(viii) Photoelectric cells are used in burglar alarm and fire alarm. In burglar alarm, ultraviolet light is continuously made to fall on the photo-cell installed at the door-way. A person entering the door interrupts the beam falling on the photo-cell. The abrupt change in photocurrent is used to start an electric bell ringing. In fire alarm, a number of photo-cells are installed at suitable places in a building. In the event of breaking out of fire, light radiations fall upon the photo-cell. This completes the electric circuit through an electric bell or a siren which starts operating as a warning signal.

7.3 Matter waves

The radiant energy has dual aspects of particle and wave, hence a natural question arises, if radiation has a dual nature, why not the matter. In 1924, a French Physicist Louis de Broglie put forward the bold hypothesis that moving particles should possess wave like

properties under suitable conditions. He reasoned this idea, from the fact, that nature is symmetrical and hence the basic physical entities – matter and energy should have symmetrical characters. If radiation shows dual aspects, so should matter.

7.3.1 de Broglie's wavelength of matter waves

de Broglie equated the energy equations of Planck (wave) and Einstein (particle).

For a wave of frequency ν , the energy associated with each photon is given by Planck's relation,

$$E = h\nu \quad \dots(1)$$

where h is Planck's constant.

According to Einstein's mass energy relation, a mass m is equivalent to energy,

$$E = mc^2 \quad \dots(2)$$

where c is the velocity of light.

If, $h\nu = mc^2$

$$\therefore \frac{hc}{\lambda} = mc^2 \text{ (or) } \lambda = \frac{h}{mc} \quad \dots(3) \quad \left(\text{since } \nu = \frac{c}{\lambda}\right)$$

For a particle moving with a velocity v , if $c = v$ from equation (3)

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \dots(4)$$

where $p = mv$, the momentum of the particle. These hypothetical matter waves will have appreciable wavelength only for very light particles.

7.3.2 de Broglie wavelength of an electron

When an electron of mass m and charge e is accelerated through a potential difference V , then the energy eV is equal to kinetic energy of the electron.

$$\frac{1}{2} mv^2 = eV \quad \text{(or) } v = \sqrt{\frac{2eV}{m}} \quad \dots(1)$$

The de Broglie wavelength is ,

$$\lambda = \frac{h}{mv}$$

Substituting the value of v ,

$$\lambda = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2meV}} \quad \dots(2)$$

Substituting the known values in equation (2),

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

If $V = 100$ volts, then $\lambda = 1.227 \text{ \AA}$ i.e., the wavelength associated with an electron accelerated by 100 volts is 1.227 \AA .

Since $E = eV$ is kinetic energy associated with the electron, the equation (2) becomes,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

7.3.3 Wave mechanical concept of atom

According to de Broglie's hypothesis, an electron of mass m in motion with a velocity v is associated with a wave whose wavelength λ is given by

$$\lambda = \frac{h}{mv} \quad \dots(1)$$

where h is Planck's constant.

On the basis of de Broglie's hypothesis, an atom model was proposed in which the stationary orbits of Bohr's model were retained, but with the difference that electron in various orbits behaves as a wave. This is shown in Fig 7.7.

It was suggested that stationary orbits are those in which orbital circumference ($2\pi r$) is an integral multiple of de Broglie wavelength λ ,

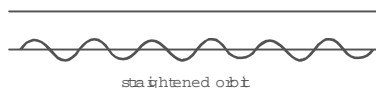
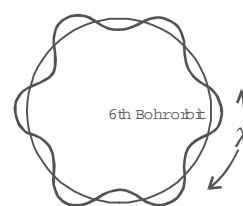


Fig 7.7 Wave concept of atom

i.e., stationary orbits for an electron are those which contain the complete waves of electron.

$$\text{Thus, } 2\pi r = n\lambda \quad \dots(2)$$

where $n = 1, 2, 3 \dots$ and r is the radius of the circular orbit.

Substituting equation (1) in equation (2),

$$2\pi r = n\lambda \left(\frac{h}{mv} \right) \quad (\text{or}) \quad mv r = \frac{nh}{2\pi} \quad \dots(3)$$

From equation (3), it is seen that the total angular momentum of the moving electron is an integral multiple of $\frac{h}{2\pi}$. Thus, de Broglie's concept confirms the Bohr's postulate.

7.4 Electron microscope

The electron microscope, like optical microscope, is an instrument principally used in the research laboratory for magnifying small objects. The wave nature of moving electron is the basis for the electron microscope. The resolving power of a microscope is the least distance between two points which can be distinguished. The resolving power of a microscope is limited by the wave length of the radiation used. In optical microscope, the visible light is used to illuminate the object and the highest magnification obtained with the best optical microscope is about 2000. Since, the wavelength of X-rays is smaller than that of the visible light, one can think of having an X-ray microscope. However, X-rays cannot be focussed as visible radiations are focussed using lenses. On the other hand, electrons having de Broglie wavelength of the order of X-rays can be focussed easily using electric and magnetic fields and one can build a high resolving power microscope using electrons.

For electrons accelerated by a potential difference of about 60,000 volts, the wavelength is about 5×10^{-12} m. This is 10^5 times smaller than that of visible light. Hence the resolving power of an electron microscope will be 1,00,000 times greater than that of an optical microscope.

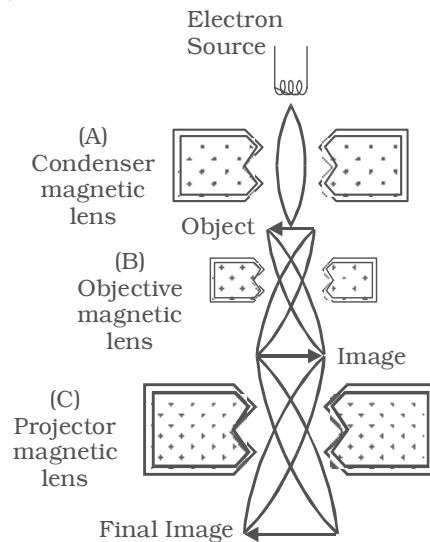


Fig 7.8 Schematic diagram of electron microscope

Construction and working

The schematic diagram of an electron microscope is shown in Fig 7.8. An electron microscope is similar in principle to an optical microscope. The modern electron microscope is usually of transmission type in which magnetic lenses of short focal length are used to obtain large magnification. An electron beam emitted by a filament is accelerated through a large potential difference in a device called electron gun. The fine beam of electrons is made to pass through the centre of the doughnut shaped electromagnet A (condenser magnetic lens). The electrons get deflected to form a parallel beam which strikes the object to be magnified. It should be noted that the electrons will be transmitted more through the transparent parts of the object and transmitted in less number through comparatively denser portions. The transmitted beam will thus have the likeness of the object traversed by it. The second electro magnet B (objective magnetic lens) causes the electron beam to diverge to produce enlarged image of the object. The electromagnet C (projector magnetic lens) focusses the electron beam from the part of the enlarged image on the fluorescent screen producing still greater

magnification. The image obtained on the fluorescent screen is made visible by scintillation for direct view. It can also be obtained on a suitable photographic plate for a permanent record. Sharp focussing is obtained by adjusting the intensity of magnetic fields produced by electro magnets. Since, the electron beam operates in vacuum, the apparatus is mounted in a chamber which is completely evacuated.

Uses:

(i) It is used in the industry, to study the structure of textile fibres, surface of metals, composition of paints etc.

(ii) In medicine and biology, it is used to study virus, and bacteria.

(iii) In Physics, it has been used in the investigation of atomic structure and structure of crystals in detail.

Limitations

An electron microscope is operated only in high vacuum. This prohibits the use of the microscope to study living organisms which would evaporate and disintegrate under such conditions.

7.5 Relativity

In Newtonian mechanics, mass, time, length and space were treated as absolute. It is regarded that space existed without any reference to objects. Space was considered absolute and at rest.

In the wave theory of light, it is assumed that a medium called ether exists in space and pervaded all space in the universe. But as per Michelson and Morley experiment, the concept of ether and absolute space become meaningless. In Einstein's view, there is no absolute space and all motions are relative. The time, mass, length and space are interdependent and are not absolute. Einstein, in 1905, profounded the special theory of relativity. The special theory of relativity deals with objects and systems, which are either moving at a constant speed with respect to one another or at rest.

Albert Einstein published his special theory of relativity and the theory of photo electric effect in the same year 1905. It was a very productive year for him - and for Physics.

7.5.1 Concept of Space

In classical mechanics, motion in absolute space led to two useful results.

(i) Fixed frame of reference by which the position or motion of any object in the universe could be measured.

(ii) The geometrical form of an object remains the same irrespective of changes in position or state of motion of the object or observer.

Concept of time

According to classical mechanics,

(i) The time interval between two events has the same value for all observers irrespective of their motion.

(ii) If two events are simultaneous for an observer, they are simultaneous for all observers, irrespective of their position or motion. This means simultaneity is absolute.

Concept of mass

In classical mechanics, the mass of the body is absolute and constant and independent of the motion of the body.

7.5.2 Frame of reference

A system of co-ordinate axes which defines the position of a particle in two or three dimensional space is called a frame of reference.

The simplest frame of reference is the Cartesian co-ordinate system in which the position of a particle is specified by three co-ordinates x, y and z . There are infinite number of reference frames in the universe. Our earth itself is a frame of reference. There are two types of frames of reference (i) inertial and (ii) non – inertial frames.

(i) Inertial (or) unaccelerated frames.

A frame of reference is said to be inertial, when the bodies in this frame obey Newton's law of inertia and other laws of Newtonian mechanics. In this frame, a body remains at rest or in continuous motion unless acted upon by an external force.

(ii) Non-inertial (or) accelerated frames

A frame of reference is said to be a non-inertial frame, when a body not acted upon by an external force, is accelerated. In this frame, Newton's laws are not valid.

7.6 Special theory of relativity

It is a direct outcome of new realisations of the nature of mass, length and time. It discards the absolute motion through space and deals with objects or observers, that are moving with uniform velocities relative to each other.

The two fundamental postulates of the special theory of relativity are :

- (i) The laws of Physics are the same in all inertial frames of reference.
- (ii) The velocity of light in free space is a constant in all the frames of reference.

7.6.1 Length contraction

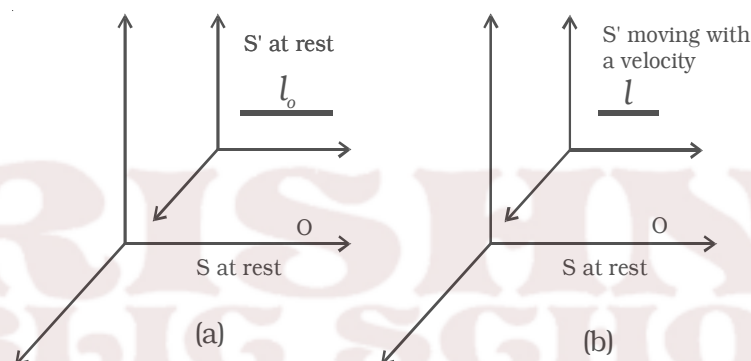


Fig 7.9 Length contraction

Consider two frames of references S and S' to be initially at rest. A rod is placed in the frame of reference S' and an observer O is in S (Fig 7.9). The length of the rod in S' as measured by the observer in S is l_0 .

Now the frame of reference S' moves with a velocity v along the positive X-axis. Now, the length of the rod is measured as l by the observer in S. Then

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

i.e., $l < l_0$

Thus the length of the rod moving with a velocity v relative to the observer at rest is contracted by a factor $\sqrt{1 - \frac{v^2}{c^2}}$ in the direction of motion. This is known as Lorentz – Fitzgerald contraction.

Example : A circular object will appear as an ellipse for a fast moving observer as shown in Fig 7.10.

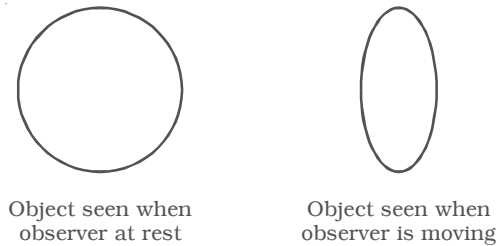


Fig. 7.10

7.6.2 Time dilation

Consider two frames S and S'. Let S' be moving with a velocity v with respect to S in the positive X-direction. Suppose a clock situated in the frame S' at a position gives out signals at an interval t_0 .

If this interval is observed by an observer in frame S, then the interval t recorded by him is

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

i.e. $t > t_0$

To a stationary observer in S, time interval appears to be lengthened by a factor $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$. In other words, a moving clock appears

to be slowed down to a stationary observer. This is known as time dilation.

Example : The clock in the moving space ships will appear to go slower than the clocks on the earth.

7.6.3 Variation of mass with velocity

According to Newtonian mechanics, the mass of a body does not change with velocity. But according to Einstein, the mass of the body changes with velocity. Einstein established the relation between the mass of a body at rest (m_0) and the mass of the same body moving with a velocity v is,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Example : Electrons accelerated in a particle accelerator, cyclotron with a very high velocity acquire increased mass, exactly as predicted by the above expression.

7.7 Einstein's mass-energy equivalence

Consider a body of rest mass m_0 . A force F is acting on it in X-direction. According to Newton's second law of motion, force is defined as the rate of change of momentum.

$$\text{i.e. } F = \frac{d}{dt} (mv) \quad \dots(1)$$

According to the theory of relativity, both mass and velocity are variable, therefore

$$F = m \frac{dv}{dt} + v \frac{dm}{dt} \quad \dots(2)$$

If a body is displaced through a distance dx due to the force F then, the increase in kinetic energy dE_k of the body is

$$\begin{aligned} dE_k &= Fdx \\ &= \left(m \frac{dv}{dt} + v \frac{dm}{dt} \right) dx \\ &= m dv \frac{dx}{dt} + v dm \frac{dx}{dt} \\ dE_k &= mv dv + v^2 dm \quad \dots(3) \end{aligned}$$

From Einstein's theory of relativity

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$m^2 = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$m^2 c^2 - m^2 v^2 = m_0^2 c^2$$

Differentiating we get,

$$c^2 2m dm - v^2 2m dm - m^2 2v dv = 0.$$

$$c^2 dm = mv dv + v^2 dm \quad \dots(4)$$

Comparing equations (3) and (4) we get,

$$dE_k = c^2 dm \quad \dots(5)$$

Thus the change in kinetic energy dE_k is directly proportional to the change in mass dm

When a body is at rest, its velocity is zero and $m = m_0$. When its velocity is v its mass becomes m . Therefore integrating equation (5)

$$\int_0^{E_k} dE_k = c^2 \int_{m_0}^m dm$$

$$E_k = c^2 (m - m_0) = mc^2 - m_0 c^2$$

This is the relativistic formula for kinetic energy. m_0 is the rest mass and $m_0 c^2$ is the internal energy (rest mass energy or rest energy).

∴ Total energy = kinetic energy of the moving body

+ rest mass energy

$$\begin{aligned} E &= E_k + m_0 c^2 \\ &= mc^2 - m_0 c^2 + m_0 c^2 \\ E &= mc^2 \end{aligned}$$

This is Einstein's mass-energy equivalence.

Implications of the equivalence between mass and energy

Particles like electron, proton, neutron have mass. If a particle has mass, it has rest energy $m_0 c^2$ and may or may not have other forms of energy such as kinetic energy and potential energy. The particle of light, the photon has zero mass but has energy.

It is possible to convert an isolated system of particles with mass into a system of particles with less mass, even zero mass. Similarly, it is possible to convert a particle of zero mass into a particle with mass.

(i.e) Rest energy is converted into other types of energy (mass is converted into energy) or other types of energy are converted into rest energy (energy is converted into mass). Hence, the statement “mass energy equivalence” comes true and the total energy is conserved in the isolated system.

Example : When an electron meets its antiparticle the positron, both of them annihilate and form two photons. Since, the total energy is conserved, the total energy associated with the electron - positron pair (kinetic energy + rest energy) is transferred to the photons that have no rest energy (no mass). Also, an energetic photon (zero mass) can create an electron - positron pair (particles with mass).

Other examples such as nuclear fission and fusion process are discussed in the next chapter nuclear physics.

Solved Problems

7.1 The work function of zinc is 6.8×10^{-19} J. What is the threshold frequency for emission of photoelectrons from zinc?

Data : $W = 6.8 \times 10^{-19}$ J $\nu_0 = ?$

Solution : Work function $W = h\nu_0$

$$\therefore h\nu_0 = 6.8 \times 10^{-19}$$

$$\nu_0 = \frac{6.8 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$\nu_0 = 1.026 \times 10^{15} \text{ Hz.}$$

7.2 A metallic surface when illuminated with light of wavelength 3333 \AA emits electrons with energies upto 0.6 eV . Calculate the work function of the metal.

Data : $\lambda = 3333 \text{ \AA}$, $K.E = 0.6 \text{ eV}$; $W = ?$

Solution : Work function, $W = h\nu - \text{kinetic energy}$

$$\text{or } W = \frac{hc}{\lambda} - K.E$$

$$= \left(\frac{6.626 \times 10^{-34} \times 3 \times 10^8}{3333 \times 10^{-10}} \right) - (0.6 \times 1.6 \times 10^{-19})$$

$$= (5.96 \times 10^{-19}) - (0.96 \times 10^{-19})$$

$$W = 5 \times 10^{-19} \text{ J}$$

$$W = \frac{5 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$W = 3.125 \text{ eV.}$$

- 7.3 Lithium has a work function of 2.3 eV. It is exposed to light of wavelength 4.8×10^{-7} m. Find the maximum kinetic energy with which the electron leaves the surface. What is the longest wavelength which can produce the photo electrons?
(Given $h = 6.626 \times 10^{-34}$ Js ; $e = 1.6 \times 10^{-19}$ C).

Data : $W = 2.3 \text{ eV}$; $h = 6.626 \times 10^{-34}$ Js ; $e = 1.6 \times 10^{-19}$ C.

longest wavelength = ? ; kinetic energy = ?

Solution : Kinetic energy of ejected electrons = $h\nu - W$

$$\text{But, } h\nu = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4.8 \times 10^{-7}} \text{ joule}$$

$$h\nu = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4.8 \times 10^{-7} \times 1.6 \times 10^{-19}}$$

$$h\nu = 2.588 \text{ eV}$$

\therefore Kinetic energy of ejected electrons

$$= 2.588 - 2.3$$

$$\text{K.E} = 0.288 \text{ eV}$$

$$\text{Work function } W = h\nu_0 = \frac{hc}{\lambda_0}$$

$$\therefore \lambda_0 = \frac{hc}{W}$$

$$\text{longest wavelength } \lambda_0 = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{2.3 \times 1.6 \times 10^{-19}}$$

$$\lambda_0 = 5.40 \times 10^{-7} \text{ m.}$$

- 7.4 Calculate the de Broglie wave length of an electron, if the speed is 10^5 ms^{-1} . (Given $m = 9.1 \times 10^{-31}$ kg; $h = 6.626 \times 10^{-34}$ Js)

Data : $m = 9.1 \times 10^{-31}$ kg ; $v = 10^5 \text{ ms}^{-1}$;

$h = 6.626 \times 10^{-34}$ Js ; $\lambda = ?$

Solution : Wave length $\lambda = \frac{h}{mv}$

$$= \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^5}$$

$$\lambda = 72.81 \text{ \AA}$$

7.5 What is the de Broglie wave length of an electron of kinetic energy 120 eV?

Data : $K.E = 120 \text{ eV} = 120 \times 1.6 \times 10^{-19} \text{ J} ; \lambda = ?$

Solution : $\lambda = \frac{h}{\sqrt{2mE}}$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 120 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1.121 \times 10^{-10} \text{ m.}$$

7.6 In the Bohr model of hydrogen atom, what is the de Broglie wave length λ for the electron when it is in the (i) $n = 1$ level and (ii) $n = 4$ level. In each case, compare the de Broglie wave length to the circumference of the orbit.

Data : $n = 1, n = 4, \lambda = ?$

Solution : By Bohr's first postulate,

$$\text{angular momentum of the electron} = \frac{nh}{2\pi}$$

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \frac{2\pi r}{n} = \frac{h}{mv} \quad \dots(1)$$

$$\text{But, } \lambda = \frac{h}{mv} \quad \dots(2) \quad (\text{de Broglie wave length})$$

From equations (1) and (2)

$$\lambda = \frac{2\pi r}{n} \quad \dots(3)$$

(i) $n = 1, r = r_1 = 0.53 \text{ \AA}$
 $\therefore \lambda_1 = 2\pi r_1 = 2\pi (0.53) \text{ \AA} = 3.328 \text{ \AA}.$

From equation (3) $\frac{\lambda}{2\pi r} = \frac{1}{n}$, for $n = 1$,

$$\frac{\lambda}{2\pi r} = 1, \text{ i.e. } \lambda = 2\pi r$$

\therefore In first orbit, de Broglie wavelength of the electron is equal to the circumference of the orbit.

(ii) $n = 4, r_4 = 4^2 r_1.$

From equation (3) $\lambda_4 = \frac{2\pi r_4}{4} = \frac{2\pi(4^2 r_1)}{4}$

i.e. $\lambda_4 = 2\pi \times 4 \times (0.53) \text{ \AA}$
 $\lambda_4 = 13.313 \text{ \AA}$

and from $\lambda_4 = \frac{2\pi r_4}{4}$, the de Broglie wavelength of the electron in the fourth orbit is equal to one fourth of the perimeter.

7.7 Find the accelerating potential of the electron, when its de Broglie wavelength is 1 \AA .

Data : $\lambda = 1 \text{ \AA} ; V = ?$

Solution : $\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}.$

$$\sqrt{V} = \frac{12.27 \times 10^{-10}}{\lambda} = \frac{12.27 \times 10^{-10}}{1 \times 10^{-10}}$$

$$V = (12.27)^2 = 150.55 \text{ V}$$

7.8 At what speed is a particle moving if the mass is equal to three times its rest mass.

Data : $m = 3 m_0 ; v = ?$

Solution :
$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v = 0.943 c$$

$$= 0.943 \times 3 \times 10^8$$

$$v = 2.829 \times 10^8 \text{ ms}^{-1}$$

7.9 How fast would a rocket have to go relative to an observer for its length to be corrected to 99% of its length at rest.

Data : $\frac{l}{l_0} = 99\% = \frac{99}{100}; \quad v = ?$

Solution :
$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = \frac{99}{100} l_0$$

$$\frac{l}{l_0} = \frac{99}{100}$$

$$\therefore \frac{99}{100} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\therefore v = 0.141 c$$

$$v = 0.141 \times 3 \times 10^8$$

$$v = 0.423 \times 10^8 \text{ ms}^{-1}$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

- 7.1 A photon of frequency ν is incident on a metal surface of threshold frequency ν_0 . The kinetic energy of the emitted photoelectron is
- (a) $h(\nu - \nu_0)$ (b) $h\nu$
(c) $h\nu_0$ (d) $h(\nu + \nu_0)$
- 7.2 The work function of a photoelectric material is 3.3 eV. The threshold frequency will be equal to
- (a) 8×10^{14} Hz (b) 8×10^{10} Hz
(c) 5×10^{20} Hz (d) 4×10^{14} Hz.
- 7.3 The stopping potential of a metal surface is independent of
- (a) frequency of incident radiation
(b) intensity of incident radiation
(c) the nature of the metal surface
(d) velocity of the electrons emitted.
- 7.4 At the threshold frequency, the velocity of the electrons is
- (a) zero (b) maximum
(c) minimum (d) infinite
- 7.5 The photoelectric effect can be explained on the basis of
- (a) corpuscular theory of light
(b) wave theory of light
(c) electromagnetic theory of light
(d) quantum theory of light
- 7.6 The wavelength of the matter wave is independent of
- (a) mass (b) velocity
(c) momentum (d) charge

7.7 If the kinetic energy of the moving particle is E , then the de Broglie wavelength is,

(a) $\lambda = \frac{h}{\sqrt{2mE}}$

(b) $\lambda = \frac{\sqrt{2mE}}{h}$

(c) $\lambda = h\sqrt{2mE}$

(d) $\lambda = \frac{h}{E\sqrt{2m}}$

7.8 The momentum of the electron having wavelength 2\AA is

(a) $3.3 \times 10^{24} \text{ kg m s}^{-1}$

(b) $6.6 \times 10^{24} \text{ kg m s}^{-1}$

(c) $3.3 \times 10^{-24} \text{ kg m s}^{-1}$

(d) $6.6 \times 10^{-24} \text{ kg m s}^{-1}$

7.9 According to relativity, the length of a rod in motion

(a) is same as its rest length

(b) is more than its rest length

(c) is less than its rest length

(d) may be more or less than or equal to rest length depending on the speed of the rod

7.10 If 1 kg of a substance is fully converted into energy, then the energy produced is

(a) $9 \times 10^{16} \text{ J}$

(b) $9 \times 10^{24} \text{ J}$

(c) 1 J

(d) $3 \times 10^8 \text{ J}$

7.11 What is photoelectric effect?

7.12 Define stopping potential.

7.13 Define threshold frequency.

7.14 Define work function.

7.15 Explain the variation of photoelectric current with applied voltage.

7.16 State the laws of photoelectric emission.

7.17 Explain Einstein's theory of photoelectric effect

7.18 What are photo-cells?

7.19 What are the applications of photo-cells?

7.20 What are matter waves?

- 7.21 Derive an expression for de Broglie wavelength of matter waves.
- 7.22 Draw a neat sketch of an electron microscope. Explain its working.
- 7.23 Mention the applications of electron microscope.
- 7.24 Define frame of reference.
- 7.25 State the postulates of special theory of relativity.
- 7.26 Discuss the concept of space, time and mass.
- 7.27 Explain length contraction.
- 7.28 Explain time dilation.
- 7.29 If a body moves with the velocity of light, what will be its mass? Comment on your result.
- 7.30 Derive Einstein's mass energy equivalence.

Problems

- 7.31 The photoelectric threshold wavelength of a metal is 5000 Å. Find (i) the work function in electron volts and (ii) the kinetic energy of the photoelectrons in electron volts, ejected by the light of wave length 4000 Å.
- 7.32 Red light of wavelength 670 nm produces photoelectrons from a certain metal which requires a stopping potential of 0.5 V. What is the work function and threshold wavelength of the metal?
- 7.33 Calculate the velocity of a photoelectron if the work function of the target material is 1.24 eV and the wave length of incident light is 4.36×10^{-7} m.
- 7.34 The rest mass of an electron is 9.1×10^{-31} kg. What will be its mass if it moves with $4/5^{\text{th}}$ of the speed of light?
- 7.35 Calculate the threshold frequency of photons which can remove photoelectrons from (i) caesium and (ii) nickel surface (work function of caesium is 1.8 eV and work function of nickel is 5.9 eV)
- 7.36 If the speed of photoelectrons is 10^4 ms^{-1} , calculate the frequency of the radiation incident on a potassium metal? Work function of potassium is 2.3 eV?
- 7.37 Ultra violet light of wave length 2271 Å from a source irradiates a photo-cell which is made of molybdenum metal. If the stopping potential is 1.3 volt, calculate the work function of the metal. How

would the photo cell respond to a high intensity (10^5 Wm^{-2}) red light of wavelength 6328 \AA produced by He - Ne Laser?

- 7.38 The work function of Iron is 4.7 eV . Calculate the cut off frequency and the corresponding cut off wave length for this metal.
- 7.39 For an observer imagined to be moving at a speed of $36 \times 10^6 \text{ km/hr}$, length of the rod measures 1m . Find the length of the rod as measured by a stationary observer.
- 7.40 The time interval measured by an observer at rest is $2.5 \times 10^{-8}\text{s}$. What is the time interval as measured by an observer moving with a velocity $v = 0.73 c$.
- 7.41 Calculate the rest energy of an electron in MeV (mass of an electron $9.11 \times 10^{-31} \text{ kg}$).
- 7.42 A proton is moving at a speed of 0.900 times the velocity of light. Find its kinetic energy in joules and MeV.

Answers

7.1 (a)

7.2 (a)

7.3 (b)

7.4 (a)

7.5 (d)

7.6 (d)

7.7 (a)

7.8 (c)

7.9 (c)

7.10 (a)

7.31 2.475 eV , 0.62 eV

7.32 1.354 eV , 917.2 nm

7.33 $7.52 \times 10^5 \text{ ms}^{-1}$

7.34 $15.16 \times 10^{-31} \text{ kg}$

7.35 $4.34 \times 10^{14} \text{ Hz}$; $1.42 \times 10^{15} \text{ Hz}$

7.36 $0.56 \times 10^{15} \text{ Hz}$

7.37 4.166 eV , As $v_0 > v$, electrons are not emitted.

7.38 $1.1 \times 10^{15} \text{ Hz}$; $2.64 \times 10^2 \text{ nm}$ 7.39 1.0006 m

7.40 $3.658 \times 10^{-8} \text{ s}$

7.41 0.512 MeV

7.42 $1.955 \times 10^{-10} \text{ J}$; 1221.8 MeV

8. Nuclear Physics

The atomic nucleus was discovered by Earnest Rutherford in 1911. Rutherford's experiment on scattering of alpha particles proved that the mass of the atom and the positive charge is concentrated in a very small central core called nucleus. The dimension of nucleus is much smaller than the overall dimension of the atom. The nucleus is surrounded by orbiting electrons.

8.1 Nucleus

The nucleus consists of the elementary particles, protons and neutrons which are known as nucleons. A proton has positive charge of the same magnitude as that of electron and its rest mass is about 1836 times the mass of an electron. A neutron is electrically neutral, whose mass is almost equal to the mass of the proton. The nucleons inside the nucleus are held together by strong attractive forces called nuclear forces.

A nucleus of an element is represented as ${}_Z X^A$, where X is the chemical symbol of the element. Z represents the atomic number which is equal to the number of protons and A, the mass number which is equal to the total number of protons and neutrons. The number of neutrons is represented as N which is equal to A-Z. For example, the chlorine nucleus is represented as ${}_{17}\text{Cl}^{35}$. It contains 17 protons and 18 neutrons.

8.1.1 Classification of nuclei

(i) Isotopes

Isotopes are atoms of the same element having the same atomic number Z but different mass number A. The nuclei ${}_1\text{H}^1$, ${}_1\text{H}^2$ and ${}_1\text{H}^3$ are the isotopes of hydrogen. In other words isotopes of an element contain the same number of protons but different number of neutrons. As the atoms of isotopes have identical electronic structure, they have identical chemical properties and placed in the same location in the periodic table.

(ii) Isobars

Isobars are atoms of different elements having the same mass number A , but different atomic number Z . The nuclei ${}_8\text{O}^{16}$ and ${}_7\text{N}^{16}$ represent two isobars. Since isobars are atoms of different elements, they have different physical and chemical properties.

(iii) Isotones

Isotones are atoms of different elements having the same number of neutrons. ${}_6\text{C}^{14}$ and ${}_8\text{O}^{16}$ are some examples of isotones.

8.1.2 General properties of nucleus

Nuclear size

According to Rutherford's α -particle scattering experiment, the distance of the closest approach of α - particle to the nucleus was taken as a measure of nuclear radius, which is approximately 10^{-15}m . If the nucleus is assumed to be spherical, an empirical relation is found to hold good between the radius of the nucleus R and its mass number A . It is given by

$$R \propto A^{1/3}$$

$$R = r_0 A^{1/3}$$

where r_0 is the constant of proportionality and is equal to 1.3 F (1 Fermi, $F = 10^{-15} \text{ m}$)

Nuclear density

The nuclear density ρ_N can be calculated from the mass and size of the nucleus.

$$\rho_N = \frac{\text{Nuclear mass}}{\text{Nuclear volume}}$$

$$\text{Nuclear mass} = Am_N$$

where, A = mass number

and m_N = mass of one nucleon and is approximately equal to $1.67 \times 10^{-27} \text{ kg}$

$$\text{Nuclear volume} = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi (r_0 A^{1/3})^3$$

$$\therefore \rho_N = \frac{A m_N}{\frac{4}{3}\pi(r_0 A^{1/3})^3} = \frac{m_N}{\frac{4}{3}\pi r_0^3}$$

Substituting the known values, the nuclear density is calculated as $1.816 \times 10^{17} \text{ kg m}^{-3}$ which is almost a constant for all the nuclei irrespective of its size.

The high value of the nuclear density shows that the nuclear matter is in an extremely compressed state.

Nuclear charge

The charge of a nucleus is due to the protons present in it. Each proton has a positive charge equal to $1.6 \times 10^{-19} \text{ C}$.

\therefore The nuclear charge = Ze , where Z is the atomic number.

Atomic mass unit

It is convenient to express the mass of a nucleus in atomic mass unit (amu), though the unit of mass is kg. One atomic mass unit is considered as one twelfth of the mass of carbon atom ${}_6\text{C}^{12}$. Carbon of atomic number 6 and mass number 12 has mass equal to 12 amu.

$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

The mass of a proton, $m_p = 1.007276 \text{ amu}$

This is equal to the difference in mass of the hydrogen atom which is 1.007825 amu and the mass of electron.

The mass of a neutron, $m_n = 1.008665 \text{ amu}$

The energy equivalence of one amu can be calculated in electron-volt

Einstein's mass energy relation is, $E = mc^2$

Here, $m = 1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\therefore E = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 \text{ J}$$

One electron-volt (eV) is defined as the energy of an electron when it is accelerated through a potential difference of 1 volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ coulomb} \times 1 \text{ volt}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ joule}$$

$$\begin{aligned} \text{Hence, } E &= \frac{1.66 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV} = 931 \times 10^6 \text{ eV} \\ &= 931 \text{ million electronvolt} = 931 \text{ MeV} \end{aligned}$$

Thus, energy equivalent of 1 amu = 931 MeV

Nuclear mass

As the nucleus contains protons and neutrons, the mass of the nucleus is assumed to be the mass of its constituents.

$$\text{Assumed nuclear mass} = Zm_p + Nm_n,$$

where m_p and m_n are the mass of a proton and a neutron respectively. However, from the measurement of mass by mass spectrometers, it is found that the mass of a stable nucleus (m) is less than the total mass of the nucleons.

$$\begin{aligned} \text{i.e. mass of a nucleus, } m &< (Zm_p + Nm_n) \\ Zm_p + Nm_n - m &= \Delta m \end{aligned}$$

where Δm is the mass defect

Thus, the difference in the total mass of the nucleons and the actual mass of the nucleus is known as the mass defect.

Note : In any mass spectrometer, it is possible to determine only the mass of the atom, which includes the mass of Z electrons. If M represents the mass of the atom, then the mass defect can be written as

$$\begin{aligned} \Delta m &= Zm_p + Nm_n + Zm_e - M \\ &= Zm_H + Nm_n - M \end{aligned}$$

where m_H represents the mass of one hydrogen atom

Binding energy

When the protons and neutrons combine to form a nucleus, the mass that disappears (mass defect, Δm) is converted into an equivalent amount of energy (Δmc^2). This energy is called the binding energy of the nucleus.

$$\begin{aligned} \therefore \text{Binding energy} &= [Zm_p + Nm_n - m] c^2 \\ &= \Delta m c^2 \end{aligned}$$

The binding energy of a nucleus determines its stability against disintegration. In other words, if the binding energy is large, the nucleus is stable and vice versa.

The binding energy per nucleon is

$$\frac{BE}{A} = \frac{\text{Binding energy of the nucleus}}{\text{Total number of nucleons}}$$

It is found that the binding energy per nucleon varies from element to element. A graph is plotted with the mass number A of the nucleus along the X-axis and binding energy per nucleon along the Y-axis (Fig 8.1).

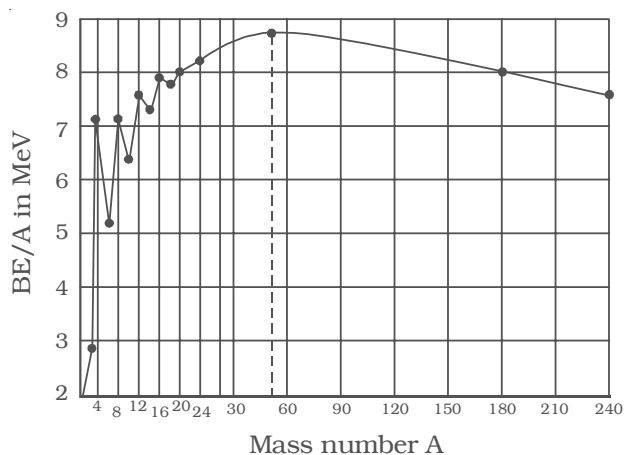


Fig 8.1 Binding energy graph

Explanation of binding energy curve

(i) The binding energy per nucleon increases sharply with mass number A upto 20. It increases slowly after $A = 20$. For $A < 20$, there exists recurrence of peaks corresponding to those nuclei, whose mass numbers are multiples of four and they contain not only equal but also even number of protons and neutrons. Example: ${}^2_2\text{He}^4$, ${}^4_2\text{Be}^8$, ${}^6_6\text{C}^{12}$, ${}^8_8\text{O}^{16}$, and ${}^{10}_{10}\text{Ne}^{20}$. The curve becomes almost flat for mass number between 40 and 120. Beyond 120, it decreases slowly as A increases.

(ii) The binding energy per nucleon reaches a maximum of 8.8 MeV at $A=56$, corresponding to the iron nucleus (${}^{56}_{26}\text{Fe}^{56}$). Hence, iron nucleus is the most stable.

(iii) The average binding energy per nucleon is about 8.5 MeV for nuclei having mass number ranging between 40 and 120. These elements are comparatively more stable and non radioactive.

(iv) For higher mass numbers the curve drops slowly and the BE/A is about 7.6 MeV for uranium. Hence, they are unstable and radioactive.

(v) The lesser amount of binding energy for lighter and heavier nuclei explains nuclear fusion and fission respectively. A large amount of energy will be liberated if lighter nuclei are fused to form heavier one (fusion) or if heavier nuclei are split into lighter ones (fission).

8.2 Bainbridge mass spectrometer - Determination of isotopic masses of nuclei

Bainbridge mass spectrometer is an instrument used for the accurate determination of atomic masses. A schematic diagram of this spectrometer is shown in Fig 8.2.

Atoms with one or more electrons removed, have a net positive charge and they become positive ions. A beam of positive ions produced in a discharge tube is collimated into a fine beam by two narrow slits S_1 and S_2 . This fine beam enters into a velocity selector. The velocity selector allows the ions of a particular velocity to come out of it, by the combined action of an electric and a magnetic field. The velocity selector consists of two plane parallel plates P_1 and P_2 , which produces a uniform electric field E and an electromagnet, to produce uniform magnetic field B (represented by the dotted circle). These two fields are at right angles to each other and to the direction of the beam.

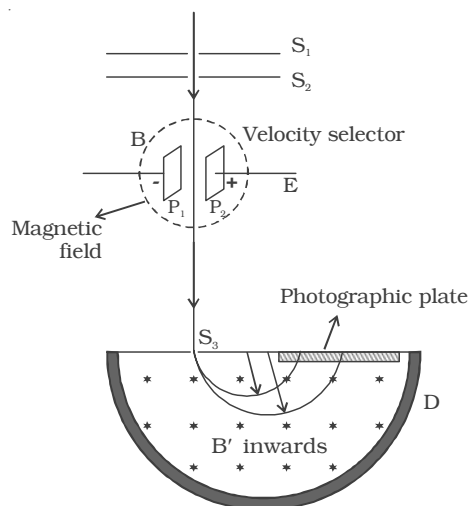


Fig 8.2 Bainbridge mass spectrometer

The electric field and magnetic field are so adjusted that the deflection produced by one field is nullified by the other, so that the ions do not suffer any deflection within the velocity selector. Let E and B be the electric field intensity and magnetic induction respectively and q be the charge of the positive ion. The force exerted by the electric field is equal to qE and the force exerted by the magnetic field is equal to Bqv where v is the velocity of the positive ion.

$$qE = Bqv$$

$$v = E/B$$

Only those ions having this velocity v , pass out of the velocity selector and then through the slit S_3 , to enter the evacuated

chamber D. These positive ions having the same velocity are subjected to another strong uniform magnetic field of induction B' at right angles to the plane of the paper acting inwards. These ions are deflected along circular path of radius R and strike the photographic plate. The force due to magnetic field $B'qv$ provides the centripetal force.

$$B'qv = \frac{mv^2}{R}$$

$$m = \frac{B'qR}{v}$$

Substituting $v = E/B$

$$m = \frac{BB'qR}{E}$$

Ions with different masses trace semi-circular paths of different radii and produce dark lines on the plate. The distance between the opening of the chamber and the position of the dark line gives the diameter $2R$ from which radius R can be calculated.

Since q , B , B' , E and R are known, the mass of the positive ions and hence isotopic masses can be calculated.

8.3 Nuclear force

The nucleus of an atom consists of positively charged protons and uncharged neutrons. According to Coulomb's law, protons must repel each other with a very large force, because they are close to each other and hence the nucleus must be broken into pieces. But this does not happen. It means that, there is some other force in the nucleus which overcomes the electrostatic repulsion between positively charged protons and binds the protons and neutrons inside the nucleus. This force is called nuclear force.

(i) Nuclear force is charge independent. It is the same for all the three types of pairs of nucleons (n-n), (p-p) and (n-p). This shows that nuclear force is not electrostatic in nature.

(ii) Nuclear force is the strongest known force in nature.

(iii) Nuclear force is not a gravitational force. Nuclear force is about 10^{40} times stronger than the gravitational force.

(iv) Nuclear force is a short range force. It is very strong between two nucleons which are less than 10^{-15} m apart and is almost negligible at a distance greater than this. On the other hand electrostatic, magnetic and gravitational forces are long range forces that can be felt easily.

Yukawa suggested that the nuclear force existing between any two nucleons may be due to the continuous exchange of particles called mesons, just as photons, the exchange particle in electromagnetic interactions.

However, the present view is that the nuclear force that binds the protons and neutrons is not a fundamental force of nature but it is secondary.

8.4 Radioactivity

The phenomenon of radioactivity was discovered by Henri Becquerel in 1896. He found that a photographic plate wrapped in a black paper was affected by certain penetrating radiations emitted by uranium salt. Rutherford showed later that the radiations from the salt were capable of ionising a gas. The current produced due to the ions was taken as a measure of activity of the compound.

A few years later Madame Marie Curie and her husband Piere Curie discovered the highly radioactive elements radium and polonium. The activity of the material has been shown to be the result of the three different kinds of radiations, α , β and γ .

The phenomenon of spontaneous emission of highly penetrating radiations such as α , β and γ rays by heavy elements having atomic number greater than 82 is called radioactivity and the substances which emit these radiations are called radioactive elements.

The radioactive phenomenon is spontaneous and is unaffected by any external agent like temperature, pressure, electric and magnetic fields etc.

8.4.1 Alpha, beta and gamma rays

The existence of the three distinct types of radiations, α , β and γ -rays can be easily found by the following experiment. A small amount of radium (R) is placed at the bottom of a small hole drilled in a lead block, which is kept in an evacuated chamber (Fig. 8.3).

A photographic plate is placed at a short distance above the lead block. A strong magnetic field is applied

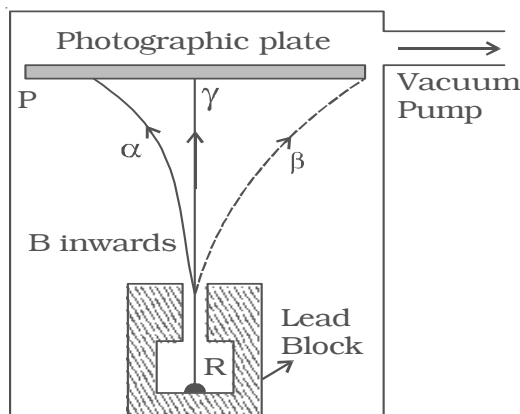


Fig 8.3 Radioactivity

at right angles to the plane of the paper and acting inwards. Three distinct traces can be seen on the photographic plate when it is developed. The trace towards left is due to positively charged particles. They are named α -particles. The trace towards the right is due to negatively charged particles. They are named β -particles. The undeviated trace is due to neutral radiations which are called γ -rays. If an electric field is applied, the α -rays are deflected towards the negative plate, β -rays towards the positive plate and γ -rays are not deflected.

Properties of α -rays

- (i) An α - particle is a helium nucleus consisting of two protons and two neutrons. It carries two units of positive charge.
- (ii) They move along straight lines with high velocities.
- (iii) They are deflected by electric and magnetic fields.
- (iv) They produce intense ionisation in the gas through which they pass. The ionising power is 100 times greater than that of β -rays and 10,000 times greater than that of γ -rays.
- (v) They affect photographic plates.
- (vi) They are scattered by heavy elements like gold.
- (vii) They produce fluorescence when they fall on substances like zinc sulphide or barium platinocyanide.

Properties of β - rays

(i) β -particles carry one unit of negative charge and mass equal to that of electron. Therefore, they are nothing but electrons.

(ii) The β -particles emitted from a source have velocities over the range of $0.3c$ to $0.99c$, where c is the velocity of light.

(iii) They are deflected by electric and magnetic fields.

(iv) The ionisation power is comparatively low

(v) They affect photographic plates.

(vi) They penetrate through thin metal foils and their penetrating power is greater than that of α -rays

(vii) They produce fluorescence when they fall on substances like barium platinocyanide.

Properties of γ - rays

(i) They are electromagnetic waves of very short wavelength.

(ii) They are not deflected by electric and magnetic fields.

(iii) They travel with the velocity of light.

(iv) They produce very less ionisation.

(v) They affect photographic plates.

(vi) They have a very high penetrating power, greater than that of β -rays.

(vii) They produce fluorescence.

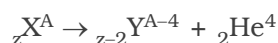
(viii) They are diffracted by crystals in the same way like X-rays are diffracted.

8.5 Radioactive displacement law

During a radioactive disintegration, the nucleus which undergoes disintegration is called a parent nucleus and that which remains after the disintegration is called a daughter nucleus. In 1913, Soddy and Fajan framed the displacement laws governing radioactivity.

α -decay

When a radioactive nucleus disintegrates by emitting an α -particle, the atomic number decreases by two and mass number decreases by four. The α -decay can be expressed as

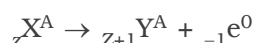


Example : Radium (${}_{88}\text{Ra}^{226}$) is converted to radon (${}_{86}\text{Rn}^{222}$) due to α -decay

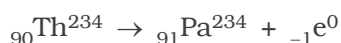


β -decay

When a radioactive nucleus disintegrates by emitting a β -particle, the atomic number increases by one and the mass number remains the same. β -decay can be expressed as



Example : Thorium (${}_{90}\text{Th}^{234}$) is converted to protoactinium (${}_{91}\text{Pa}^{234}$) due to β -decay



At a time, either α or β -particle is emitted. Both α and β particles are not emitted during a single decay.

γ -decay

When a radioactive nucleus emits γ -rays, only the energy level of the nucleus changes and the atomic number and mass number remain the same.

During α or β -decay, the daughter nucleus is mostly in the excited state. It comes to ground state with the emission of γ -rays.

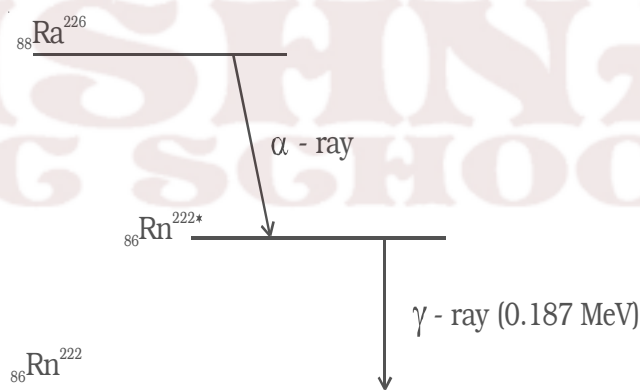


Fig 8.4 Gamma decay

Example : During the radioactive disintegration of radium (${}_{88}\text{Ra}^{226}$) into radon (${}_{86}\text{Rn}^{222}$), gamma ray of energy 0.187 MeV is emitted, when radon returns from the excited state to the ground state (Fig 8.4).

Radioactive law of disintegration

Rutherford and Soddy found that the rate of disintegration is independent of physical and chemical conditions. The rate of disintegration at any instant is directly proportional to the number of

atoms of the element present at that instant. This is known as radioactive law of disintegration.

Let N_0 be the number of radioactive atoms present initially and N , the number of atoms at a given instant t . Let dN be the number of atoms undergoing disintegration in a small interval of time dt . Then the rate of disintegration is

$$-\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N \quad \dots(1)$$

where λ is a constant known as decay constant or disintegration constant. The negative sign indicates that N decreases with increase in time.

Equation (1) can be written as

$$\frac{dN}{N} = -\lambda dt$$

Integrating, $\log_e N = -\lambda t + C \quad \dots(2)$

where C is a constant of integration.

At $t = 0, N = N_0$

$\therefore \log_e N_0 = C$

Substituting for C , equation (2) becomes

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-\lambda t} \quad \dots(3)$$

Equation (3) shows that the number of atoms of a radioactive substance decreases exponentially with increase in time (Fig 8.5).

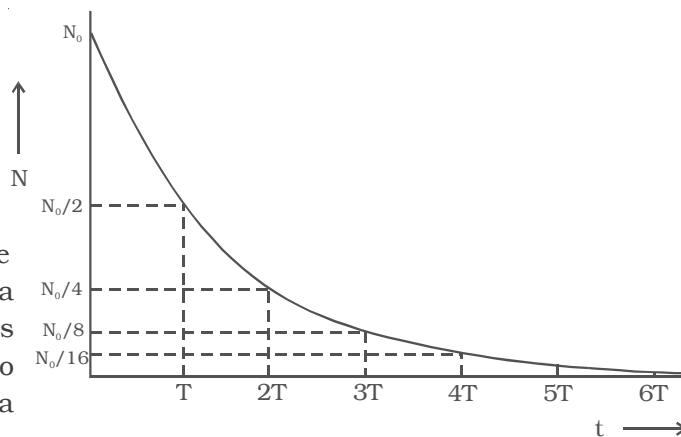


Fig 8.5 Radioactive law of disintegration

Initially the disintegration takes place at a faster rate. As time increases, N gradually decreases exponentially. Theoretically, an infinite time is required for the complete disintegration of all the atoms.

Half life period

Since all the radioactive elements have infinite life period, in order to distinguish the activity of one element with another, half life period and mean life period are introduced.

The half life period of a radioactive element is defined as the time taken for one half of the radioactive element to undergo disintegration.

From the law of disintegration

$$N = N_0 e^{-\lambda t}$$

Let $T_{1/2}$ be the half life period. Then, at $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\log_e 2 = \lambda T_{1/2}$$

$$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{\log_{10} 2 \times 2.3026}{\lambda} = \frac{0.6931}{\lambda}$$

The half life period is inversely proportional to its decay constant.

The concept of half time period can be understood from Fig 8.5.

For a radioactive substance, at the end of $T_{1/2}$, 50% of the material remain unchanged. After another $T_{1/2}$ i.e., at the end of $2T_{1/2}$, 25% remain unchanged. At the end of $3T_{1/2}$, 12.5% remain unchanged and so on.

Mean life

When the radioactive substance is undergoing disintegration, the atom which disintegrates at first has zero life and that disintegrates at last has infinite life. The actual life of each atom ranges from zero to infinity.

The mean life of a radioactive substance is defined as the ratio

Note :

Fraction of radioactive substance left undecayed is, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$ where n is the number of half lives.

$$n = \frac{\text{Total time}}{\text{Half life}}$$

of total life time of all the radioactive atoms to the total number of atoms in it.

$$\therefore \text{Mean life} = \frac{\text{Sum of life time of all the atoms}}{\text{Total number of atoms}}$$

The mean life is calculated from the law of disintegration and it can be shown that the mean life is the reciprocal of the decay constant.

$$\tau = \frac{1}{\lambda}$$

The half life and mean life are related as

$$T_{1/2} = \frac{0.6931}{\lambda} = 0.6931 \tau$$

Activity

The activity of a radioactive substance is defined as the rate at which the atoms decay. If N is the number of atoms present at a certain time t , the activity R is given by

$$R = -\frac{dN}{dt}$$

The unit of activity is becquerel named after the scientist Henri Becquerel

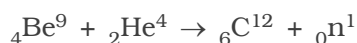
$$\therefore 1 \text{ becquerel} = 1 \text{ disintegration per second}$$

The activity of a radioactive substance is generally expressed in curie. Curie is defined as the quantity of a radioactive substance which gives 3.7×10^{10} disintegrations per second or 3.7×10^{10} becquerel. This is equal to the activity of one gram of radium.

8.6 Neutron - Discovery

In 1930, two German Physicists Bothe and Becker found that when beryllium was bombarded with α -particles, a highly penetrating radiation was emitted. This radiation was capable of traversing through a thick layer of lead and was unaffected by electric and magnetic fields. At that time, the radiation was thought to be γ -rays.

In 1932, Irene Curie and F. Joliot found that those radiations were able to knock out protons from paraffin and similar substances that are rich in hydrogen. Chadwick in the same year discovered that the emitted radiation consists of particles of mass nearly equal to proton and no charge. He called them as neutrons. The above reaction can be written as



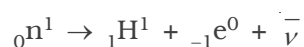
where ${}_0\text{n}^1$ represents neutron.

8.6.1 Properties of neutrons

(i) Neutrons are the constituent particles of all nuclei, except hydrogen.

(ii) Neutrons are neutral particles with no charge and mass slightly greater than that of protons. Hence, they are not deflected by electric and magnetic fields.

(iii) Neutrons are stable inside the nucleus. But outside the nucleus they are unstable. The free neutron decays with an emission of proton, electron and antineutrino, with half life of 13 minutes.



(iv) As neutrons are neutral, they can easily penetrate any nucleus.

(v) Neutrons are classified according to their kinetic energy as (a) slow neutrons and (b) fast neutrons. Both are capable of penetrating a nucleus causing artificial transmutation of the nucleus.

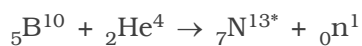
Neutrons with energies from 0 to 1000 eV are called slow neutrons. The neutrons with an average energy of about 0.025 eV in thermal equilibrium are called thermal neutrons.

Neutrons with energies in the range between 0.5 MeV and 10 MeV are called fast neutrons. In nuclear reactors, fast neutrons are converted into slow neutrons using moderators.

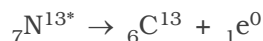
8.7 Artificial radioactivity

Artificial radioactivity or induced radioactivity was discovered by Irene Curie and F. Joliot in 1934. This is also known as man-made radioactivity. The phenomenon by which even light elements are made radioactive by artificial or induced methods is called artificial radioactivity.

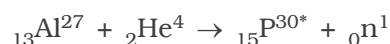
Curie and Joliot showed that when lighter elements such as boron and aluminium were bombarded with α -particles, there was a continuous emission of radioactive radiations, even after the α -source had been removed. They showed that the radiation was due to the emission of a particle carrying one unit positive charge with mass equal to that of an electron. This particle is known as positron and the reaction is



The nitrogen atom is radioactive and decays with a half life of about 10.1 minutes into a stable isotope of carbon with the emission of a positron



In the case of Aluminium,



Radio-phosphorous decays into a stable isotope of silicon with the emission of a positron. The half life of radioactive phosphorous is about 3 minutes.



Artificial radioactivity follows the same laws of natural radioactivity. Artificial radioactive elements emit electrons, positrons, neutrons and γ rays.

Production of artificial radio-isotopes

(i) Artificial radio-isotopes are produced by placing the target element in the nuclear reactor, where plenty of neutrons are available.



(ii) Another method of production of artificial radio-isotope is to bombard the target element with particles from particle accelerators like cyclotron.



8.7.1 Applications of radio-isotopes

The radio-isotopes have wide applications in medicine, agriculture, industry and research. A radio-isotope is added to a particular system and the course of the isotope is studied to understand the system.

(i) Medical applications

In medical field, radio-isotopes are used both in diagnosis and therapy. Radio cobalt (Co^{60}) emitting γ -rays is used in the treatment of cancer. Gamma rays destroy cancer cells to a greater extent.

Radio-sodium (Na^{24}) is used to detect the presence of blocks in blood vessels, to check the effective functioning of heart in pumping blood and maintaining circulation. Radio-iodine (I^{131}) is used in the detection of the nature of thyroid gland and also for treatment. Radio-iodine is also used to locate brain tumours.

Radio-iron (Fe^{59}) is used to diagnose anaemia. An anaemic patient retains iron in the blood longer than normal patient.

Radio-phosphorous (P^{32}) is used in the treatment of skin diseases.

(ii) Agriculture

In agriculture, radio-isotopes help to increase the crop yields. Radio-phosphorous (P^{32}) incorporated with phosphate fertilizer is added to the soil. The plant and soil are tested from time to time. Phosphorous is taken by the plant for its growth and radio-phosphorous is found to increase the yield.

Sprouting and spoilage of onions, potatoes, grams etc. are prevented by exposure to a very small amount of radiation. Certain perishable cereals remain fresh beyond their normal life span when exposed to radiation.

(iii) Industry

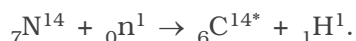
In Industry, the lubricating oil containing radio-isotopes is used to study the wear and tear of the machinery.

(iv) Molecular biology

In molecular biology radio-isotopes are used in sterilising pharmaceutical and surgical instruments.

(v) Radio-carbon dating

In the upper atmosphere, C^{14} is continually formed from N^{14} due to the bombardment by neutrons produced from cosmic rays.



The C^{14} is radioactive with half life of 5570 years.

The production and the decay of C^{14} are in equilibrium in atmosphere. The ratio of C^{14} and C^{12} atoms in atmosphere is $1 : 10^6$. Hence, carbon dioxide present in the atmosphere contains a small portion of C^{14} .

Living things take C^{14} from food and air. However with death, the intake of C^{14} stops, and the C^{14} that is already present begins to decay. Hence the amount of C^{14} in the sample will enable the calculation of time of death i.e, the age of the specimen could be estimated. This is called radio-carbon dating. This method is employed in the dating of wooden implements, leather clothes, charcoal used in oil paintings, mummies and so on.

8.7.2 Biological hazards of nuclear radiations

When γ -ray or any high energy nuclear particle passes through human beings, it disrupts the entire normal functioning of the biological system and the effect may be either pathological or genetic.

The biological effects of nuclear radiation can be divided into three groups

- (i) Short term recoverable effects
- (ii) long term irrecoverable effects and
- (iii) genetic effect

The extent to which the human organism is damaged depends upon (i) the dose and the rate at which the radiation is given and (ii) the part of the body exposed to it.

Smaller doses of radiation exposure produce short term effects such as skin disorder and loss of hair.

If the exposure is 100 R*, it may cause diseases like leukemia (death of red blood corpuscle in the blood) or cancer. When the body is exposed to about 600 R, ultimately it causes death. Safe limit of receiving the radiations is about 250 milli roentgen per week.

The genetic damage is still worse. The radiations cause injury to genes in the reproductive cells. This gives rise to mutations which pass on from generation to generation.

The following precautions are to be taken for those, who are working in radiation laboratories.

(1) Radioactive materials are kept in thick-walled lead container.

(2) Lead aprons and lead gloves are used while working in hazardous area.

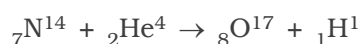
(3) All radioactive samples are handled by a remote control process.

(4) A small micro-film badge is always worn by the person and it is checked periodically for the safety limit of radiation.

* The radiation exposure is measured by the unit called roentgen (R). One roentgen is defined as the quantity of radiation which produces 1.6×10^{12} pairs of ions in 1 gram of air.

8.7.3 Artificial transmutation

Artificial transmutation is the conversion of one element into another by artificial methods. The first successful artificial transmutation was carried out by Rutherford. When nitrogen was bombarded with α -particles of sufficient energy, a rare isotope of oxygen (${}^8\text{O}^{17}$) and a proton were formed.



This process is called nuclear reaction.

In a nuclear reaction, the sum of initial atomic numbers (charges) is equal to the sum of the final atomic numbers, i.e conservation of charges is satisfied. In the same way, conservation of nucleons is also satisfied. But the initial rest mass is not equal to the final rest mass. The energy equivalence of the difference between the rest masses is equal to the nuclear reaction energy Q . The nuclear reaction can be written as



where A is the target nucleus and a , the bombarding particle. B and b are the product nucleus and particle.

8.7.4 Particle accelerators

A particle accelerator is a device used to accelerate the charged particles, which are required in the study of artificial transmutation of elements. Hence the accelerator is the basic device in high energy particle physics.

Accelerators can be divided broadly into two types.

(i) The first type belongs to electrostatic accelerators in which charged particles are accelerated by applying a constant potential difference. The final energy is determined by the amount of the potential difference. The Cockcroft–Walton and Van de Graaff generators belong to this class. These accelerators can accelerate particles only upto a few million electron-volts.

(ii) The second type is the cyclic or synchronous accelerator, which accelerate particles in multiple steps imparting a small energy in each successive step. Example : Linear accelerator, cyclotron, betatron, synchrocyclotron and synchrotron. They can accelerate particles to

energy of the order of 10^9 eV (GeV), of course, with each machine having its own energy limit.

The action of Van de Graaff generator and cyclotron have been already dealt in previous chapters.

8.8 Geiger - Muller counter

Geiger - Muller counter is used to measure the intensity of the radioactive radiation. When nuclear radiations pass through gas, ionisation is produced. This is the principle of this device.

Construction

The G.M tube consists of a metal tube with glass envelope (C) acting as the cathode and a fine tungsten wire (W) along the axis of the tube, which acts as anode (Fig 8.6). The tube is well insulated from the anode wire.

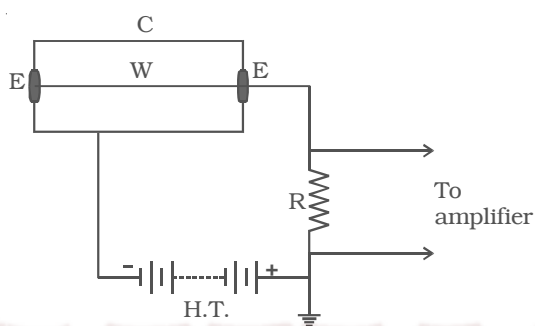


Fig 8.6 G.M Counter

The tube is filled with an inert gas like argon at a low pressure. One end is fitted with a thin mica sheet and this end acts as a window through which radiations enter the tube. A high potential difference of about 1000 V is applied between the electrodes through a high resistance R of about 100 mega ohm.

Operation

When an ionising radiation enters the counter, primary ionisation takes place and a few ions are produced. These ions are accelerated with greater energy due to the high potential difference and they cause further ionisation and these ions are multiplied by further collisions. Thus an avalanche of electrons is produced in a short interval of time. This avalanche of electrons on reaching the anode generates a current pulse, which when passing through R develops a potential difference. This is amplified by electronic circuits and is used to operate an electronic counter. The counts in the counter is directly proportional to the intensity of the ionising radiation.

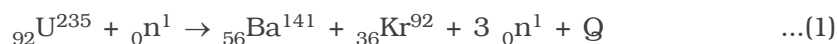
The ionisation of the gas is independent of the type of the incident radiation. Hence, G.M. counter does not distinguish the type of radiation that enters the chamber.

Wilson's cloud chamber is another type of particle detector. This was the first instrument to record the visual observation of the tracks of the charged particles, when they pass through matter.

8.9 Nuclear fission

In 1939, German scientists Otto Hahn and F. Strassman discovered that when uranium nucleus is bombarded with a neutron, it breaks up into two fragments of comparable masses with the release of energy.

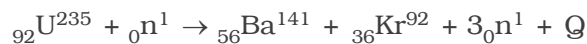
The process of breaking up of the nucleus of a heavier atom into two fragments with the release of large amount of energy is called nuclear fission. The fission is accompanied of the release of neutrons. The fission reactions with ${}_{92}\text{U}^{235}$ are represented as



In the above examples the fission reaction is taking place with the release of 3 neutrons and 2 neutrons. On an average 2.5 neutrons per fission are actually released. Of the many possible fission reactions of ${}_{92}\text{U}^{235}$, the reaction given in equation (1) is the most favourable.

Energy released in fission

Let us calculate the amount of energy released during the fission of ${}_{92}\text{U}^{235}$ with a neutron. The fission reaction is



Mass of ${}_{92}\text{U}^{235}$	=	235.045733	amu
---------------------------------	---	------------	-----

Mass of ${}_0\text{n}^1$	=	1.008665	amu
--------------------------	---	----------	-----

Total mass of the reactant	=	236.054398	amu
----------------------------	---	------------	-----

Mass of ${}_{56}\text{Ba}^{141}$	=	140.9177	amu
----------------------------------	---	----------	-----

Mass of ${}_{36}\text{Kr}^{92}$	=	91.8854	amu
---------------------------------	---	---------	-----

Mass of 3 neutrons	=	3.025995	amu
--------------------	---	----------	-----

(3 × 1.008665)

Total mass of the products	=	235.829095	amu
----------------------------	---	------------	-----

∴ Mass defect = 236.054398 – 235.829095 = 0.225303 amu

As, 1 amu = 931 MeV, energy released in a fission =

$$0.225303 \times 931 \approx 200 \text{ MeV}$$

Niels Bohr and John A. Wheeler explained the nuclear fission process with the help of liquid drop model. A liquid drop has a spherical shape due to surface tension. On applying external force the sphere changes into ellipsoid, which may change into a dumb bell shape when the force is larger. This may break at the narrow end into two portions.

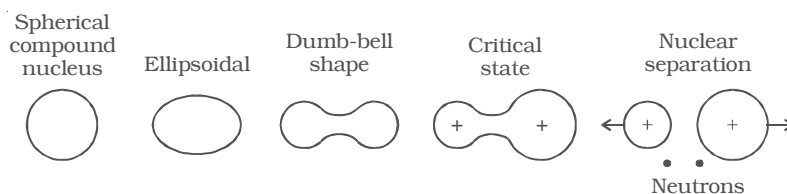


Fig 8.7 Bohr – Wheeler’s explanation of Nuclear fission

In the same way, when the heavier nucleus absorbs a neutron, a compound nucleus is formed and is left in an excited state. The excitation energy sets up a series of rapid oscillations. The compound nucleus undergoes distortion from spherical to dumb bell shape. Each portion of the dumb bell has a positive charge and one repels the other. This results in fission and the formation of fission fragments (Fig 8.7)

8.9.1 Chain reaction

Consider a neutron causing fission in a uranium nucleus producing three neutrons. The three neutrons in turn may cause fission in three uranium nuclei producing nine neutrons. These nine neutrons in turn may produce twenty seven neutrons and so on. A chain reaction is a self propagating process in which the number of neutrons goes on multiplying rapidly almost in a geometrical progression.

Two types of chain reactions are possible. In the uncontrolled chain reaction, the number of neutrons multiply indefinitely and the entire amount of energy is released within a fraction of a second. This type of chain reaction takes place in atom bombs.

In the controlled chain reaction the number of fission producing neutron is kept constant and is always equal to one. The reaction is sustained in a controlled manner (Fig 8.8). Controlled chain reaction is taking place in a nuclear reactor.

When a thermal neutron bombards U^{235} nucleus, it breaks into two fission fragments and three fast neutrons (Fig 8.8). One neutron may escape and one neutron may be captured by U^{238} which decays to

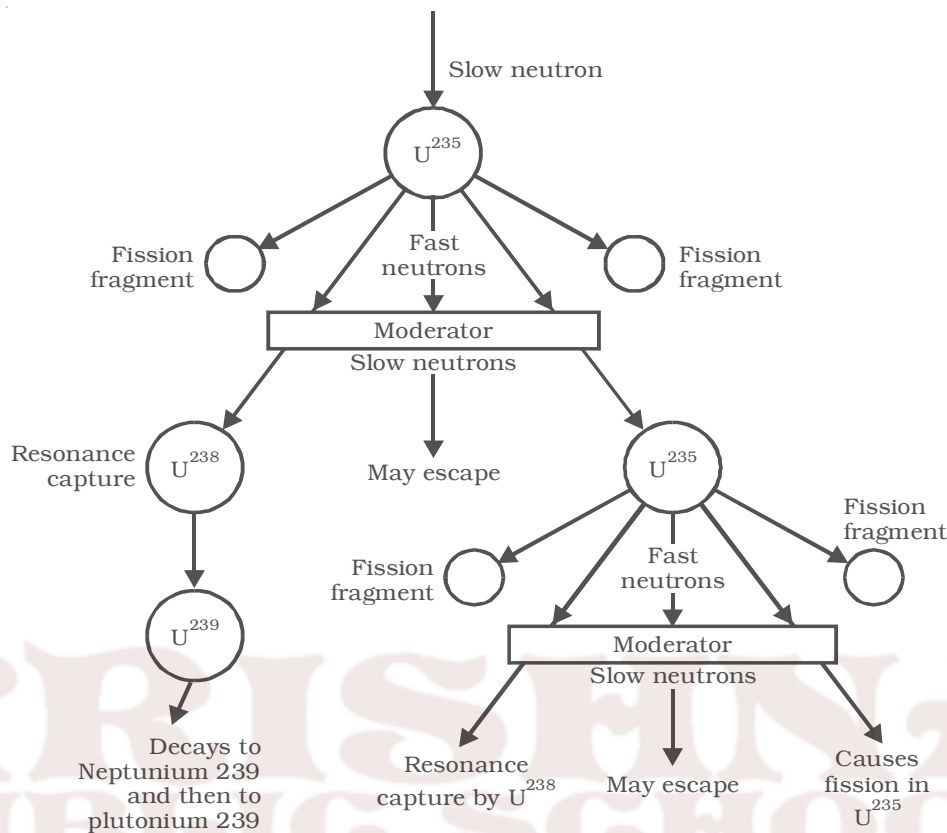


Fig 8.8 Controlled chain reaction

Np^{239} and then to Pu^{239} . One neutron is available for carrying out chain reaction. The chain reaction is possible, only when the loss of neutrons is less than the neutrons produced.

Critical size

Critical size of a system containing a fissile material is defined as the minimum size in which atleast one neutron is available for further fission reaction. The mass of the fissile material at the critical size is called critical mass. The chain reaction is not possible if the size is less than the critical size.

8.9.2 Atom bomb

Atom bomb is based on the principle of uncontrolled fission chain reaction. Natural uranium consists of 99.28% of U^{238} and 0.72% of U^{235} . U^{238} is fissionable only by fast neutrons. Hence, it is essential in

a bomb that either U^{235} or Pu^{239} should be used, because they are fissionable by neutrons of all energies.

An atom bomb consists of two hemispheres of U^{235} (or ${}_{94}Pu^{239}$), each smaller than the critical size and are kept apart by a separator aperture (Fig 8.9).

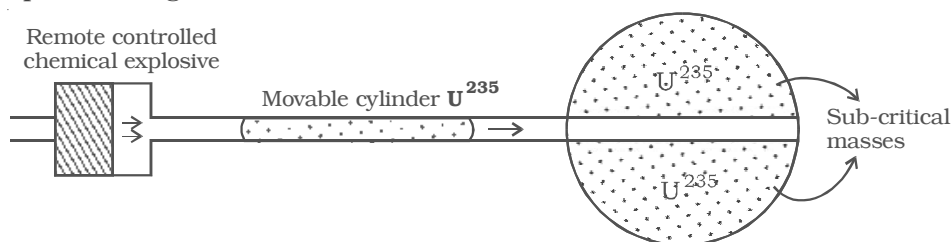


Fig 8.9 Atom bomb

When the bomb has to be exploded, a third well fitting cylinder of U^{235} (or ${}_{94}Pu^{239}$) whose mass is also less than the critical mass, is propelled so that it fuses together with the other two pieces. Now the total quantity is greater than the critical mass and an uncontrolled chain reaction takes place resulting in a terrific explosion.

The explosion of an atom bomb releases tremendous amount of energy in the form of heat, light and radiation. Temperature of millions of degree celsius and pressure of millions of atmosphere are produced. Such explosions produce shock waves. The release of dangerously radioactive γ -rays, neutrons and radioactive materials produce a health hazards over the surroundings for a long time. These bombs were used in world war II and were exploded over Hiroshima and Nagasaki in Japan. We are quite familiar with the amount of destruction and hazard.

8.9.3 Nuclear reactor

A nuclear reactor is a device in which the nuclear fission reaction takes place in a self sustained and controlled manner. The first nuclear reactor was built in 1942 at Chicago USA.

Depending on the purpose for which the reactors are used, they may be calssified into research reactors, production reactors and power reactors. Research reactors are used primarily to supply neutrons for research purpose and for production of radio-isotopes. The purpose of production reactors is to convert fertile (non-fissile but abundant) material into fissile material. The power reactor converts nuclear fission

energy into electric power. The power reactors can be further classified into boiling water reactor, pressurised water reactor, pressurised heavy water reactor and fast breeder reactor depending upon the choice of the moderator and the coolant used.

Numerous reactors of different designs have been constructed all over the world for a variety of purposes, but there exists a number of general features common to all the reactors. The schematic diagram of a nuclear reactor is shown in Fig 8.10.

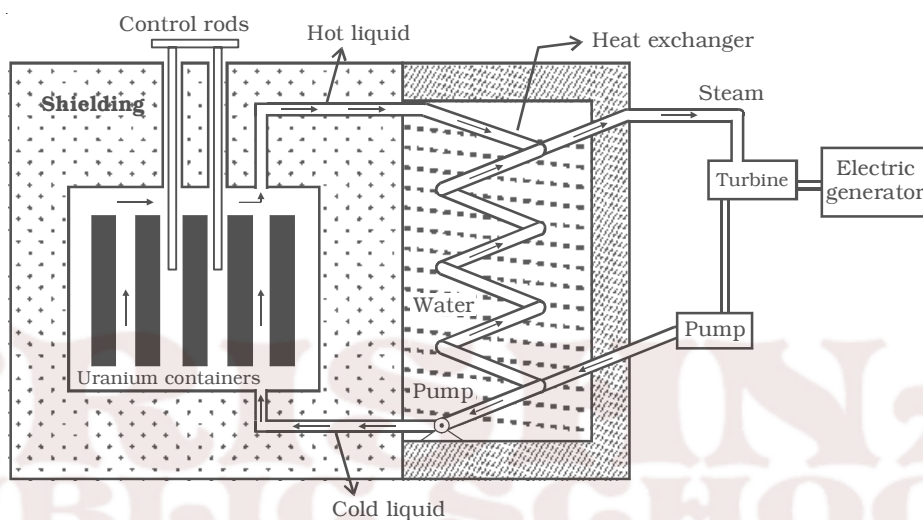


Fig 8.10 Nuclear reactor

(i) Fissile material or fuel

The fissile material or nuclear fuel generally used is ${}_{92}\text{U}^{235}$. But this exists only in a small amount (0.7%) in natural uranium. Natural uranium is enriched with more number of ${}_{92}\text{U}^{235}$ (2 – 4%) and this low enriched uranium is used as fuel in some reactors. Other than U^{235} , the fissile isotopes U^{233} and Pu^{239} are also used as fuel in some of the reactors.

In the pressurised heavy water reactors (PHWR) built in our country, natural uranium oxide is used as fuel. Tiny pellets of uranium oxide are packed in thin tubes of zirconium alloy and sealed to form a fuel rod. Nineteen such rods are tied together to form a fuel bundle. The reactor vessel which goes by the name ‘Calandria’ has about three hundred tubes passing through it. The fuel bundles are placed in these

tubes. The part of the reactor vessel which contains the fuel rods is known as reactor core.

In the pressurised light water reactors (PWR), low enriched uranium is used. In the fast breeder test reactor (FBTR) at Kalpakkam, a mixture of the carbides of uranium and plutonium is used as fuel. The fuel of the prototype fast breeder reactor (PFBR) that is being built at Kalpakkam, is a mixture of oxides of plutonium and uranium. In the fast breeder reactors, the fuel pellets are packed in special stainless steel tubes to withstand the high temperature. Kamini [Kalpakkam mini reactor] is the only operating reactor in the world which uses ${}_{92}\text{U}^{233}$ as fuel. In this reactor, the fuel is an alloy of uranium and aluminium and is in the form of plates.

(ii) Moderator

The function of a moderator is to slow down fast neutrons produced in the fission process having an average energy of about 2 MeV to thermal neutrons with an average energy of about 0.025 eV, which are in thermal equilibrium with the moderator. Ordinary water and heavy water are the commonly used moderators. A good moderator slows down neutrons by elastic collisions and it does not remove them by absorption. The moderator is present in the space between the fuel rods in a channel. Graphite is also used as a moderator in some countries.

In fast breeder reactors, the fission chain reaction is sustained by fast neutrons and hence no moderator is required.

(iii) Neutron source

A source of neutron is required to initiate the fission chain reaction for the first time. A mixture of beryllium with plutonium or radium or polonium is commonly used as a source of neutron.

(iv) Control rods

The control rods are used to control the chain reaction. They are very good absorbers of neutrons. The commonly used control rods are made up of elements like boron or cadmium. The control rods are inserted into the core and they pass through the space in between the fuel tubes and through the moderator. By pushing them in or pulling out, the reaction rate can be controlled. In our country, all the power reactors use boron carbide (B_4C), a ceramic material as control rod.

(v) The cooling system

The cooling system removes the heat generated in the reactor core. Ordinary water, heavy water and liquid sodium are the commonly used coolants. A good coolant must possess large specific heat capacity and high boiling point. The coolant passes through the tubes containing the fuel bundle and carries the heat from the fuel rods to the steam generator through heat exchanger. The steam runs the turbines to produce electricity in power reactors.

The coolant and the moderator are the same in the PHWR and PWR. In fast breeder reactors, liquid sodium is used as the coolant. A high temperature is produced in the reactor core of the fast breeder reactors. Being a metal substance, liquid sodium is a very good conductor of heat and it remains in the liquid state for a very high temperature as its boiling point is about 1000°C .

(vi) Neutron reflectors

Neutron reflectors prevent the leakage of neutrons to a large extent, by reflecting them back. In pressurised heavy water reactors the moderator itself acts as the reflector.

In the fast breeder reactors, the reactor core is surrounded by depleted uranium (uranium which contains less than 0.7% of ${}_{92}\text{U}^{235}$) or thorium (${}_{90}\text{Th}^{232}$) which acts as neutron reflector. Neutrons escaping from the reactor core convert these materials into Pu^{239} or U^{233} respectively.

(vii) Shielding

As a protection against the harmful radiations, the reactor is surrounded by a concrete wall of thickness about 2 to 2.5 m.

Breeder reactor

${}_{92}\text{U}^{238}$ and ${}_{90}\text{Th}^{232}$ are not fissile materials but are abundant in nature. In the reactor, these can be converted into a fissile material ${}_{94}\text{Pu}^{239}$ and ${}_{92}\text{U}^{233}$ respectively by absorption of neutrons. The process of producing more fissile material in a reactor in this manner than consumed during the operation of the reactor is called breeding. A fast reactor can be designed to serve as a good breeder reactor.

Uses of reactors

- (1) Nuclear reactors are mostly aimed at power production, because of the large amount of energy evolved with fission.
- (2) Nuclear reactors are useful to produce radio-isotopes.

(3) Nuclear reactor acts as a source of neutrons, hence used in the scientific research.

Nuclear power programme in India

The atomic energy commission was established in 1948 with Dr. Homi J. Bhabha as its first chairman. The Department of Atomic Energy has constituted many research reactors and critical facilities of different designs and with varying power levels. Research reactors are powerful tools for carrying out basic and applied research in a number of disciplines. The research reactors in India include Apsara, Cirus, Dhruva and Purnima in Bhabha Atomic Research Centre (BARC) Trombay, Maharashtra, and Kamini at Indira Gandhi Centre for Atomic Research (IGCAR), Kalpakkam, Tamil Nadu. They support research work, that requires the use of neutrons and also to produce radio isotopes for use in research, medicine, agriculture, industry, food preservation, etc. One more research reactor at Kalpakkam, Fast Breeder Test Reactor (FBTR) is mainly used to test fuel and other materials used in the fast reactors and to develop sodium technology.

Nuclear Power Corporation of India Ltd. (NPCIL) is the public sector company which owns, constructs and operates nuclear power plants in India. NPCIL plans to put up a total installed nuclear power capacity of 20,000 MW by the year 2020.

India's nuclear power programme has fourteen reactors in operation. There are twelve pressurised heavy water reactors (PHWR) and two boiling water reactors (BWR) with a total power generation capacity of 2770 MW. The operating nuclear power reactors are given in the table 8.1

Table 8.1 Operating nuclear power reactors (NOT FOR EXAMINATION)

Location	Type	Capacity	Number of units
Tarapur, Maharashtra Kota, Rajasthan	BWR	160 MW	2
	PHWR	100 MW	1
		200 MW	1
		220 MW	2
Kalpakkam, Tamil Nadu	PHWR	170 MW	1
		220 MW	1
Narora, Uttarpradesh	PHWR	220 MW	2
Kakrapara, Gujarat	PHWR	220 MW	2
Kaiga, Karnataka	PHWR	220 MW	2

The nuclear power reactors under construction are given in the table 8.2

**Table 8.2 Nuclear power reactors under construction
(NOT FOR EXAMINATION)**

Location	Type	Capacity	Number of units	Expected criticality date
Tarapur, Maharashtra (3 & 4)	PHWR	540 MW	2	Unit 3 - July 2006 Unit 4 - Dec. 2004
Kaiga, Karnataka (3 & 4)	PHWR	220 MW	2	Unit 3 - Dec. 2006 Unit 4 - June 2007
Rajasthan (5 & 6)	PHWR	220 MW	2	Unit 5 - May 2007 Unit 6 - Nov. 2007
Koodankulam, Tamil Nadu (1 & 2)	PWR	1000 MW	2	Unit 1 - 2007 Unit 2 - 2008
Kalpakkam, Tamil Nadu (3)	PFBR	500 MW	1	Unit 3 - 2010

8.10 Nuclear fusion

Nuclear fusion is a process in which two or more lighter nuclei combine to form a heavier nucleus. The mass of the product nucleus is always less than the sum of the masses of the individual lighter nuclei. The difference in mass is converted into energy. The fusion process can be carried out only at a extremely high temperature of the order of 10^7 K because, only at these very high temperatures the nuclei are able to overcome their mutual repulsion. Therefore before fusion, the lighter nuclei must have their temperature raised by several million degrees. The nuclear fusion reactions are known as thermo-nuclear reactions.

8.10.1 Hydrogen bomb

The principle of nuclear fusion is used in hydrogen bomb. It is an explosive device to release a very large amount of energy by the fusion of light nuclei. The temperature required for the purpose of fusion is produced by fission reactions. The explosion of an atom bomb produces temperature of the order of 50 million degree celcius. A suitable assembly of deuteron and triton is arranged at the sight of the explosion of the atom bomb. Favourable temperature initiates the fusion of light nuclei in an uncontrolled manner. This releases enormous amount of heat energy.

The fusion reaction in hydrogen bomb is

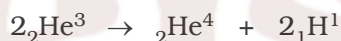
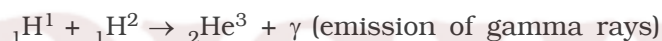


8.10.2 Stellar energy

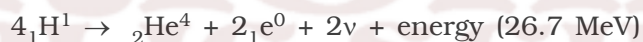
Fusion is the source of stellar energy. The temperature of the interior of the sun is about 1.4×10^7 K and the temperature of some stars is of the order 10^8 K. It has been estimated that the total energy radiated by sun is about 3.8×10^{26} joule per second. The origin of such a tremendous amount of energy is neither chemical nor gravitational. The fusion of protons into helium is supposed to release energy in sun and stars. All the elements like hydrogen and helium are in a highly ionised state called plasma at such a high temperature. The energy produced in fusion is responsible for the maintenance of high temperature of the sun and stars and also for the emission of energy in the form of heat and light.

Proton–proton cycle and carbon–nitrogen cycle are the two important types in which nuclear fusion takes place in sun and stars.

Proton – Proton cycle



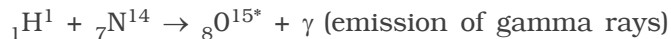
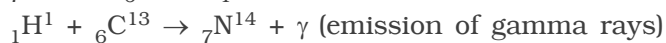
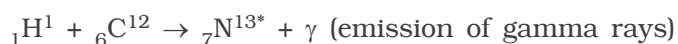
The reaction cycle is written as



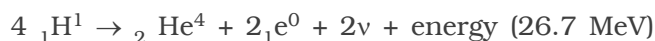
Thus four protons fuse together to form an alpha particle and two positrons with a release of large amount of energy.

Carbon – Nitrogen Cycle

The following cycle of reactions take place in carbon – nitrogen cycle in which carbon acts as a catalyst.



The overall reaction of the above cycle is given as



8.11 Cosmic Rays

The study of cosmic rays started with gold leaf electroscope experiment. It was found that if a properly insulated and charged gold leaf electroscope is left to itself, the charge leaks gradually, which happened in all the places on the land, mines and sea. Ionising radiation coming from the outer space is the reason for leakage of charges. The ionising radiation many times stronger than γ -rays entering the earth from all the directions from cosmic or interstellar space is known as cosmic rays. The name, cosmic rays was given by Millikan.

The cosmic rays can be broadly classified into primary and secondary cosmic rays. The primary cosmic rays are those coming from outer space and enter the outer boundary of the earth's atmosphere. The primary cosmic rays consist of 90% protons, 9% helium nuclei and remaining heavy nuclei. The energy of the primary cosmic rays is of the order 10^8 MeV.

The secondary cosmic rays are produced when primary cosmic rays interact with gases in the upper layers of the atmosphere. They are made up of particles like α -particles, protons, electrons, positrons, mesons, photons, etc. in different proportions.

Latitude effect

The experiments to study the variation of cosmic ray intensity (I) with geomagnetic latitude (θ) showed that the intensity is maximum at the poles ($\theta = 90^\circ$), minimum at the equator ($\theta = 0$) and constant between latitudes of 42° and 90° . The variation of cosmic ray intensity with geomagnetic latitude is known as latitude effect and is shown in Fig 8.11.

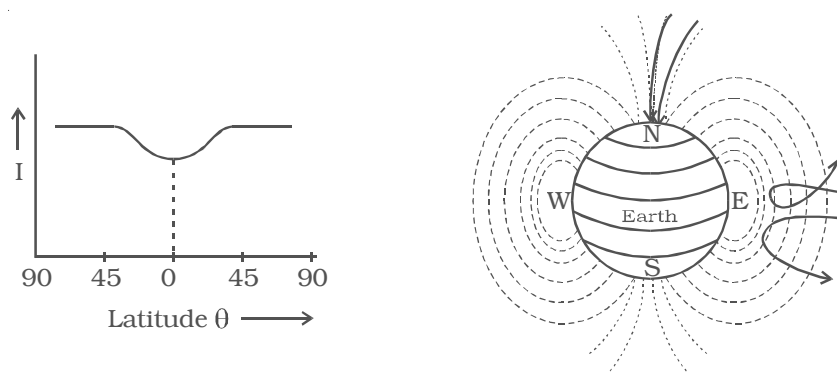


Fig 8.11 Latitude effect

The decrease in cosmic ray intensity at the earth's equator is explained to be due to the earth's magnetic field. The charged particles approaching the earth near the poles travel almost along the direction of the magnetic lines of force. They experience no force and easily reach the surface of the earth and hence maximum intensity at poles. But the charged particles that approach at the equator have to travel in a perpendicular direction to the field and are deflected away. Only particles with sufficient energy can reach the equator, while the slow particles are deflected back into cosmos and hence minimum intensity at the equator.

Altitude effect

The study of variation of cosmic ray intensity (I) with altitude (h) is known as altitude effect, which is represented in Fig 8.12. It is seen that the intensity increases with altitude and reaches a maximum at a height of about 20 km. Above this height there is a fall in intensity. The experimental results are similar at different places of the earth.

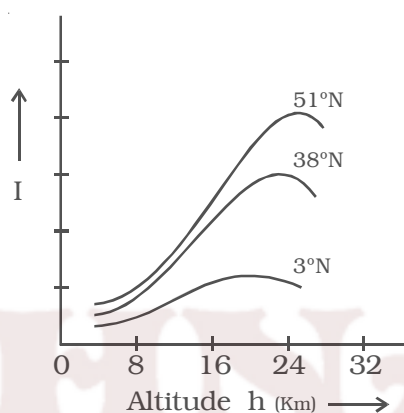


Fig 8.12 Altitude effect

Cosmic ray showers

When a detecting device is used to study cosmic ray intensities, it is observed that the intensity rises momentarily to several times its normal value, which indicates sudden burst of radiation.

The cascade theory of cosmic ray shower shows that the shower production involves two processes, radiative collision and pair production (Fig. 8.13). An energetic electron or positron present in cosmic rays loses energy, when it collides with the nuclei of atoms in earth's atmosphere. This energy loss appears as high energy photon. This photon interacts with an atomic nucleus and produce an electron positron pair. The electron and positron, so produced have sufficient energies to produce more photons on interaction with nuclei, which are further capable of bringing about pair production. The result is the generation of a large number of photons, electrons and positrons having a common origin like a shower and hence it is known as cosmic ray

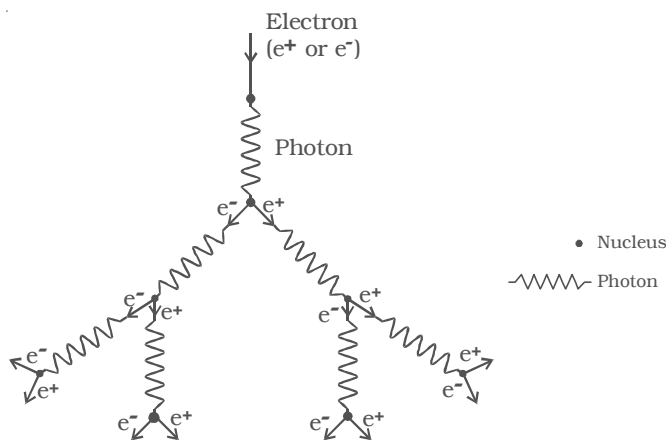


Fig 8.13 Cosmic ray shower

shower. The multiplication will continue until the initial energy becomes divided and the individual energy of the particles fall below the 'critical energy' when photon emission and pair production can no longer occur.

Pair production and annihilation of matter

The conversion of a photon into an electron-positron pair on its interaction with the strong electric field surrounding a nucleus is called pair production.

The converse of pair production in which an electron and positron combine to produce a photon is known as annihilation of matter.

8.12 Elementary particles

The study of the structure of atom reveals that the fundamental particles electron, proton and neutron are the building blocks of an atom. But the extensive studies on cosmic rays have revealed the existence of numerous new nuclear particles like mesons. These particles are classified into four major groups as photons, leptons mesons and baryons.

Photon

Photon is a quantum of radiation with no charge and no mass, but it is a carrier of energy. It travels with velocity of light.

Lepton

Leptons are lighter particles having mass equal to or less than about 207 times the mass of an electron except neutrino and antineutrino. This group contains particles such as electron, positron, neutrino, antineutrino, positive and negative muons. The electron and positron are the antiparticles. Neutrino and antineutrino are also

associated with β -ray emission. The neutrinos and antineutrinos are massless and chargeless particles, but carrier of energy and spin. Muons were discovered in cosmic ray studies.

Mesons

Mesons are fundamental particles carrying a single unit of charge and possessing mass intermediate between electron and proton (m_e and m_p). The name meson was given by Yukawa in 1935. The three types of mesons are (i) π -meson (pion) (ii) K-meson (kaon) and (iii) η -meson. The mesons are the interaction agents between nucleons. The rest mass of mesons vary between $250 m_e$ and $1000 m_e$.

Baryons

Baryons form the heavier particle group. Protons and neutrons are called nucleons and the rest of the heavier particles other than nucleons are known as hyperons. There are four types of hyperons which are lambda, sigma, xi and omega hyperons. Protons and neutrons are around 1836 times the mass of the electron, whereas the mass of the hyperons vary from $2180 m_e$ and $3275 m_e$.

The classification of elementary particles is tabulated in Table 8.3.

**Table 8.3 Classification of elementary particles
(NOT FOR EXAMINATION)**

Group	Name	Symbol	Mass in terms of rest mass of electron (m_e)
Photons	Photons	γ	0
Leptons	Neutrino	$\nu, \bar{\nu}$	0
	Electron	e^\pm	1
Mesons	Muon	μ^\pm	207
	Pion	π^\pm, π^0	273
	Kaon	K^\pm, K^0	851
	Eta meson	η^0	1072
Baryons	Proton	p^\pm	1836
	Neutron	n	1840
	Lambda hyperon	λ^0	2180
	Sigma hyperon	Σ^\pm, Σ^0	2340
	Xi hyperon	Ξ^0, Ξ^-	2583
	Omega hyperon	Ω^-	3272

Solved Problems

8.1 Calculate the binding energy and binding energy per nucleon of ${}_{26}\text{Fe}^{56}$ nucleus. Given,

$$\text{mass of } {}_{26}\text{Fe}^{56} \text{ nucleus} = 55.9349 \text{ amu}$$

$$\text{mass of 1 proton} = 1.007825 \text{ amu}$$

$$\text{mass of 1 neutron} = 1.008665 \text{ amu}$$

Data : Mass of ${}_{26}\text{Fe}^{56}$ nucleus = 55.9349 amu

$$\text{Mass of 1 proton} = 1.007825 \text{ amu}$$

$$\text{Mass of 1 neutron} = 1.008665 \text{ amu}$$

$$\text{Number of protons } A = 26$$

$$\text{Number of neutrons } N = 30$$

$$\text{Binding energy and binding energy per nucleon} = ?$$

Solution : Mass of 26 Protons = 26×1.007825

$$= 26.20345 \text{ amu}$$

$$\text{Mass of 30 neutrons} = 30 \times 1.008665$$

$$= 30.25995 \text{ amu}$$

$$\text{Total mass of the nucleons} = 26.20345 + 30.25995$$

$$= 56.46340 \text{ amu}$$

$$\text{Actual mass of the } {}_{26}\text{Fe}^{56} \text{ Nucleus} = 55.9349 \text{ amu}$$

$$\therefore \text{Mass defect} = 56.4634 - 55.9349$$

$$= 0.5285 \text{ amu}$$

$$\text{But 1 amu} = 931 \text{ MeV}$$

$$\text{Binding energy} = 0.5285 \times 931 = 492.0335 \text{ MeV}$$

$$\text{Binding energy per nucleon} = \frac{492.0335}{56} \text{ MeV}$$

$$= 8.786 \text{ MeV}$$

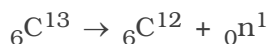
8.2 The binding energy per nucleon for ${}_{6}\text{C}^{12}$ nucleus is 7.68 MeV and that for ${}_{6}\text{C}^{13}$ is 7.47 MeV. Calculate the energy required to remove a neutron from ${}_{6}\text{C}^{13}$ nucleus.

Data : Binding energy per nucleon of ${}_{6}\text{C}^{13} = 7.47 \text{ MeV}$

Binding energy per nucleon of ${}_6\text{C}^{12} = 7.68 \text{ MeV}$

Binding energy of neutron = ?

Solution : We can write the reaction as



$$\begin{aligned}\text{Total binding energy of } {}_6\text{C}^{13} &= 7.47 \times 13 \\ &= 97.11 \text{ MeV}\end{aligned}$$

$$\begin{aligned}\text{Total binding energy of } {}_6\text{C}^{12} &= 7.68 \times 12 \\ &= 92.16 \text{ MeV}\end{aligned}$$

Total binding energy of the reactant

= Total binding energy of the product

$$\therefore 97.11 \text{ MeV} = 92.16 \text{ MeV} + \text{Binding energy of a neutron}$$

$$\begin{aligned}\therefore \text{Binding energy of a neutron} &= 97.11 - 92.16 \\ &= 4.95 \text{ MeV}\end{aligned}$$

- 8.3 Assuming that the energy released by the fission of a single ${}_{92}\text{U}^{235}$ nucleus is 200 MeV, calculate the number of fissions per second required to produce 1 watt of power.

Data : Energy per fission = 200 MeV

Required power = 1 watt = 1 J/s

Number of fissions per second = ?

Solution :

Since the two energies are in different units, we must convert them into the same unit.

$$\therefore \text{Energy released per fission} = 200 \text{ MeV.}$$

$$= 200 \times 10^6 \text{ eV}$$

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \quad [\because 1\text{eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$= 320 \times 10^{-13} \text{ joules}$$

Let N be the number of fissions per second, producing 1 W.

Energy per fission \times N = Total energy released per second.

$$\therefore 320 \times 10^{-13} \times N = 1 \text{ J/s.}$$

$$\begin{aligned}\therefore N &= \frac{1}{320 \times 10^{-13}} \\ &= 3.125 \times 10^{10} \text{ fissions}\end{aligned}$$

Number of fissions per second to produce the required power is 3.125×10^{10}

- 8.4 Determine the amount of Po^{210} required to provide a source of α particles of activity 5 milli curie. Given $T_{1/2}$ of polonium is 138 days.

Data : $T_{1/2}$ for polonium = 138 days

$$\frac{dN}{dt} = 5 \text{ milli curie} = 5 \times 3.7 \times 10^{10} \times 10^{-3} \text{ disintegrations per second}$$

Amount of polonium sample = ?

$$\begin{aligned}\text{Solution : } \lambda &= \frac{0.6931}{138} \text{ per day} \\ &= \frac{0.6931}{138 \times 24 \times 60 \times 60} \text{ per second}\end{aligned}$$

$$\begin{aligned}\text{The activity } \frac{dN}{dt} &= 5 \text{ milli curie} \\ &= 5 \times 10^{-3} \times 3.7 \times 10^{10} \text{ disintegration per second} \\ &= 5 \times 3.7 \times 10^7 \text{ disintegration per second}\end{aligned}$$

From law of disintegration,

$$\begin{aligned}\frac{dN}{dt} &= \lambda N \\ \therefore N &= \frac{dN}{dt} \cdot \frac{1}{\lambda} \\ &= \frac{5 \times 3.7 \times 10^7 \times 24 \times 60 \times 60 \times 138}{0.6931} \\ &= 3.1825 \times 10^{15} \text{ atoms}\end{aligned}$$

According to Avagadro's principle, 6.023×10^{23} atoms = 210 g of polonium

$$\therefore 3.1825 \times 10^{15} \text{ atoms} = \frac{210 \times 3.1825 \times 10^{15}}{6.023 \times 10^{23}} = 1.11 \times 10^{-6}$$

\therefore Amount required is, 1.11×10^{-6} g

8.5 Calculate the time required for 60% of a sample of radon to undergo decay. Given $T_{1/2}$ of radon = 3.8 days

Data : Half life of radon ($T_{1/2}$) = 3.8 days

Amount of sample disintegrated = 60%

Time required = ?

Solution : $\lambda = \frac{0.6931}{3.8}$ per day

Amount of sample disintegrated = 60%

Amount of sample present = 40%

Let N_0 be the original amount of the sample present.

From law of disintegration,

$$N = N_0 e^{-\lambda t}$$

Substituting for $N = 40\%$ of N_0 ,

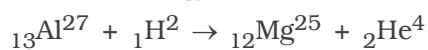
$$\frac{40}{100} N_0 = N_0 e^{-\lambda t}$$

$$\therefore e^{\lambda t} = \frac{10}{4}$$

$$\log_e 2.5 = \lambda \times t$$

$$\therefore t = \frac{3.8}{0.6931} \times \log_{10} 2.5 \times 2.3026 = 5.022 \text{ days}$$

8.6 Calculate the energy released in the reaction



Given : Mass of ${}_{13}\text{Al}^{27} = 26.981535$ amu

Mass of ${}_1\text{H}^2 = 2.014102$ amu

Mass of ${}_{12}\text{Mg}^{25} = 24.98584$ amu

Mass of ${}_2\text{He}^4 = 4.002604$ amu

Data : Mass of $_{13}\text{Al}^{27}$ = 26.981535 amu
 Mass of $_{1}\text{H}^2$ = 2.014102 amu
 Mass of $_{12}\text{Mg}^{25}$ = 24.98584 amu
 Mass of $_{2}\text{He}^4$ = 4.002604 amu
 Energy released = ?

Solution : Mass of the reactants = 26.981535 + 2.014102
 = 28.995637 amu
 Mass of the products = 24.98584 + 4.002604
 = 28.988444 amu

\therefore Difference in mass in the reaction
 = 28.995637 - 28.988444
 = 0.007193 amu

1 amu = 931 MeV

\therefore Energy released in the reaction = 0.007193 \times 931
 = 6.697 MeV

8.7 Calculate the energy released when 1 kg of $_{92}\text{U}^{235}$ undergoes nuclear fission. Assume, energy per fission is 200 MeV. Avagadro number = 6.023×10^{23} . Express your answer in kilowatt hour also.

Data : Energy produced per fission of $_{92}\text{U}^{235}$ = 200 MeV
 Amount of $_{92}\text{U}^{235}$ = 1 kg
 Avagadro number = 6.023×10^{23}
 Energy released = ?

Solution : Energy released in one fission = 200 MeV

According to Avagadro's hypothesis,

Number of atoms in 235 g of uranium = 6.023×10^{23}

\therefore Number of atoms in 1 kg (1000 gm) of Uranium

$$= \frac{6.023 \times 10^{26}}{235}$$

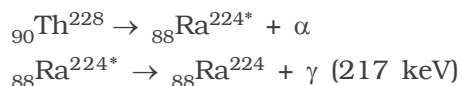
\therefore Energy produced by 1 kg of uranium during fission, E = $\frac{6.023 \times 10^{26}}{235} \times 200$ MeV

$$\begin{aligned}
&= 5.126 \times 10^{26} \text{ MeV} \\
&= 5.126 \times 10^{26} \times 10^6 \text{ eV} \\
&= 5.126 \times 10^{32} \times 1.6 \times 10^{-19} \text{ J } [\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]
\end{aligned}$$

1 kilowatt hour = 3.6×10^6 joule

$$\begin{aligned}
\therefore E &= \frac{5.126 \times 10^{32} \times 1.6 \times 10^{-19}}{3.6 \times 10^6} \text{ kWh} \\
&= 2.2782 \times 10^7 \text{ kWh}
\end{aligned}$$

8.8 Thorium (${}_{90}\text{Th}^{228}$) emits an α -particle to reduce to ${}_{88}\text{Ra}^{224}$. Calculate the kinetic energy of the α -particle emitted in the following decay.



Given : mass of ${}_{90}\text{Th}^{228}$ = 228.028726 amu

mass of ${}_{88}\text{Ra}^{224}$ = 224.020196 amu

mass of ${}_{2}\text{He}^4$ = 4.002604 amu

Data : mass of ${}_{90}\text{Th}^{228}$ = 228.028726 amu

mass of ${}_{88}\text{Ra}^{224}$ = 224.020196 amu

mass of ${}_{2}\text{He}^4$ = 4.002604 amu

Energy of γ -ray = 217 keV

Energy of α -particle = ?

Solution :

The α -decay of thorium can be represented as in the figure.

$$\therefore \text{Mass of the products} = 224.020196 + 4.002604$$

$$= 228.022800 \text{ amu}$$

$$\therefore \text{Mass defect} = 228.028726 - 228.022800$$

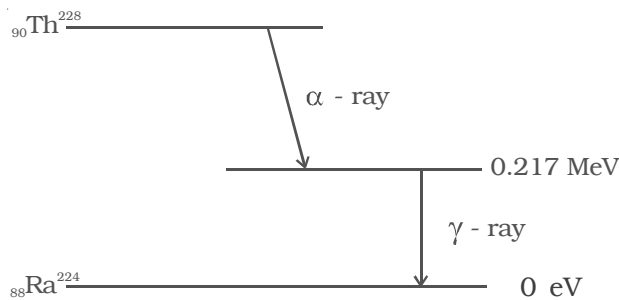
$$= 0.005926 \text{ amu}$$

$$1 \text{ amu} = 931 \text{ MeV}$$

$$\therefore \text{Maximum energy of } \alpha\text{-particle}$$

$$= 0.005926 \times 931$$

$$= 5.517 \text{ MeV}$$



In this case energy of α -particle emitted = maximum energy of α -particle - Energy of γ -ray

$$= 5.517 - 0.217$$

$$= 5.300 \text{ MeV}$$

- 8.9 Calculate the mass of coal required to produce the same energy as that produced by the fission of 1 kg of U^{235} . Given ; heat of combustion of coal = $33.6 \times 10^6 \text{ J/kg}$, 1 ton = 1000 kg. Energy per fission of U^{235} = 200 MeV. $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$. Avagadro number $N = 6.023 \times 10^{23}$

Data : Energy per fission = 200 MeV;

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\text{Avagadro number } N = 6.023 \times 10^{23}$$

$$\text{Heat of combustion of coal} = 33.6 \times 10^6 \text{ J/kg}$$

Mass of coal = ?

Solution : Energy per fission of U^{235} = 200 MeV

$$= 200 \times 10^6 \text{ eV}$$

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{Number of atoms in 235 g of } \text{U}^{235} = 6.023 \times 10^{23}$$

$$\therefore \text{Number of atoms in 1 kg (1000 g)} = \frac{6.023}{235} \times 10^{23} \times 1000$$

$$= \frac{6.023 \times 10^{26}}{235}$$

\therefore Energy produced by the fission of 1 kg of U^{235}

$$= \frac{6.023 \times 10^{26}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 8.2016 \times 10^{13} \text{ J}$$

Heat of combustion of coal = 33.6×10^6 J /kg.

Let M be the mass in kg of the coal required to produce equivalent energy produced by 1 kg of U^{235}

$$\therefore 33.6 \times 10^6 \times M = 8.2016 \times 10^{13} \text{ J}$$

$$\begin{aligned}\therefore M &= \frac{8.2016 \times 10^{13}}{33.6 \times 10^6} \\ &= 0.2441 \times 10^7 \text{ Kg} \\ &= 2.441 \times 10^6 \text{ kg}\end{aligned}$$

$$1000 \text{ kg} = 1 \text{ ton}$$

$$\begin{aligned}\therefore 2.441 \times 10^6 \text{ kg} &= 2441 \times 10^3 \text{ kg} \\ &= 2441 \text{ ton}\end{aligned}$$

\therefore 2441 ton of coal is required to produce the same energy as that produced by fission of 1 kg of U^{235} .

8.10 A piece of bone from an archaeological site is found to give a count rate of 15 counts per minute. A similar sample of fresh bone gives a count rate of 19 counts per minute. Calculate the age of the specimen. Given : $T_{1/2} = 5570$ years

Data : Count rate of fresh sample, $N_0 = 19$ counts per minute

Count rate of bone $N = 15$ counts per minute

$T_{1/2} = 5570$ years ; Age of the sample, $t = ?$

Solution : $N = N_0 e^{-\lambda t}$

$$\lambda = \frac{0.6931}{5570}$$

$$15 = 19 e^{-\lambda t} \quad (\text{or}) \quad e^{\lambda t} = \frac{19}{15}$$

$$\log e^{\frac{19}{15}} = \lambda t \quad (\text{or}) \quad t = \log e^{\frac{19}{15}} \times \frac{1}{\lambda}$$

$$t = \frac{5570}{0.6931} \times 2.3026 \times \log_{10} \frac{19}{15}$$

$$= \frac{5570}{0.6931} \times 2.3026 \times 0.1026$$

$$t = 1899 \text{ years}$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

- 8.1 The nuclear radius of ${}_4\text{Be}^8$ nucleus is
(a) 1.3×10^{-15} m (b) 2.6×10^{-15} m
(c) 1.3×10^{-13} m (d) 2.6×10^{-13} m
- 8.2 The nuclei ${}_{13}\text{Al}^{27}$ and ${}_{14}\text{Si}^{28}$ are example of
(a) isotopes (b) isobars
(c) isotones (d) isomers
- 8.3 The mass defect of a certain nucleus is found to be 0.03 amu. Its binding energy is
(a) 27.93 eV (b) 27.93 KeV
(c) 27.93 MeV (d) 27.93 GeV
- 8.4 Nuclear fission can be explained by
(a) shell model (b) liquid drop model
(c) quark model (d) Bohr atom model
- 8.5 The nucleons in a nucleus are attracted by
(a) gravitational force (b) electrostatic force
(c) nuclear force (d) magnetic force
- 8.6 The ionisation power is maximum for
(a) neutrons (b) α - particles
(c) γ - rays (d) β - particles
- 8.7 The half life period of a certain radioactive element with disintegration constant 0.0693 per day is
(a) 10 days (b) 14 days
(c) 140 days (d) 1.4 days
- 8.8 The radio-isotope used in agriculture is
(a) ${}_{15}\text{P}^{31}$ (b) ${}_{15}\text{P}^{32}$
(c) ${}_{11}\text{Na}^{23}$ (d) ${}_{11}\text{Na}^{24}$

- 8.9 The average energy released per fission is
(a) 200 eV (b) 200 MeV
(c) 200 meV (d) 200 GeV
- 8.10 The explosion of atom bomb is based on the principle of
(a) uncontrolled fission reaction (b) controlled fission reaction
(c) fusion reaction (d) thermonuclear reaction
- 8.11 Anaemia can be diagnosed by
(a) $_{15}\text{P}^{31}$ (b) $_{15}\text{P}^{32}$
(c) $_{26}\text{Fe}^{59}$ (d) $_{11}\text{Na}^{24}$
- 8.12 In the nuclear reaction $_{80}\text{Hg}^{198} + X \rightarrow _{79}\text{Au}^{198} + _1\text{H}^1$, X-stands for
(a) proton (b) electron
(c) neutron (d) deuteron
- 8.13 In β - decay
(a) atomic number decreases by one
(b) mass number decreases by one
(c) proton number remains the same
(d) neutron number decreases by one
- 8.14 Isotopes have
(a) same mass number but different atomic number
(b) same proton number and neutron number
(c) same proton number but different neutron number
(d) same neutron number but different proton number
- 8.15 The time taken by the radioactive element to reduce to $1/e$ times is
(a) half life (b) mean life
(c) half life/2 (d) twice the mean life
- 8.16 The half life period of N^{13} is 10.1 minute. Its life time is
(a) 5.05 minutes (b) 20.2 minutes
(c) $\frac{10.1}{0.6931}$ minutes (d) infinity

- 8.17 Positive rays of the same element produce two different traces in a Bainbridge mass spectrometer. The positive ions have
- (a) same mass with different velocity
 - (b) same mass with same velocity
 - (c) different mass with same velocity
 - (d) different mass with different velocity
- 8.18 The binding energy of ${}_{26}\text{Fe}^{56}$ nucleus is
- (a) 8.8 MeV
 - (b) 88 MeV
 - (c) 493 MeV
 - (d) 41.3 MeV
- 8.19 The ratio of nuclear density to the density of mercury is about
- (a) 1.3×10^{10}
 - (b) 1.3
 - (c) 1.3×10^{13}
 - (d) 1.3×10^4
- 8.20 With example explain the classification of the nuclei in terms of its proton number and neutron number.
- 8.21 Explain mass defect and binding energy.
- 8.22 Calculate the energy equivalence of 1 atomic mass unit.
- 8.23 Show that nuclear density is almost a constant for all the nuclei.
- 8.24 Explain the variation of binding energy with mass number by a graph and discuss its features.
- 8.25 Discuss the principle and action of a Bainbridge mass spectrometer to determine the isotopic masses.
- 8.26 Explain the different characteristics of nuclear forces.
- 8.27 Define radioactivity.
- 8.28 Explain the Soddy-Fajan's radioactive displacement law.
- 8.29 Obtain an expression to deduce the amount of the radioactive substance present at any moment.
- 8.30 Obtain the relation between half-life period and decay constant.
- 8.31 Define curie.
- 8.32 What do you mean by artificial radioactivity?
- 8.33 What are the applications of radio-isotopes?

- 8.34 Explain the construction and working of a Geiger–Muller Counter.
- 8.35 How do you classify the neutrons in terms of its kinetic energy?
- 8.36 What is artificial transmutation?
- 8.37 Explain how liquid drop model of the nucleus can account for nuclear fission.
- 8.38 With a neat sketch, explain the working of a nuclear reactor.
- 8.39 What is meant by breeder reactor?
- 8.40 What are thermonuclear reactions?
- 8.41 Explain how carbon–nitrogen cycle can account for the production of stellar energy.
- 8.42 What are cosmic rays?
- 8.43 Explain the latitude effect of cosmic rays.
- 8.44 Explain how the intensity of the cosmic rays changes with altitude.
- 8.45 Explain how a cosmic ray shower is formed.
- 8.46 How do you classify the elementary particles into four groups?

Problems :

- 8.47 The half-life of ${}_{84}\text{Po}^{218}$ is 3 minute. What percentage of the sample has decayed in 15 minutes?
- 8.48 Calculate the radius of ${}_{13}\text{Al}^{27}$ nucleus.
- 8.49 Find the energy released when two ${}_{1}\text{H}^2$ nuclei fuse together to form a single ${}_{2}\text{He}^4$ nucleus. Given, the binding energy per nucleon of ${}_{1}\text{H}^2$ and ${}_{2}\text{He}^4$ are 1.1 MeV and 7.0 MeV respectively.
- 8.50 Calculate the binding energy and binding energy per nucleon of ${}_{20}\text{Ca}^{40}$ nucleus. Given, mass of 1 proton = 1.007825 amu ; mass of 1 neutron = 1.008665 amu ; mass of ${}_{20}\text{Ca}^{40}$ nucleus = 39.96259 amu
- 8.51 Calculate the energy released in the following reaction.
- $${}_{3}\text{Li}^6 + {}_0\text{n}^1 \rightarrow {}_{2}\text{He}^4 + {}_{1}\text{H}^3$$
- Given mass of ${}_{3}\text{Li}^6$ nucleus = 6.015126 amu
 Mass of ${}_{1}\text{H}^3$ nucleus = 3.016049 amu
 Mass of ${}_{2}\text{He}^4$ nucleus = 4.002604 amu
 Mass of ${}_0\text{n}^1$ = 1.008665 amu
- 8.52 Tritium has a half life of 12.5 years. What fraction of the sample of

will be left over after 25 years?

- 8.53 Show that the mass of radium (${}_{88}\text{Ra}^{226}$) with an activity of 1 curie is almost a gram. Given $T_{1/2} = 1600$ years;
1 curie = 3.7×10^{10} disintegrations per second.
- 8.54 The disintegration constant λ of a radioactive element is 0.00231 per day. Calculate its half life and mean life.
- 8.55 A reactor is developing energy at the rate of 32 MW. Calculate the required number of fissions per second of ${}_{92}\text{U}^{235}$. Assume that energy per fission is 200 MeV.
- 8.56 A carbon specimen found in a cave contained a fraction of 1/8 of C^{14} to that present in a living system. Calculate the approximate age of the specimen. Given $T_{1/2}$ for ${}_{6}\text{C}^{14} = 5560$ years.
- 8.57 The radioactive isotope ${}_{84}\text{Po}^{214}$ undergoes a successive disintegration of two α -decays and two β -decays. Find the atomic number and mass number of the resulting isotope.
- 8.58 The half life of radon is 3.8 days. Calculate its mean life
- 8.59 If 50% of a radioactive sample decays in 5 days, how much of the original sample will be left over after 20 days?
- 8.60 The isotope ${}_{92}\text{U}^{238}$ successively undergoes three α -decays and two β -decays. What is the resulting isotope?
- 8.61 Singly ionized magnesium atoms enter into the velocity selector of a Bainbridge mass spectrograph having electric and magnetic fields 30 kV/m and 0.1 tesla respectively. Calculate the radii of the path followed by the three isotopes of mass numbers 24, 25 and 26 when the deflecting magnetic field is 0.5 tesla. Mass of a nucleon = 1.67×10^{-27} kg ; Charge of the ion, $e = 1.6 \times 10^{-19}$ C.

Answers

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| 8.1 (b) | 8.2 (c) | 8.3 (c) | 8.4 (b) |
| 8.5 (c) | 8.6 (b) | 8.7 (a) | 8.8 (b) |
| 8.9 (b) | 8.10 (a) | 8.11 (c) | 8.12 (c) |
| 8.13 (d) | 8.14 (c) | 8.15 (b) | 8.16 (d) |
| 8.17 (c) | 8.18 (c) | 8.19 (c) | |

8.47 96.875 %

8.48 $3.9 \times 10^{-15} \text{ m}$

8.49 23.6 MeV

8.50 341.8725 MeV; 8.5468 MeV

8.51 4.783 MeV

8.52 1/4

8.53 1.0107 g

8.54 $T_{1/2} = 300 \text{ days}; \tau = 432.9 \text{ days}$

8.55 $1 \times 10^{18} \text{ fissions per second}$

8.56 16681 year

8.57 Atomic number 82 and mass number 206

8.58 5.482 days

8.59 6.25%

8.60 ${}_{88}\text{Ra}^{226}$

8.61 $R_1 = 0.1503 \text{ m}; R_2 = 0.1565 \text{ m}; R_3 = 0.1628 \text{ m}$.

9. Semiconductor Devices and their Applications

In years before the second world war, vacuum tubes were used in the process of electrical signal generation, amplification and transmission. After the invention of transistor in 1944 and subsequent improvement of the transistor and other solid state electronic devices, vacuum tubes were replaced by these solid state devices in many fields of applications. A solid state electronic device mainly consists of a semiconducting material. Some semiconductor devices and their applications are dealt in this chapter.

9.1 Semiconductors

It has been observed that certain materials like germanium, silicon etc. have resistivity between good conductors like copper and insulators like glass. These materials are known as semiconductors. A material which has resistivity between conductors and insulators is known as semiconductor. The resistivity of a semiconductor lie approximately between 10^{-2} and $10^4 \Omega \text{ m}$ at room temperature. The resistance of a semiconductor decreases with increase in temperature over a particular temperature range. This behaviour is contrary to that of a metallic conductor for which the resistance increases with increase in temperature.

The elements that are classified as semiconductors are Si, Ge, In, etc. Germanium and silicon are most widely used as semiconductors.

9.1.1 Energy band in solids

In the case of a single isolated atom, there are various discrete energy levels. In solids, the atoms are arranged in a systematic space lattice and each atom is influenced by neighbouring atoms. The closeness of atoms results in the intermixing of electrons of neighbouring atoms. Due to this, number of permissible energy levels increases. Hence in the case of a solid, instead of a single energy level

associated with single atom, there will be bands of energy levels. A set of such closely packed energy levels is called an energy band. The bands of energy levels are referred to the entire solid as a whole and not to the single atom.

The concept of energy bands can be understood from Fig 9.1a and Fig 9.1b. The energy levels of a single isolated atom of silicon are shown in Fig 9.1a. Each silicon atom has 14 electrons, two of which

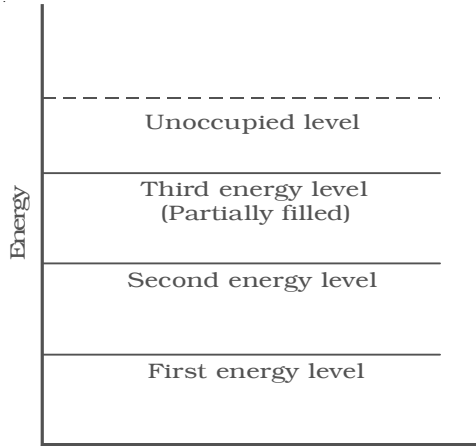


Fig 9.1a Energy levels of a single isolated atom

occupy K shell, 8 occupy the L shell and 4 occupy the M shell. The electrons in the M shell are distributed as 2 electrons in the subshell 3s and 2 electrons in the subshell 3p. This subshell 3p is partially filled because it can accommodate a total of 6 electrons. The completely filled levels are known as core levels and the electrons filling these levels are called core electrons. The electrons in the outermost level are called valence electrons. The partially filled outermost level is valence level and the permitted levels which are vacant are known as conduction levels.

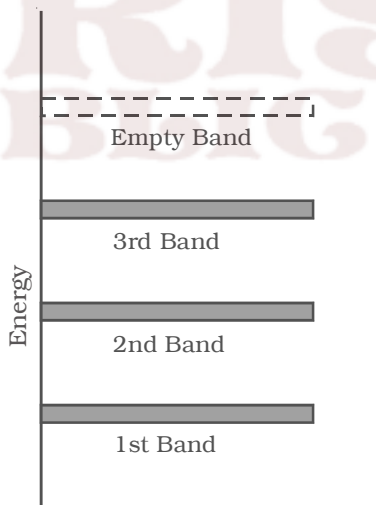


Fig 9.1b Energy bands in a solid

In a solid, there are large number of atoms, which are very close to each other. The energy of s or p level is of the order of eV, therefore the levels are very closely spaced. The first orbit electrons form a band called first energy band. Similarly second orbit electrons form

second energy band and so on as shown in Fig 9.1b.

9.1.2 Valence band, conduction band and forbidden energy gap

The atoms of a solid are arranged in a regular repeated geometric pattern and the electrons

of the atom revolve around the nucleus in certain permitted energy levels. The electrons in the inner shells are strongly bound to the nucleus. A band which is occupied by the valence electrons or a band having highest energy is defined as

valence band (Fig 9.2). The valence band may be partially or completely filled. This band can never be empty.

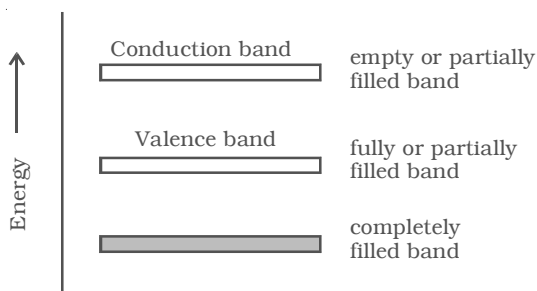


Fig 9.2 Valence band and conduction band

In some materials, the valence electrons are loosely attached to the nucleus. Even at room temperature, some of the valence electrons can leave the valence band. These are called as free electrons. They are responsible for conduction of current in a conductor and are henceforth called as conduction electrons. The band occupied by these electrons is called conduction band. This band may be an empty band or partially filled band.

The separation between valence band and conduction band is known as forbidden energy gap. If an electron is to be transferred from valence band to conduction band, external energy is required, which is equal to the forbidden energy gap.

9.1.3 Insulators, semiconductors and conductors

Insulators

In an insulator, the forbidden energy gap is very large (Fig 9.3a). In general, the forbidden energy gap is more than 3eV and almost no electrons are available for conduction. Therefore, a very large amount of energy must be supplied to a valence electron to enable it to move to the conduction band. In the case of materials like glass, the valence band is completely filled at 0 K. The energy gap between valence band and conduction band is of the order of 10 eV. Even in the presence of high electric field, the electrons cannot move from

valence band to conduction band. If the electron is supplied with high energy, it can jump across the forbidden gap. When the temperature is increased, some electrons will move to the conduction band. This is the reason, why certain materials, which are insulators at room temperature become conductors at high temperature. The resistivity of insulator approximately lies between 10^{11} and $10^{16} \Omega \text{ m}$.

Semiconductors

In semiconductors (Fig 9.3b), the forbidden gap is very small. Germanium and silicon are the best examples of semiconductors. The forbidden gap energy is of the order of 0.7eV for Ge and 1.1eV for Si. There are no electrons in the conduction band. The valence band is completely filled at 0 K. With a small amount of energy that is supplied, the electrons can easily jump from the valence band to the conduction band. For example, if the temperature is raised, the forbidden gap is decreased and some electrons are liberated into the conduction band. The conductivity of a semiconductor is of the order of 10^2 mho m^{-1} .

Conductors

In conductors, there is no forbidden gap available, the valence and conduction band overlap each other (Fig 9.3c). The electrons from valence band freely enter into the conduction band. Due to the overlapping of the valence and conduction bands, a very low potential difference can cause the continuous flow of current.

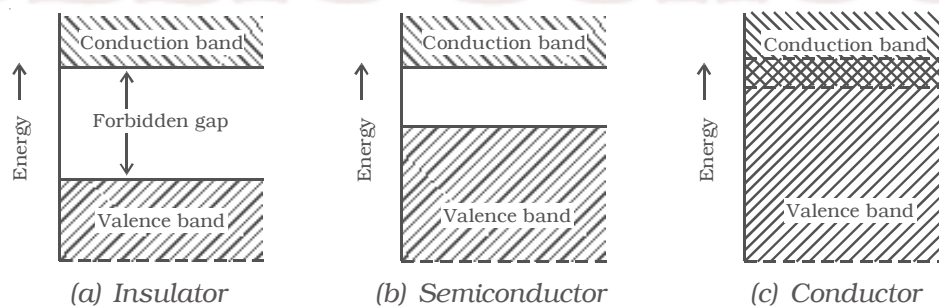


Fig 9.3 Energy band of solids

9.1.4 Electrons and holes in semiconductors

Fig 9.3b shows the energy band diagram of an intrinsic semiconductor (pure semiconductor). Fig 9.4a and Fig 9.4b represent charge carriers at absolute zero temperature and at room temperature respectively.

The electrons in an intrinsic semiconductor, which move in to the conduction band at high temperatures are called as intrinsic carriers. In the valence band, a vacancy is created at the place where the electron was present, before it had moved in to the conduction band. This vacancy is called hole. Fig 9.4c helps in understanding the creation of a hole. Consider the case of pure germanium crystal. It has four electrons in its outer or valence orbit. These electrons are known as valence electrons. When two atoms of germanium are brought close to each other, a covalent bond is formed between the atoms. If some additional energy is received, one of the electrons contributing to a covalent bond breaks and it is free to move in the crystal lattice.

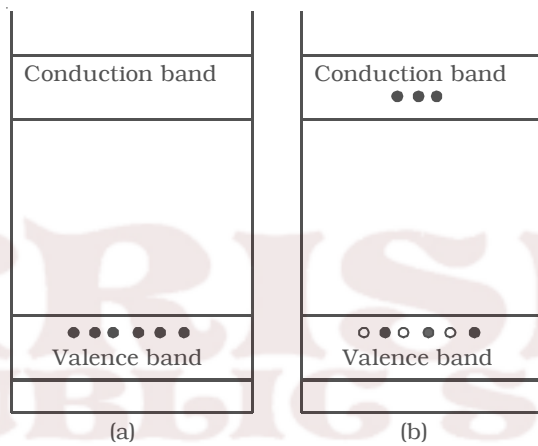


Fig 9.4a&b Electrons and holes in semiconductors

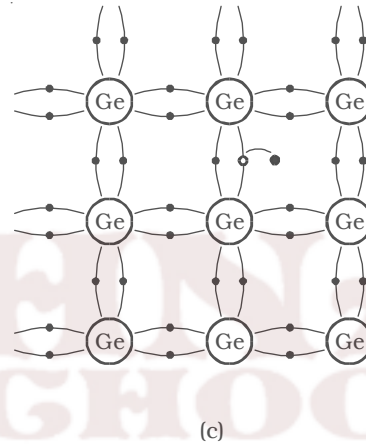


Fig 9.4c Formation of a hole

While coming out of the bond, a hole is said to be created at its place, which is usually represented by a open circle. An electron from the neighbouring atom can break the covalent bond and can occupy this hole, creating a hole at another place. Since an electron has a unit negative charge, the hole is associated with a unit positive charge. The importance of hole is that, it may serve as a carrier of electricity in the same manner as the free electron, but in the opposite direction.

9.1.5 Intrinsic semiconductor

A semiconductor which is pure and contains no impurity is known as an intrinsic semiconductor. In an intrinsic semiconductor, the number of free electrons and holes are equal. Common examples of intrinsic semiconductors are pure germanium and silicon.

The forbidden energy gap is so small that even at ordinary room temperature, there are many electrons which possess sufficient energy to cross the forbidden energy gap and enter into the conduction band. Schematic band diagram of an intrinsic semiconductor at room temperature is represented in Fig 9.5.

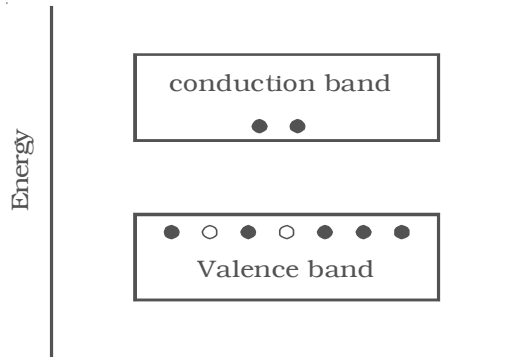


Fig 9.5 Energy band diagram of an intrinsic semiconductor

9.1.6 Doping a semiconductor

Electrons and holes can be generated in a semiconductor crystal with heat energy or light energy. But in these cases, the conductivity remains very low. The efficient and convenient method of generating free electrons and holes is to add very small amount of selected impurity inside the crystal. The impurity to be added is of the order of 100 ppm (parts per million). The process of addition of a very small amount of impurity into an intrinsic semiconductor is called doping. The impurity atoms are called dopants. The semiconductor containing impurity atoms is known as impure or doped or extrinsic semiconductor.

There are three different methods of doping a semiconductor.

- (i) The impurity atoms are added to the semiconductor in its molten state.
- (ii) The pure semiconductor is bombarded by ions of impurity atoms.
- (iii) When the semiconductor crystal containing the impurity atoms is heated, the impurity atoms diffuse into the hot crystal.

Usually, the doping material is either pentavalent atoms (bismuth, antimony, phosphorous, arsenic which have five valence electrons) or trivalent atoms (aluminium, gallium, indium, boron which have three valence electrons). The pentavalent doping atom is known as donor atom, since it donates one electron to the conduction band of pure semiconductor. The trivalent atom is called an acceptor atom, because it accepts one electron from the pure semiconductor atom.

9.1.7 Extrinsic semiconductor

An extrinsic semiconductor is one in which an impurity with a valency higher or lower than the valency of the pure semiconductor is added, so as to increase the electrical conductivity of the semiconductor.

Depending upon the type of impurity atoms added, an extrinsic semiconductor can be classified as N-type or P-type.

(a) N-type semiconductor

When a small amount of pentavalent impurity such as arsenic is added to a pure germanium semiconductor crystal, the resulting crystal is called N-type semiconductor.

Fig 9.6a shows the crystal structure obtained when pentavalent arsenic impurity is added with pure germanium crystal. The four valence electrons of arsenic atom form covalent bonds with electrons of neighbouring four germanium atoms. The fifth electron of arsenic atom is loosely bound. This electron can move about almost as freely as an electron in a conductor and hence it will be the carrier of current. In the energy band picture, the energy state corresponding to the fifth valence electron is in the forbidden gap and lies slightly below the conduction band (Fig 9.6b). This level is known as the donor level.

When the fifth valence electron is transferred to the conduction band, the arsenic atom becomes positively charged immobile ion. Each impurity atom donates one free electron to the semiconductor. These impurity atoms are called donors.

In N-type semiconductor material, the number of electrons increases, compared to the available number of charge carriers in the intrinsic semiconductor. This is because, the available larger number of

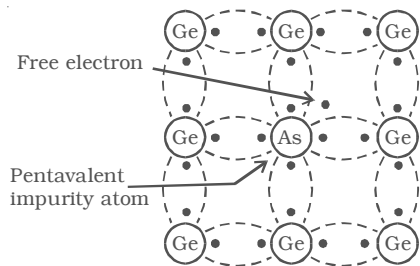


Fig 9.6a N-type semiconductor

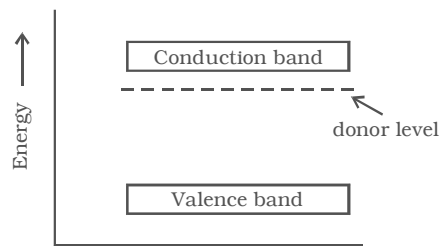


Fig 9.6b Energy band diagram of N-type semiconductor

electrons increases the rate of recombination of electrons with holes. Hence, in N-type semiconductor, free electrons are the majority charge carriers and holes are the minority charge carriers.

(b) P-type semiconductor

When a small amount of trivalent impurity (such as indium, boron or gallium) is added to a pure semiconductor crystal, the resulting semiconductor crystal is called P-type semiconductor.

Fig 9.7a shows the crystal structure obtained, when trivalent boron impurity is added with pure germanium crystal. The three valence electrons of the boron atom form covalent bonds with valence electrons of three neighbourhood germanium atoms. In the fourth covalent bond, only one valence electron is available from germanium atom and there is deficiency of one electron which is called as a hole. Hence for each boron atom added, one hole is created. Since the holes can accept electrons from neighbourhood, the impurity is called acceptor. The hole, may be filled by the electron from a neighbouring atom, creating a hole in that position from where the electron moves. This process continues and the hole moves about in a random manner due to thermal effects. Since the hole is associated with a positive charge moving from one position to another, this is called as P-type semiconductor. In the P-type semiconductor, the acceptor impurity produces an energy level just above the valence band. (Fig 9.7b). Since, the energy difference between acceptor energy level and the valence band is much smaller, electrons from the valence band can easily jump into the acceptor level by thermal agitation.

In P-type semiconductors, holes are the majority charge carriers and free electrons are the minority charge carriers.

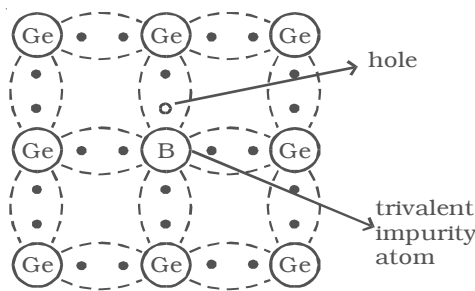


Fig 9.7a P-type semiconductor

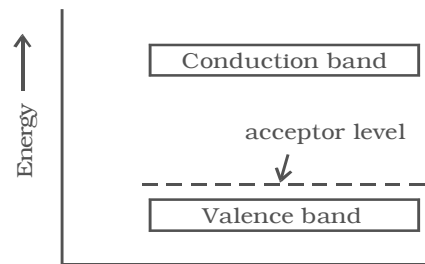


Fig 9.7b Energy band diagram of a P-type semiconductor

9.2 PN Junction diode

If one side of a single crystal of pure semiconductor (Ge or Si) is doped with acceptor impurity atoms and the other side is doped with donor impurity atoms, a PN junction is formed as shown in Fig 9.8. P region has a high concentration of holes and N region contains a large number of electrons.

As soon as the junction is formed, free electrons and holes cross through the junction by the process of diffusion. During this process, the electrons crossing the junction from N-region into the P region, recombine with holes in the P-region very close to the junction. Similarly holes crossing the junction from the P-region into the N-region, recombine

with electrons in the N-region very close to the junction. Thus a region is formed, which does not have any mobile charges very close to the junction. This region is called depletion region. In this region, on the left side of the junction, the acceptor atoms become negative ions and on the right side of the junction, the donor atoms become positive ions (Fig 9.8).

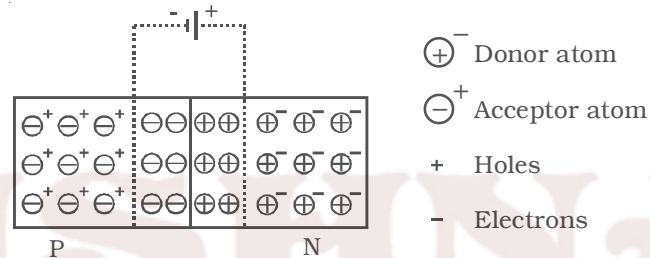


Fig 9.8 P N Junction diode

An electric field is set up, between the donor and acceptor ions in the depletion region. The potential at the N-side is higher than the potential at P-side. Therefore electrons in the N-side are prevented to go to the lower potential of P-side. Similarly, holes in the P-side find themselves at a lower potential and are prevented to cross to the N-side. Thus, there is a barrier at the junction which opposes the movement of the majority charge carriers. The difference of potential from one side of the barrier to the other side is called potential barrier. The potential barrier is approximately 0.7V for a silicon PN junction and 0.3V for a germanium PN junction. The distance from one side of the barrier to the other side is called the width of the barrier, which depends upon the nature of the material.

9.2.1 Forward biased PN junction diode

When the positive terminal of the battery is connected to P-side and negative terminal to the N-side, so that the potential difference acts in opposite direction to the barrier potential, then the PN junction diode is said to be forward biased.

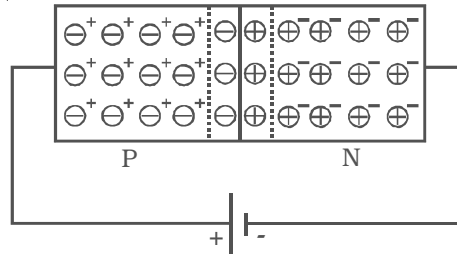


Fig 9.9 Forward biased PN junction diode

When the PN junction is forward biased (Fig 9.9), the applied positive potential repels the holes in the P-region, and the applied negative potential repels the electrons in the N-region, so the charges move towards the junction. If the applied potential difference is more than the potential barrier, some holes and free electrons enter the depletion region.

Hence, the potential barrier as well as the width of the depletion region are reduced. The positive donor ions and negative acceptor ions within the depletion region regain electrons and holes respectively. As a result of this, the depletion region disappears and the potential barrier also disappears. Hence, under the action of the forward potential difference, the majority charge carriers flow across the junction in opposite direction and constitute current flow in the forward direction.

9.2.2 Reverse biased PN junction diode

When the positive terminal of the battery is connected to the N-side and negative terminal to the P-side, so that the applied potential difference is in the same direction as that of barrier potential, the junction is said to be reverse biased.

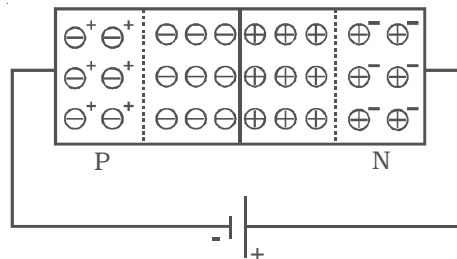


Fig 9.10 Reverse biased PN junction diode.

When the PN junction is reverse biased (Fig 9.10), electrons in the N region and holes in the P-region are attracted away from the junction.

Because of this, the number of negative ions in the P-region and positive ions in the N-region increases. Hence the depletion region becomes wider and the potential barrier is increased.

Since the depletion region does not contain majority charge carriers, it acts like an insulator. Therefore, no current should flow in the external circuit. But, in practice, a very small current of the order of few microamperes flows in the reverse direction. This is due to the minority carriers flowing in the opposite direction. This reverse current is small, because the number of minority carriers in both regions is very small. Since the major source of minority carriers is, thermally broken covalent bonds, the reverse current mainly depends on the junction temperature.

9.2.3 Symbol for a semiconductor diode

The diode symbol is shown in Fig 9.11. The P-type and N-type regions are referred to as P-end and N-end respectively. The arrow on the diode points the direction of conventional current.

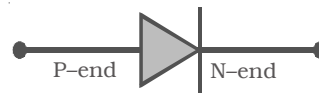
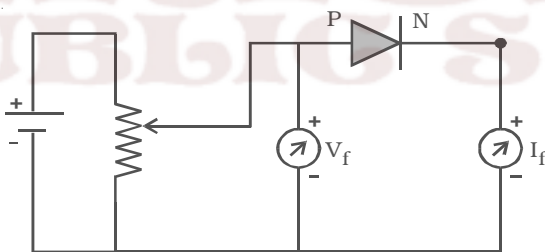
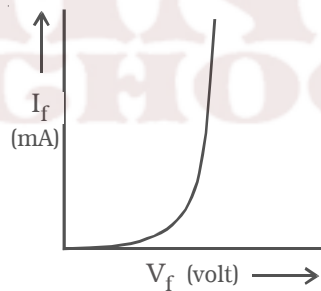


Fig 9.11 Circuit symbol for a semiconductor diode

9.2.4 Forward bias characteristics



(a) Diode circuit-Forward bias



(b) Forward characteristics

Fig 9.12 Forward bias characteristics of a diode

The circuit for the study of forward bias characteristics of PN junction diode is shown in Fig 9.12a. The voltage between P-end and N-end is increased from zero in suitable equal steps and the corresponding currents are noted down. Fig 9.12b shows the forward bias characteristic curve of the diode. Voltage is the independent variable. Therefore, it is plotted along X-axis. Since, current is the dependent variable, it is plotted against Y-axis. From the

characteristic curve, the following conclusions can be made. (i) The forward characteristic is not a straight line. Hence the ratio V/I is not a constant (i.e) the diode does not obey Ohm's law. This implies that the semiconductor diode is a non-linear conductor of electricity. (ii) It can be seen from the characteristic curve that initially, the current is very small. This is because, the diode will start conducting, only when the external voltage overcomes the barrier potential (0.7V for silicon diode). As the voltage is increased to 0.7 V, large number of free electrons and holes start crossing the junction. Above 0.7V, the current increases rapidly. The voltage at which the current starts to increase rapidly is known as cut-in voltage or knee voltage of the diode.

9.2.5 Reverse bias characteristics

The circuit for the study of reverse bias characteristics of PN junction diode is shown in Fig 9.13a. The voltage is increased from zero in suitable steps. For each voltage, the corresponding current readings are noted down. Fig 9.13b shows the reverse bias characteristic curve of the diode. From the characteristic curve, it can be concluded that, as voltage is increased from zero, reverse current (in the order of microamperes) increases and reaches the maximum value at a small value of the reverse voltage. When the voltage is further increased, the current is almost independent of the reverse voltage upto a certain critical value. This reverse current is known as the reverse saturation current or leakage current. This current is due to the minority charge carriers, which depends on junction temperature.

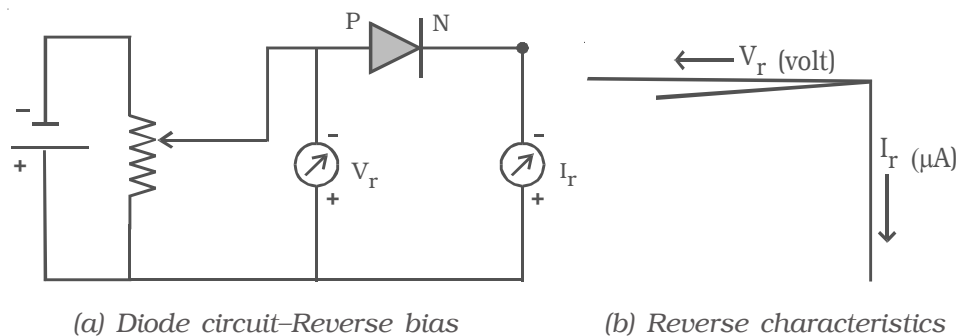


Fig 9.13 Reverse bias characteristics of a diode

9.3 PN junction diode as rectifier

The process in which alternating voltage or alternating current is converted into direct voltage or direct current is known as rectification. The device used for this process is called as rectifier. The junction diode has the property of offering low resistance and allowing current to flow through it, in the forward biased condition. This property is used in the process of rectification.

9.3.1 Half wave rectifier

A circuit which rectifies half of the a.c wave is called half wave rectifier.

Fig 9.14 shows the circuit for half wave rectification. The a.c. voltage (V_s) to be rectified is

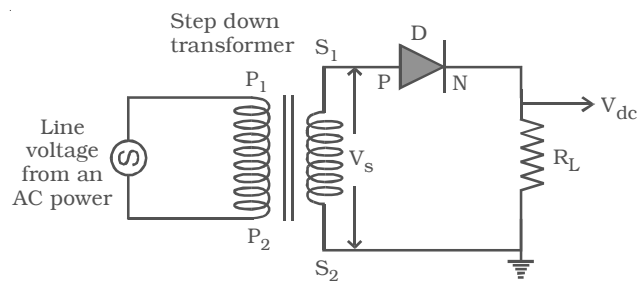


Fig 9.14 Half wave rectifier

obtained across the secondary ends S_1 S_2 of the transformer. The P-end of the diode D is connected to S_1 of the secondary coil of the transformer. The N-end of the diode is connected to the other end S_2 of the secondary coil of the transformer, through a load resistance R_L . The rectified output voltage V_{dc} appears across the load resistance R_L .

During the positive half cycle of the input a.c. voltage V_s , S_1 will be positive and the diode is forward biased and hence it conducts.

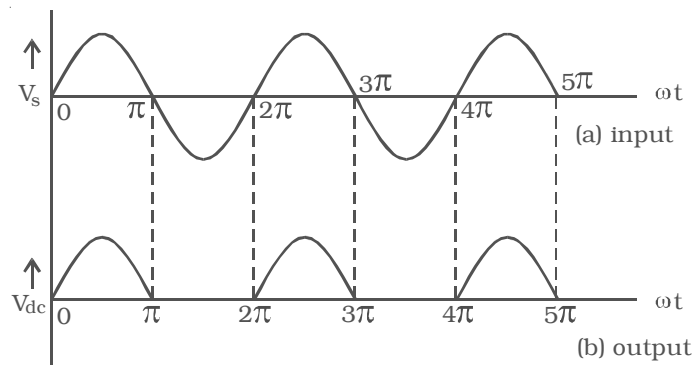


Fig 9.15 Half wave rectifier signals

Therefore, current flows through the circuit and there is a voltage drop across R_L . This gives the output voltage as shown in Fig 9.15.

During the negative half cycle of the input a.c. voltage (V_s), S_1 will be negative and the diode D is reverse biased. Hence the diode does not conduct. No current flows through the circuit and the voltage drop across R_L will be zero. Hence no output voltage is obtained. Thus corresponding to an alternating input signal, unidirectional pulsating output is obtained.

The ratio of d.c. power output to the a.c. power input is known as rectifier efficiency. The efficiency of half wave rectifier is approximately 40.6%

9.3.2 Bridge rectifier

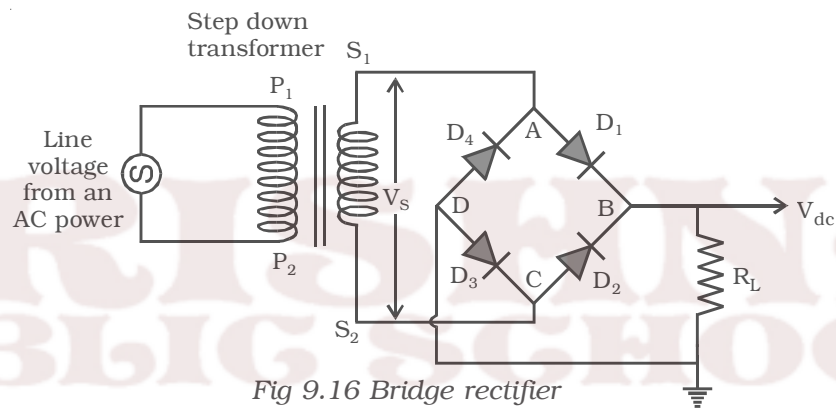


Fig 9.16 Bridge rectifier

A bridge rectifier is shown in Fig 9.16. There are four diodes D_1 , D_2 , D_3 and D_4 used in the circuit, which are connected to form a network. The input ends A and C of the network are connected to the secondary ends S_1 and S_2 of the transformer. The output ends B and D are connected to the load resistance R_L .

During positive input half cycle of the a.c. voltage, the point A is positive with respect to C. The diodes D_1 and D_3 are forward biased and conduct, whereas the diodes D_2 and D_4 are reverse biased and do not conduct. Hence current flows along S_1ABDCS_2 through R_L . During negative half cycle, the point C is positive with respect to A. The diodes D_2 and D_4 are forward biased and conduct, whereas the diodes D_1 and D_3 are reverse biased and they do not conduct. Hence current flows along S_2CBDAS_1 through R_L . The same process is repeated for subsequent half cycles. It can be seen that, current flows through R_L

in the same direction, during both half cycles of the input a.c. signals. The output signal corresponding to the input signal is shown in Fig 9.17. The efficiency of the bridge rectifier is approximately 81.2%.

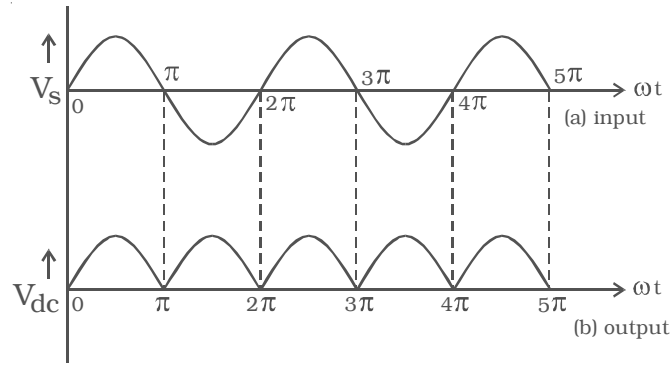


Fig 9.17 Full wave bridge rectifier signals

9.3.3 Filter circuits and regulation property of the power supply

Both in half wave and full wave rectifiers, it is observed that the output voltage across R_L varies from zero to a maximum value. Eventhough, unidirectional current through R_L is obtained, the output voltage fluctuates. This fluctuation in output voltage is not desirable, when pure d.c. voltage is required. Hence they must be removed or smoothened. This can be achieved with the help of suitable networks called filters such as capacitor filter, inductor filter etc., and we can get almost a steady d.c. voltage. But this steady d.c. output voltage from a rectifier is not constant due to the following reasons.

(i) As the load varies, the d.c. output voltage is not constant. That is, as the current drawn from the rectifier increases, the output voltage decreases and vice versa. The variation of d.c. output voltage as a function of d.c. load current is called regulation.

$$\text{The percentage of regulation} = \frac{V_{no\ load} - V_{load}}{V_{load}} \times 100$$

(ii) The d.c. output voltage varies directly as the a.c. input voltage to the rectifier. The line voltage from a.c. power (220 V) may not be a constant and may vary from 200 V to 240 V. Hence the d.c. output voltage will also vary. To overcome these difficulties, Zener diodes are used as regulators and are used along with rectifier and filter circuits. They are called 'regulated power supplies'.

9.4 Breakdown mechanisms

There are two mechanisms which give rise to the breakdown of a PN junction under reverse bias condition. They are (i) avalanche breakdown and (ii) zener breakdown.

(i) Avalanche breakdown : When both sides of the PN junction are lightly doped and the depletion layer becomes large, avalanche breakdown takes place. In this case, the electric field across the depletion layer is not so strong. The minority carriers accelerated by the field, collide with the semiconductor atoms in the crystal. Because of this collision with valence electrons, covalent bonds are broken and electron hole pairs are generated. These charge carriers, so produced acquire energy from the applied potential and in turn produce more and more carriers. This cumulative process is called avalanche multiplication and the breakdown is called avalanche breakdown.

(ii) Zener breakdown : When both sides of the PN junction are heavily doped, consequently the depletion layer is narrow. Zener breakdown takes place in such a thin narrow junction. When a small reverse bias is applied, a very strong electric field is produced across the thin depletion layer. This field breaks the covalent bonds, extremely large number of electrons and holes are produced, which give rise to the reverse saturation current (Zener current). Zener current is independent of applied voltage.

9.5 Zener diode

Zener diode is a reverse biased heavily doped semiconductor (silicon or germanium) PN junction diode, which is operated exclusively in the breakdown region.

The symbol of a Zener diode is shown in Fig 9.18. For normal operation of a Zener diode, in breakdown region, the current through the diode should be limited by an external circuit. Hence the power dissipated across the junction is within its power-handling capacity. Unless this precaution is observed, a large current will destroy the diode.

The V-I characteristic curve for the Zener diode is shown in Fig 9.19. It can be seen from the figure, that, as the reverse voltage applied to the PN junction is increased, at a particular voltage, the

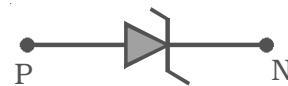


Fig 9.18 Symbol for Zener diode

current increases enormously from its normal cut off value. This voltage is called zener voltage or breakdown voltage (V_z).

9.6 Zener diode as voltage regulator

To maintain a constant voltage across the load, even if the input voltage or load current varies, voltage regulation is to be made. A Zener diode working in the breakdown region can act as voltage regulator.

The circuit in which a Zener diode is used for maintaining a constant voltage across the load R_L is shown in Fig 9.20. The Zener diode in reverse biased condition is connected in parallel with the load R_L . Let V_{dc} be the unregulated dc voltage and V_z be Zener voltage (regulated output voltage). R_s is the current limiting resistor. It is chosen in such a way that the diode operates in the breakdown region.

In spite of changes in the load current or in the input voltage, the Zener diode maintains a constant voltage across the load. The action of the circuit can be explained as given below.

(i) load current varies, input voltage is constant : Let us consider that the load current increases. Zener current hence decreases, and the current through the resistance R_s is a constant. The output voltage is $V_z = V_{dc} - IR_s$, since the total current I remains constant, output voltage remains constant.

(ii) input voltage varies : Let us consider that the input voltage V_{dc} increases. Now the current through Zener increases and voltage drop across R_s increases in such a way that the load voltage remains the same. Thus the Zener diode acts as a voltage regulator.

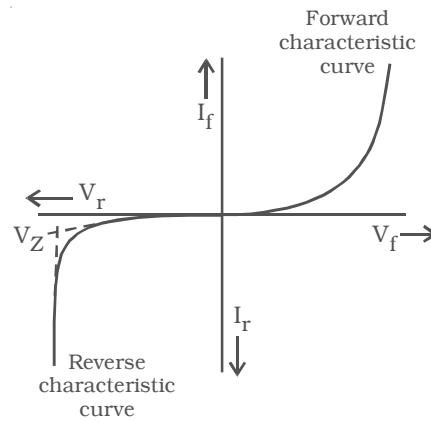


Fig 9.19 V - I characteristics of a Zener diode.

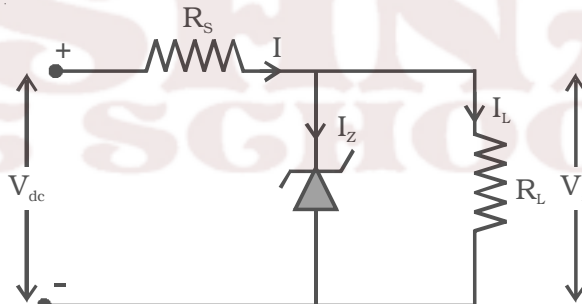


Fig 9.20 Zener diode as a voltage regulator

9.7 Light Emitting Diode (LED)

A light emitting diode (LED) is a forward biased PN junction diode, which emits visible light when energized.

When a junction diode is forward biased, electrons from N-side and holes from P-side move towards the depletion region and recombination takes place. When an electron in the conduction band recombines with a hole in the valence band, energy is released. In the case of semiconducting materials like gallium arsenide (GaAs), gallium phosphide (GaP) and gallium - arsenide phosphide (GaAsP), a greater percentage of energy is given out in the form of light. If the semiconductor material is translucent, light is emitted and the junction becomes a light source (turned ON). The LED is turned ON, when it is forward biased and it is turned OFF, when it is reverse biased. The colour of the emitted light will depend upon the type of the material used. By using gallium arsenide phosphide and gallium phosphide, a manufacturer can produce LEDs that radiate red, green, yellow and orange. Fig 9.21 shows the symbol of LED. LEDs are used for instrument displays, calculators and digital watches.

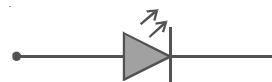


Fig 9.21 Symbol of LED

9.8 Junction transistor

A junction transistor is a solid state device. It consists of silicon or germanium crystal containing two PN junctions. The two PN junctions are formed between the three layers. These are called base, emitter and collector.

(i) Base (B) layer : It is a very thin layer, the thickness is about 25 microns. It is the central region of the transistor.

(ii) Emitter (E) and Collector (C) layers : The two layers on the opposite sides of B layer are emitter and collector layers. They are of the same type of the semiconductor.

An ohmic contact is made to each of these layers. The junction between emitter and base is called emitter junction. The junction between collector and base is called collector junction.

In a transistor, the emitter region is heavily doped, since emitter has to supply majority carriers. The base is lightly doped. The collector region is lightly doped. Since it has to accept majority charge

carriers, it is physically larger in size. Hence, emitter and collector cannot be interchanged.

The construction of PNP and NPN transistors are shown in Fig 9.22a and Fig 9.22b respectively.

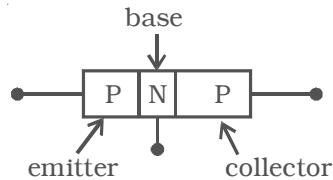


Fig 9.22a Construction of PNP transistor

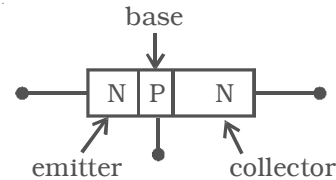


Fig 9.22b Construction of NPN transistor

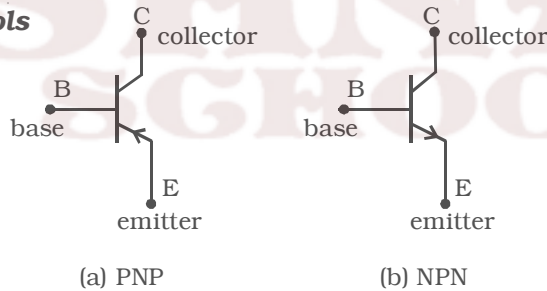
For a transistor to work, the biasing to be given are as follows :

(i) The emitter-base junction is forward biased, so that majority charge carriers are repelled from the emitter and the junction offers very low resistance to the current.

(ii) The collector-base junction is reverse biased, so that it attracts majority charge carriers and this junction offers a high resistance to the current.

9.9 Transistor circuit symbols

The circuit symbols for a PNP and NPN transistors are shown in Fig 9.23. The arrow on the emitter lead pointing towards the base represents a PNP transistor. When the emitter-base junction of a PNP transistor



(a) PNP (b) NPN

Fig 9.23 Symbol for transistors.

is forward biased, the direction of the conventional current flow is from emitter to base. NPN transistor is represented by arrow on the emitter lead pointing away from the base. When the emitter base junction of a NPN transistor is forward biased, the direction of the conventional current is from base to emitter.

9.9.1 Working of a PNP transistor

A PNP transistor is like two PN junction diodes, which are placed back-to-back. At each junction, there is a depletion region which gives rise to a potential barrier. The external biasing of the junction is

provided by the batteries V_{EE} and V_{CC} as shown in Fig. 9.24. The emitter base junction is forward biased and the collector base junction is reverse biased.

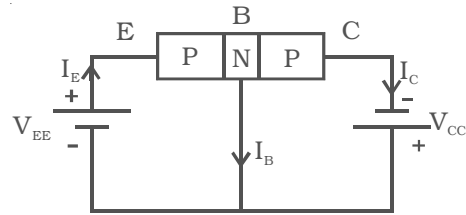


Fig 9.24 PNP Transistor Action

Since the emitter-base junction is forward biased, a large number of holes cross the junction and enters the base. At the same time, very few electrons flow from the base to the emitter. These electrons, when they reach emitter, recombine with an equal number of holes in the emitter. The loss of total number of holes in the emitter is made by flow of an equal number of electrons from the emitter to the positive terminal of the battery. The flow of holes from the emitter to base gives rise to emitter current I_E . In the emitter, I_E is due to the flow of holes. But in the external circuit the current is due to the flow of electrons from the emitter to the positive terminal of the battery V_{EE} . The holes diffuse through the base. These holes take a very small time to flow through this region before they reach the depletion region. During this time, a very small number of holes recombine with an equal number of electrons in the base. Because the base is lightly doped and very thin, this number is very small. The loss of total number of electrons per second is made up by the flow of an equal number of electrons from the negative terminal of V_{EE} into the base. The flow of these electrons contribute the base current I_B .

The remaining numbers of holes, which do not undergo recombination process in the base, reach the collector. These are neutralised by an equal number of electrons flowing from the negative terminal of the battery V_{CC} into the collector. At the same time, an equal number of electrons flows from the negative terminal of V_{EE} and reach the positive terminal of V_{CC} . The flow of holes per second from the base to the collector gives rise to the collector current I_c from the base to the collector. In the external circuit, it is due to the flow of electrons from the negative terminal of the battery V_{CC} into the collector.

Applying Kirchoff's current law to the circuit, the emitter current is the sum of collector current and base current.

$$\text{i.e } I_E = I_B + I_C$$

This equation is the fundamental relation between the currents in a transistor circuit.

This equation is true regardless of transistor type or transistor configuration.

The action of NPN transistor (Fig 9.25) is similar to that of PNP transistor.

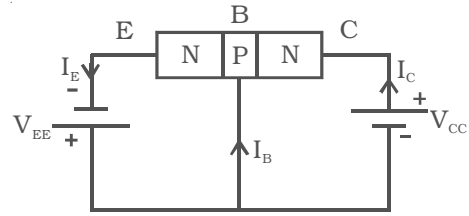


Fig 9.25 NPN transistor action

9.9.2 Transistor circuit configurations

There are three types of circuit connections (called configurations or modes) for operating a transistor. They are (i) common base (CB) mode (ii) common emitter (CE) mode and (iii) common collector (CC) mode.

The term common is used to denote the lead that is common to the input and output circuits. The different modes are shown in Fig 9.26 for NPN transistor.

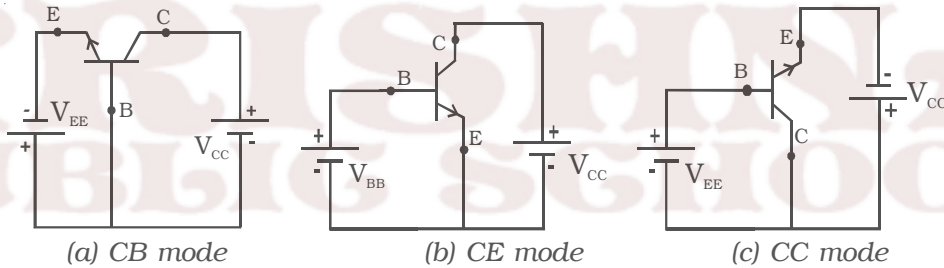


Fig 9.26 Three modes of transistor circuit

In a similar way, three configurations can be drawn for PNP transistor.

9.9.3 Current amplification factors α and β and the relation between them

The current amplification factor or current gain of a transistor is the ratio of output current to the input current. If the transistor is connected in common base mode, the current gain $\alpha = \frac{I_C}{I_E}$ and if the transistor is connected in common emitter mode, the current gain $\beta = \frac{I_C}{I_B}$. Fig 9.27 shows a NPN transistor connected in the common base and common emitter configurations. Since, 95% of the

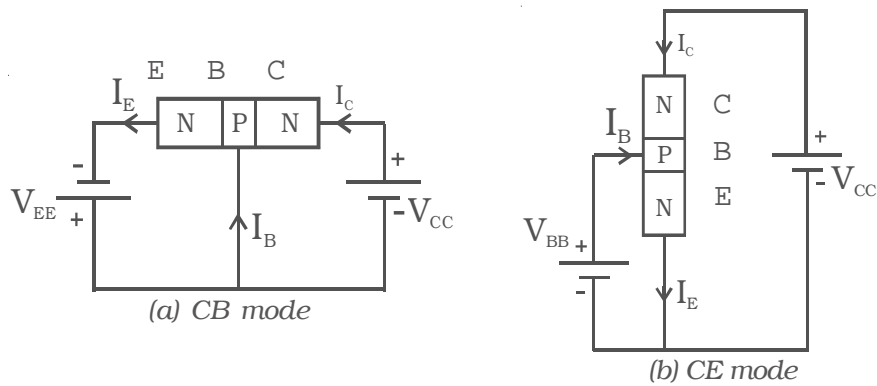


Fig 9.27 CB and CE modes of an NPN transistor

injected electrons reach the collector, the collector current is almost equal to the emitter current. Almost all transistors have α , in the range 0.95 to 0.99.

We know that

$$\alpha = \frac{I_C}{I_E} = \frac{I_C}{I_B + I_C} \quad (\because I_E = I_B + I_C)$$

$$\frac{1}{\alpha} = \frac{I_B + I_C}{I_C} = \frac{I_B}{I_C} + 1$$

$$\frac{1}{\alpha} - 1 = \frac{1}{\beta}$$

$$\therefore \beta = \frac{\alpha}{1 - \alpha}$$

Usually β lies between 50 and 300. Some transistors have β as high as 1000.

9.9.4 Characteristics of an NPN transistor in common emitter configuration

The three important characteristics of a transistor in any mode are (i) input characteristics (ii) output characteristics and (iii) transfer characteristics.

The circuit to study the characteristic curves of NPN transistor in common emitter mode is as shown in Fig 9.28.

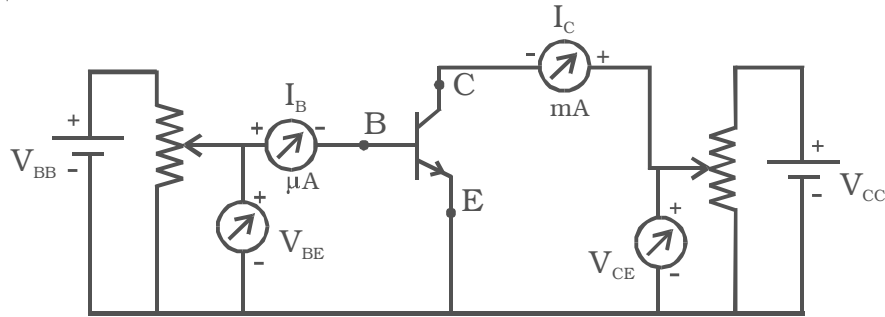


Fig 9.28 Transistor circuit in CE mode.

(i) Input characteristics

Input characteristic curve is drawn between the base current (I_B) and voltage between base and emitter (V_{BE}), when the voltage between collector and emitter (V_{CE}) is kept constant at a particular value. V_{BE} is increased in suitable equal steps and corresponding base current is noted. The procedure is repeated for different values of V_{CE} . I_B values are plotted against V_{BE} for constant V_{CE} . The input characteristic thus obtained is shown in Fig 9.29.

The input impedance of the transistor is defined as the ratio of small change in base - emitter voltage to the corresponding change in base current at a given V_{CE} .

$$\therefore \text{Input impedance, } r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE}}$$

The input impedance of the transistor in CE mode is very high.

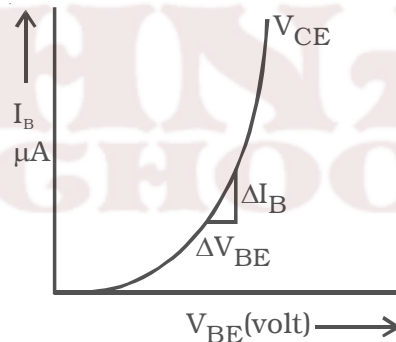


Fig 9.29 Input characteristics

(ii) Output characteristics

Output characteristic curves are drawn between I_C and V_{CE} , when I_B is kept constant at a particular value.

The base current I_B is kept at a constant value, by adjusting the base emitter voltage V_{BE} . V_{CE} is increased in suitable equal steps and the corresponding collector current is noted. The procedure is repeated for different values of I_B . Now, I_C versus V_{CE} curves are drawn for different values of I_B . The output characteristics thus obtained are

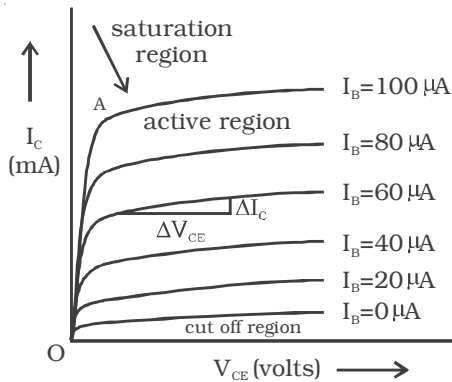


Fig 9.30 Output characteristics

represented in Fig 9.30. The three regions of the characteristics can be discussed as follows :

Saturation region : The initial part of the curve (ohmic region, OA) is called saturation region. (i.e) The region in between the origin and knee point. (Knee point is the point, where I_C is about to become a constant).

Cut off region : There is very small collector current in the transistor, even when the base current is zero ($I_B = 0$). In the output characteristics, the region below the curve for $I_B = 0$ is called cut off region. Below the cut off region, the transistor does not function.

Active region : The central region of the curves is called active region. In the active region, the curves are uniform. In this region, E-B junction is forward biased and C-B junction is reverse biased.

The output impedance r_o is defined as the ratio of variation in the collector emitter voltage to the corresponding variation in the collector current at a constant base current in the active region of the transistor characteristic curves.

$$\therefore \text{output impedance, } r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C} \right)_{I_B}$$

The output impedance of a transistor in CE mode is low.

(iii) Transfer characteristics

The transfer characteristic curve is drawn between I_C and I_B , when V_{CE} is kept constant at a particular value. The base current I_B is increased in suitable steps and the collector current I_C is noted down for each value of I_B . The transfer characteristic curve is shown in Fig 9.31.

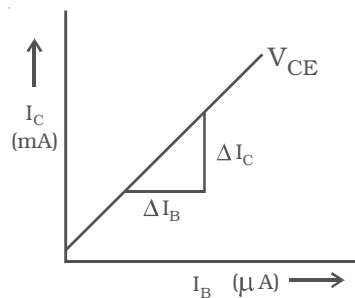


Fig 9.31 Transfer characteristic curve

The current gain is defined as the ratio of a small change in the collector current to the corresponding change in the base current at a constant V_{CE} .

$$\therefore \text{current gain, } \beta = \left(\frac{\Delta I_C}{\Delta I_B} \right)_{V_{CE}}$$

The common emitter configuration has high input impedance, low output impedance and higher current gain when compared with common base configuration.

9.10 Transistor as a switch

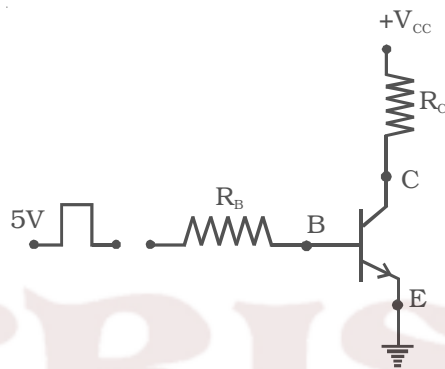


Fig 9.32 NPN Transistor as a switch.

Transistors are widely used in switching operations. In the Fig 9.32, NPN transistor is connected in common emitter configuration and a resistor R_B is connected in series with the base. The load resistance R_C is connected in series with the collector. A pulse type waveform is applied as the input to the transistor through R_B . When the input is high, base emitter junction is forward biased and current flows through R_B into the base. The values of R_B and R_C are chosen in such a manner that the base current flowing, is enough to saturate the transistor. When the transistor is saturated, it is said to be ON (maximum current). When the input is low (i.e.) at 0 V, the base emitter junction is not forward biased. So, no base current flows. Hence the transistor is said to be OFF.

9.11 Transistor amplifier

The important function of a transistor is the amplification. An amplifier is a circuit capable of magnifying the amplitude of weak signals. The important parameters of an amplifier are input impedance, output impedance, current gain and voltage gain. A good design of an amplifier circuit must possess high input impedance, low output impedance and high current gain.

9.11.1 Operating point

For the given values of the load resistance R_C and supply voltage V_{CC} , two points A (V_{CC} , 0) and B $\left(0, \frac{V_{CC}}{R_C}\right)$ are located on the axes of V_{CE}

and I_C respectively, of the output characteristics of the transistor (Fig 9.33). Joining A and B, load line AB is obtained. The point of intersection Q of this line in the active region of the output characteristics with a suitable value of the base current I_B , such that the output voltage is symmetrical is called operating point or quiescent point for the amplifier. $I_{B(Q)}$ is the input base current at the operating point. $V_{CE(Q)}$ and $I_{C(Q)}$ are the collector to emitter voltage and the collector current respectively at the operating point.

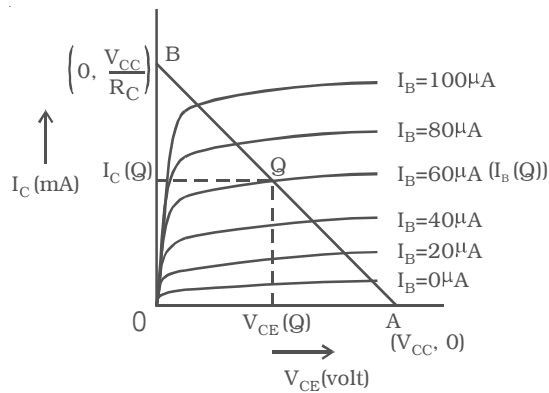


Fig 9.33 load line and operating point

9.11.2 Working

A basic circuit of an amplifier in common emitter mode with NPN transistor is shown in Fig 9.34. The emitter-base junction is forward biased by a supply voltage V_{BB} . The input ac signal to be amplified is applied between base and emitter of the transistor. R_C is the load resistance.

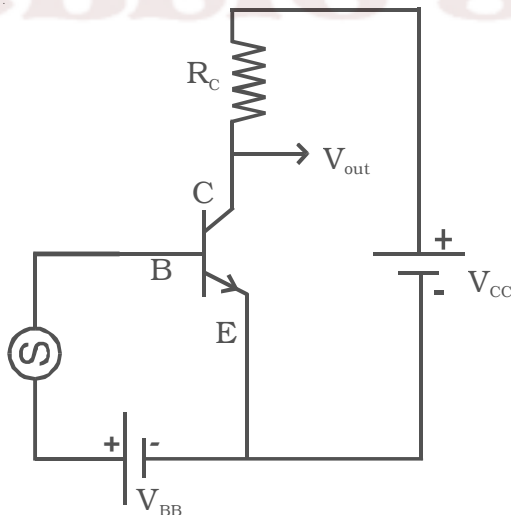


Fig 9.34 Transistor amplifier

The amplifying action of a transistor can be explained as follows. When the a.c. signal is not applied, the base current is available in small quantity in microamperes, which is represented by OP and the corresponding collector current in milliamperes is represented by PQ (Q is the operating point). When the ac signal voltage is applied, the potential difference between the base and emitter changes continuously. This results in

increase of base current (I_B) from OP to OA, then decrease of base current from OA to OP (during positive half cycles of the input a.c. voltage) and then to PB and once again increase from OB to OP (during negative half cycle of the input a.c. voltage) for each cycle of the input signal voltage. This variation in base current is reflected in the collector current as shown in Fig 9.35. The collector current (I_C) increases from PQ to AA₁, falls from AA₁ to BB₁ and again increases from BB₁ to PQ. Thus a variation in the base current in microamperes produces a corresponding variation in the collector current in milliamperes. This produces a corresponding potential difference across R_C . The increase of potential difference across R_C makes a decrease in the output voltage.

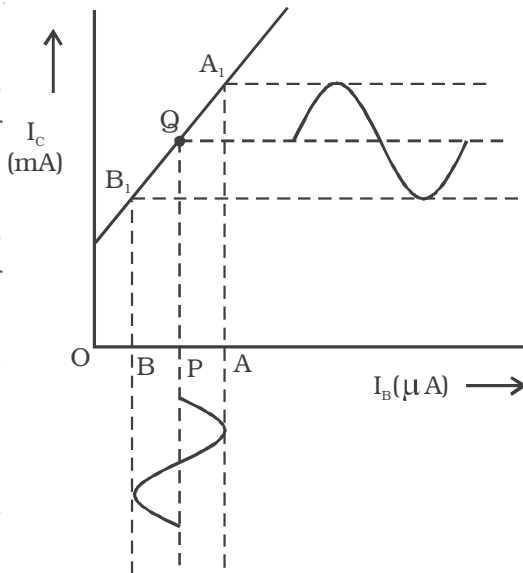


Fig 9.35 Variation in output current (I_C) with variation in input current (I_B)

Therefore, there is always a phase reversal of 180° between the input and output voltages in CE amplifier.

9.12 Transistor biasing

In order to amplify the input signal using a transistor, the signal is to be applied at an operating point called Q point in the active region. Once the operating point is established, its position should not change. If the Q point shifts near the saturation line or near cut off region of the output characteristics, the signal will be distorted after amplification.

The proper selection of operating point of a transistor and maintenance of proper emitter voltage during the passage of the signal is known as transistor biasing.

The most commonly used methods of obtaining transistor biasing are (i) base bias, (ii) base bias with emitter feedback, (iii) base bias

with collector feedback and (iv) voltage divider bias.

The principle involved in all these types is to obtain the required base current corresponding to the operating point under zero signal conditions.

In all the bias circuits except voltage divider bias, the collector current depends on the current gain (β) of the transistor. But β of a transistor is very sensitive to temperature changes. For this reason, it is desirable to have a bias circuit whose action is independent of β . The requirement is met by the voltage divider bias circuit.

9.12.1 Voltage divider bias

This is the most widely used method of providing bias and stabilization to a transistor. In this method, two resistances R_1 and R_2 are connected across the supply voltage V_{CC} (Fig 9.36) and provide biasing. The emitter resistance R_E provides stabilization. The voltage drop across R_2 forward biases the base emitter junction. This causes the base current and hence collector current to flow in zero signal conditions.

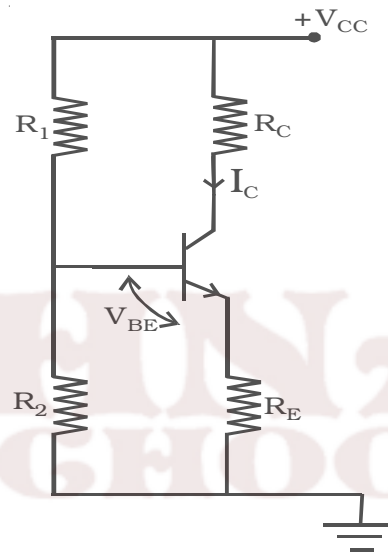


Fig 9.36 Voltage divider bias

The stabilization provided by R_E can be explained as follows. Since β is very sensitive to temperature changes, the collector current I_C increases with rise in temperature. Consequently, it can be seen that I_E increases. This will cause the voltage drop across emitter resistance R_E to increase. The voltage drop across $R_2 = V_{BE} + V_{R_E}$. As voltage drop across R_2 is independent of I_C , V_{BE} decreases. This decreases I_B and the reduced value of I_B tends to bring back I_C to the original value. Hence any variation of β will have no effect on the operating point.

9.13 Single stage CE amplifier

Fig 9.37 shows a single stage CE amplifier. The different circuit elements and their functions are described as follows.

(i) Biasing circuit : The resistances R_1 , R_2 and R_E form the biasing

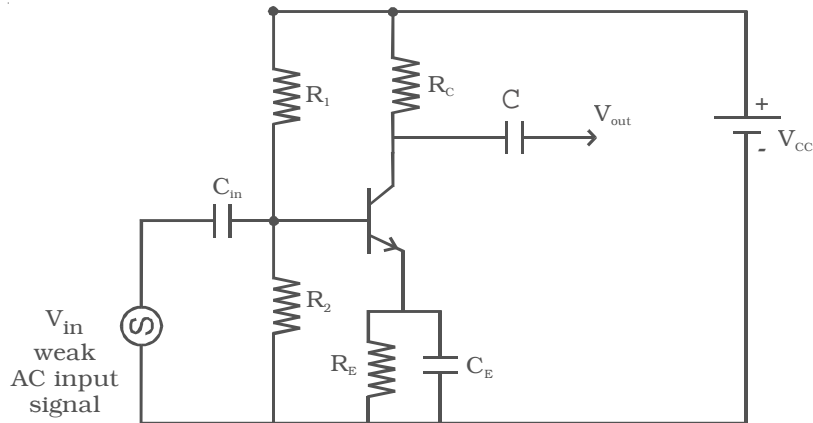


Fig 9.37 Single stage CE amplifier

and stabilization circuit.

(ii) Input capacitance C_{in} : This is used to couple the signal to the base of the transistor. If this is not used, the signal source resistance will come across R_2 and thus change the bias. The capacitor C_{in} allows only a.c. signal to flow.

(iii) Emitter bypass capacitor C_E : This is connected in parallel with R_E to provide a low reactance path to the amplified a.c. signal. If it is not used, then amplified a.c. signal flowing through R_E will cause a voltage drop across it, thereby shifting the output voltage.

(iv) Coupling capacitor C : This is used to couple the amplified signal to the output device. This capacitor C allows only a.c. signal to flow.

Working

When a weak input a.c. signal is applied to the base of the transistor, a small base current flows. Due to transistor action, a much larger a.c. current flows through collector load R_C , a large voltage appears across R_C and hence at the output. Therefore, a weak signal applied to the base appears in amplified form in the collector circuit. Voltage gain (A_v) of the amplifier is the ratio of the amplified output voltage to the input voltage.

Frequency response and bandwidth

The voltage gain (A_v) of the amplifier for different input frequencies can be determined. A graph can be drawn by taking

frequency (f) along X-axis and voltage gain (A_v) along Y-axis. The frequency response curve obtained will be of the form as shown in Fig 9.38. It can be seen that the gain decreases at very low and very high frequencies, but it remains constant over a wide range of mid-frequency region.

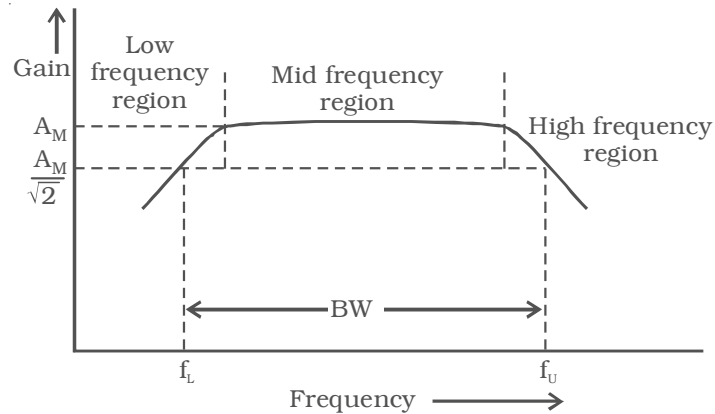


Fig 9.38 Frequency response curve

Lower cut off frequency (f_L) is defined as the frequency in the low frequency range at which the gain of the amplifier is $\frac{1}{\sqrt{2}}$ times the mid frequency gain (A_M). Upper cut off frequency (f_U) is defined as the frequency in the high frequency range at which the gain of the amplifier is $\frac{1}{\sqrt{2}}$ times the mid frequency gain (A_M).

Bandwidth is defined as the frequency interval between lower cut off and upper cut off frequencies.

$$\therefore \text{BW} = f_U - f_L$$

9.14 Multistage amplifiers

The amplification of a signal by a single amplifier may not be enough in most of the practical cases. Hence in these cases, two or more amplifiers are used in series to get sufficient amplified signal. The amplifiers are coupled in such a manner that the output of the first stage becomes the input for the next stage. Connecting the amplifiers in this manner is known as cascading the stages. When a number of amplifiers are connected in cascade, the overall voltage gain is equal to the product of voltage gain of individual stages.

In general the output terminals of one stage cannot be connected directly to the input of the next stage. A suitable coupling network is required to connect two stages. This coupling network should not permit the dc voltage at the output of one stage to pass into the input of the next stage. Otherwise the biasing condition of the second stage will be upset. Further the loss of voltage, when the signal passes from one stage to the next must be minimum. The three coupling devices generally used are

- (i) Resistance - Capacitance (RC) coupling
- (ii) Transformer coupling
- (iii) Direct coupling

RC coupling scheme consist of a series capacitor C and a parallel resistor R. C acts as a short circuit for ac signals while it behaves like an open circuit for dc signals. In transformer coupling scheme, the transformer ensures that the ac signal is transformed from the amplifier to the load, while at the same time, insulating the load from the dc signals. When dc signals have to be amplified, direct coupling is the simplest and the best to use.

9.15 Feedback in amplifiers

The characteristics of an amplifier are highly dependent on transistor parameters like current gain, input impedance and output impedance etc. The transistor parameters exhibit variations due to ageing of transistors. Manufacturing processes cause variations in parameters of transistors of the same type. To overcome any adverse effect on the overall performance of an amplifier, feedback is used. Feedback is said to exist in an amplifier circuit, when a fraction of the output signal is returned or fed back to the input and combined with the input signal. If the magnitude of the input signal is reduced by the feed back, the feed back is called negative or degenerative. If the magnitude of the input signal is increased by the feed back, such feed back is called positive or regenerative.

9.15.1 Principle of feedback amplifier

For an ordinary amplifier i.e. without feedback, let V_0 and V_i be the output voltage and input voltage respectively. If A be the voltage gain of the amplifier, then

$$A = \frac{V_0}{V_i}$$

The gain A is often called as open-loop gain.

The general theory of feedback can be explained with the help of block diagram shown in Fig 9.39. The feedback amplifier has two parts (i.e) amplifier and feedback circuit. The feedback circuit usually consists of passive components (resistor, capacitor, inductor). A fraction (say β) of the output voltage is fed back to the input through the feedback circuit. Let V'_o be the output voltage with feedback.

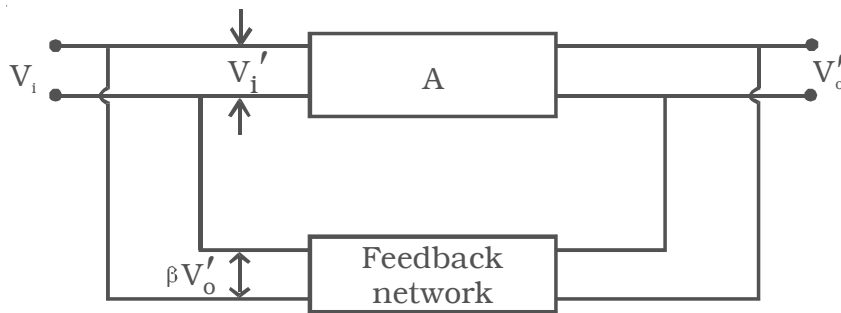


Fig 9.39 Feedback amplifier

Therefore, after feedback the input voltage V'_i becomes,

$$V'_i = V_i \pm \beta V'_o \quad \dots (1)$$

For positive feedback, β is taken as positive. For negative feedback, β is taken as negative.

For positive feedback, the input voltage will be $V_i + \beta V'_o$. When this is amplified A times by the amplifier, the output voltage after feedback (V'_o) will be $A(V_i + \beta V'_o)$

$$\therefore V'_o = A (V_i + \beta V'_o) \quad \dots (2)$$

$$V'_o (1 - \beta A) = AV_i \quad \dots (3)$$

Then the voltage gain of the amplifier with feedback is

$$A_f = \frac{V'_o}{V_i} = \frac{A}{1 - \beta A} \quad \dots (4)$$

Since $|1 - \beta A| < 1$, $A_f > A$. The positive feedback increases the amplifier gain.

For negative feedback, the feedback fraction is $-\beta$

$$\therefore A_f = \frac{A}{1 - (-A\beta)} = \frac{A}{1 + A\beta}$$

Since $|1 + \beta A| > 1$, $A_f < A$. Therefore negative feedback reduces the amplifier gain.

The term $A\beta$ is called loop gain and β is called feedback ratio.

9.15.2 Advantages of negative feedback

Following are the advantages of negative feedback.

- (i) Highly stabilised gain.
- (ii) Reduction in the noise level.
- (iii) Increased bandwidth
- (iv) Increased input impedance and decreased output impedance.
- (v) Less distortion.

9.16 Transistor oscillators

An oscillator may be defined as an electronic circuit which converts energy from a d.c. source into a periodically varying output. Oscillators are classified according to the output voltage, into two types viz. sinusoidal and non-sinusoidal oscillators. If the output voltage is a sine wave function of time, the oscillator is said to be sinusoidal oscillator. If the oscillator generates non-sinusoidal waveform, such as square, rectangular waves, then it is called as non-sinusoidal oscillator (multivibrator). The oscillators can be classified according to the range of frequency as audio-frequency (AF) and radio-frequency (RF) oscillators.

Sinusoidal oscillators may be any one of the following three types:

- (i) LC oscillators
- (ii) RC oscillators
- (iii) Crystal oscillators

9.16.1 Barkhausen condition for oscillation

The gain of the amplifier with positive feedback is given by

$$A_f = \frac{A}{1 - A\beta}, \text{ where } A \text{ is the voltage gain without feedback, } \beta \text{ is}$$

the feedback ratio and $A\beta$ is the loop gain. When $A\beta = 1$, then $A_f \rightarrow \infty$. This means that output voltage is obtained, even if input voltage is zero, (i.e) it becomes an oscillator. The essential condition for the maintenance of oscillation is $A\beta = 1$.

This condition means that (i) the loop gain $A\beta = 1$ and (ii) the net phase shift round the loop is 0° or integral multiples of 2π .

These are called the Barkhausen conditions for oscillations.

9.16.2 Generation of sinusoidal waves by a tuned LC circuit

Sinusoidal oscillators consist of two main sections : a frequency determining device and maintaining device. A resonant LC network can be used as frequency determining device. The frequency maintaining device is a transistor amplifier with its power supply. The amplifier must have sufficient gain to compensate for the attenuation of the frequency determining section and must introduce required phase shift for positive feedback.

If a capacitor of capacitance C and an inductor of inductance L are connected in parallel, then such a circuit represents an oscillatory circuit.

Let us consider a fully charged capacitor C connected with an inductance L as shown in Fig 9.40a. When the charged capacitor is connected to inductance L , the capacitor will discharge, sending current through L and induce magnetic field as shown in Fig 9.40b. Thus the electrostatic energy stored in the capacitor has been converted into electromagnetic energy associated with inductance L .

When the capacitor is completely discharged, the induced magnetic field begins to collapse, sending current in the same direction. The capacitor C is now charged with opposite polarity (Fig 9.40c). In this case, energy associated with magnetic field is converted

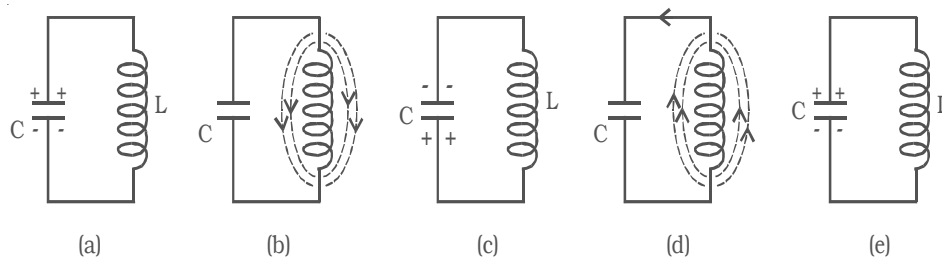


Fig 9.40 Tuned LC circuit

into electrostatic energy. This energy is stored in the capacitor. Once the capacitor is completely charged, it begins to discharge in the reverse direction producing again a magnetic field across L in the opposite direction (Fig 9.40d). Again the magnetic field will collapse and will charge the capacitor. The circuit returns to the original state. (Fig 9.40e). This charging and discharging process results in oscillating current and hence electrical oscillations are set up in the LC circuit. When a LC circuit is used to store energy, it is called tank circuit. The frequency of oscillations is given by,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

If there are no power losses in the LC circuit, then the electrical oscillations will continue for indefinite time. But, in practice, there is some power loss during each cycle of oscillation, as some resistance is always associated with a given LC circuit. Hence the amplitude of oscillations decreases gradually and becomes zero, when all the energy is consumed as losses. Therefore, damped oscillations are produced in the circuit (Fig 9.41a). I_{\max} represents the maximum current flowing through the circuit.

In order to make the oscillations undamped, energy must be supplied to the circuit at the same rate, at which it is dissipated. The energy supplied should be in phase with oscillations set up in LC circuit. The applied energy should have the same frequency as that of

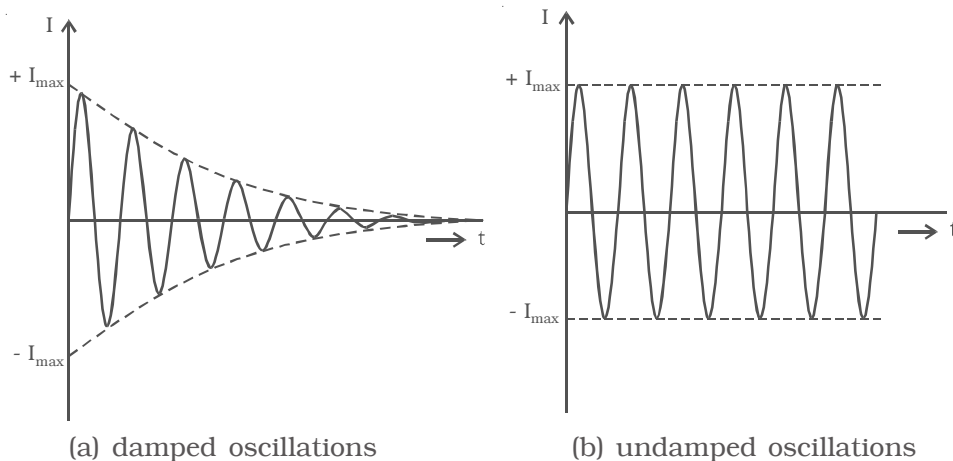


Fig 9.41 Oscillations

oscillations in the tank circuit. If these conditions are fulfilled, the circuit will produce continuous undamped oscillations (Fig 9.41b).

9.16.3 Essentials of LC oscillator

Fig 9.42 shows the block diagram of an oscillator. Its essential components are (i) tank circuit, (ii) amplifier and (iii) feedback circuit.

(i) Tank circuit : It consists of inductance coil (L) connected in parallel with capacitor (C). The frequency of oscillations in the circuit depends upon the values of inductance coil and capacitance of the capacitor.

(ii) Amplifier : The transistor amplifier receives d.c. power from the battery and changes it into a.c. power for supplying to the tank circuit.

(iii) Feedback circuit : It provides positive feedback (i.e.) this circuit transfers a part of output energy to LC circuit in proper phase, to maintain the oscillations.

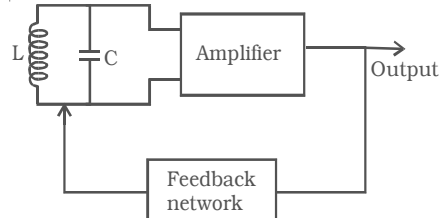


Fig 9.42 Oscillator block diagram

9.16.4 LC oscillators

A transistor can work as an LC oscillator to produce undamped oscillations of any desired frequency, if tank and feedback circuits are properly connected to it. There are different LC oscillators used in electronic circuits, of which, the working principle of Colpitt's oscillator is discussed here.

Colpitt's oscillator

The circuit diagram of Colpitt's oscillator is shown in Fig 9.43. The resistance R_1 , R_2 and R_E provide the sufficient bias for the circuit.

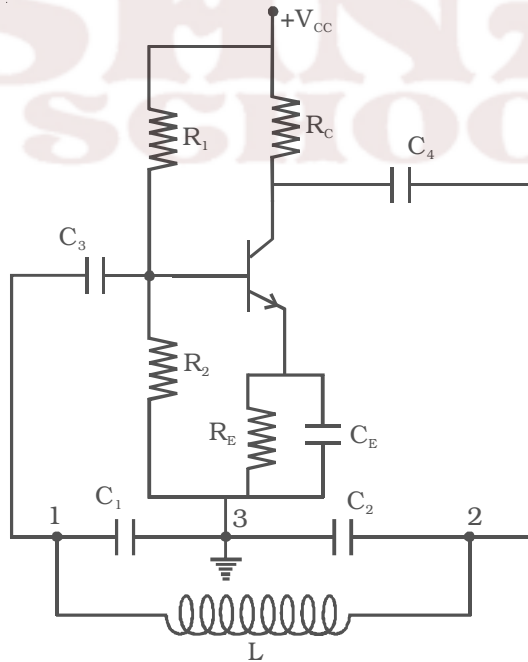


Fig 9.43 Colpitt's oscillator

The frequency determining network is the parallel resonant circuits consisting of capacitors C_1 , C_2 and the inductor L . The junction of C_1 and C_2 is earthed. The function of the capacitor C_4 is to block d.c and provide an a.c. path from the collector to the tank circuit. The voltage developed across C_1 provides the positive feedback for sustained oscillations.

Working

When the collector supply voltage is switched on, a transient current is produced in the tank circuit and damped harmonic oscillations are produced. The oscillations across C_1 are applied to the base emitter junction and appear in the amplified form in the collector circuit. If terminal 1 is at positive potential with respect to terminal 3 at any instant, then terminal 2 will be at negative potential with respect to 3, since 3 is grounded. Hence points 1 and 2 are 180° out of phase. The amplifier produces further phase shift of 180° . Thus the total phase shift is 360° . In other words, energy supplied to the tank circuit is in phase with the oscillations and if $A\beta = 1$, oscillations are sustained in the circuit.

The frequency of oscillations is given by $f = \frac{1}{2\pi\sqrt{LC}}$

$$\text{where } C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\therefore f = \frac{1}{2\pi} \sqrt{\frac{(C_1 + C_2)}{LC_1 C_2}}$$

9.17 Integrated circuit (IC)

An integrated circuit (IC) consists of a single – crystal chip of silicon, containing both active (diodes and transistors) and passive (resistors, capacitors) elements and their interconnections. ICs have the following advantages over the discrete components:

- (i) Extremely small in size
- (ii) Low power consumption
- (iii) Reliability
- (iv) Reduced cost
- (v) Very small weight
- (vi) Easy replacement

ICs offer a wide range of applications and they are broadly classified as digital ICs and linear ICs*. Two distinctly different IC technologies have been employed which are monolithic and hybrid technology.

In monolithic integrated circuits, all circuit components both active and passive elements and their inter connections are made on the top of a single silicon chip. The monolithic circuit is ideal for applications in the situations, where identical currents are received in large quantities. Hence it provides lowest cost per unit and highest order of reliability. In hybrid circuits, separate component parts are attached to a ceramic substrate and the components are interconnected by means of either metallization pattern or wire bonds.

Typical chip sizes range from about 40×40 mils (a mil is 0.001 inch) to about 300×300 mils depending on the complexity of the circuit. Any number of components from very few in number to thousands can be fabricated on a single chip. The integrated circuits are available in Dual-in-line package (DIP).

9.18 Digital electronics

The term digital is derived from the way in which computers perform operations using digits. Initially, applications of digital electronics were confined to computer systems. Nowadays, digital techniques are applied in many areas, such as telephony, radar, medical instruments, navigation and military systems etc. Digital Electronics involves circuits and systems in which there are only two possible states which are represented by voltage levels. Other circuit conditions such as current levels, open or closed switches can also represent the two states.

Analog signal

The signal current or voltage is in the form of continuous, time varying voltage or current (sinusoidal). Such signals are called continuous or analog signals. A typical analog signal is shown in Fig 9.44.

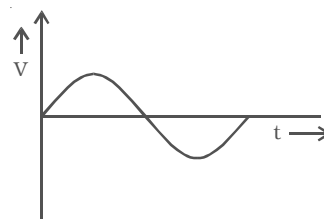


Fig 9.44 Analog signal

Digital ICs : The integrated circuits which process the digital signals are called digital ICs.

Linear ICs : The integrated circuits which process the analog signals are called linear ICs.

Digital signal and logic levels

A digital signal (pulse) is shown in Fig 9.45. It has two discrete levels, 'High' and 'Low'. In most cases, the more positive of the

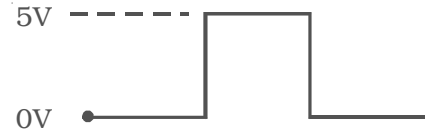


Fig 9.45 Digital Signal

two levels is called HIGH and is also referred to as logic 1. The other level becomes low and also called logic 0. This method of using more positive voltage level as logic 1 is called a positive logic system. A voltage 5V refers to logic 1 and 0 V refers to logic 0. On the other hand, in a negative logic system, the more negative of the two discrete levels is taken as logic 1 and the other level as logic 0. Both positive and negative logic are used in digital systems. But, positive logic is more common of logic gates. Hence we consider only positive logic for studying the operation of logic gates.

9.18.1 Logic gates

Circuits which are used to process digital signals are called logic gates. They are binary in nature. Gate is a digital circuit with one or more inputs but with only one output. The output appears only for certain combination of input logic levels. Logic gates are the basic building blocks from which most of the digital systems are built up. The numbers 0 and 1 represent the two possible states of a logic circuit. The two states can also be referred to as 'ON and OFF' or 'HIGH and LOW' or 'TRUE and FALSE'.

9.18.2 Basic logic gates using discrete components

The basic elements that make up a digital system are 'OR', 'AND' and 'NOT' gates. These three gates are called basic logic gates. All the possible inputs and outputs of a logic circuit are represented in a table called TRUTH TABLE. The function of the basic gates are explained below with circuits and truth tables.

(i) OR gate

An OR gate has two or more inputs but only one output. It is known as OR gate, because the output is high if any one or all of the inputs are high. The logic symbol of a two input OR gate is shown in Fig 9.46a.

The Boolean expression to represent OR gate is given by $Y = A + B$ (+ symbol should be read as OR)

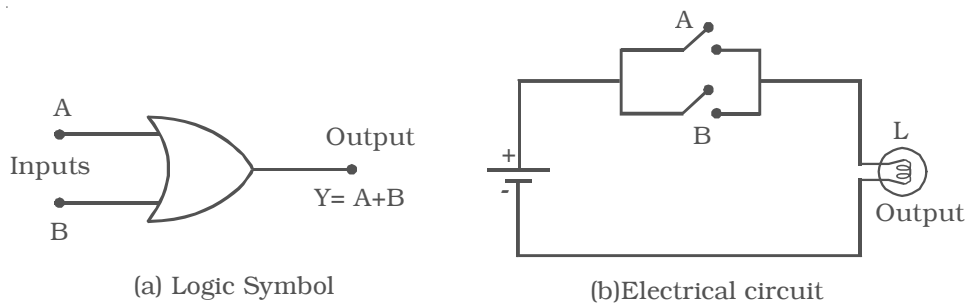


Fig 9.46 OR gate

The OR gate can be thought of like an electrical circuit shown in Fig 9.46b, in which switches are connected in parallel with each other. The lamp will glow if both the inputs are closed or any one of them is closed.

Diode OR gate

Fig 9.47 shows a simple circuit using diodes to build a two input OR gate. The working of this circuit can be explained as follows.

Case (i) $A = 0$ and $B = 0$

When both A and B are at zero level, (i.e.) low, the output voltage will be low, because the diodes are non-conducting.

Case (ii) $A = 0$ and $B = 1$

When A is low and B is high, diode D_2 is forward biased so that current flows through R_L and output is high.

Case (iii) $A = 1$ and $B = 0$

When A is high and B is low, diode D_1 conducts and the output is high.

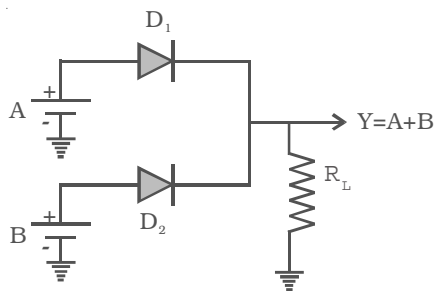


Fig 9.47 OR gate using diodes

Table 9.1 Truth table of OR gate

Inputs		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Case (iv) $A = 1$ and $B = 1$

When A and B both are high, both diodes D_1 and D_2 are conducting and the output is high. Therefore Y is high. The OR gate operations are shown in Table 9.1.

(ii) AND gate

An AND gate has two or more inputs but only one output. It is known as AND gate because the output is high only when all the inputs are high. The logic symbol of a two input AND gate is shown in Fig 9.48a.

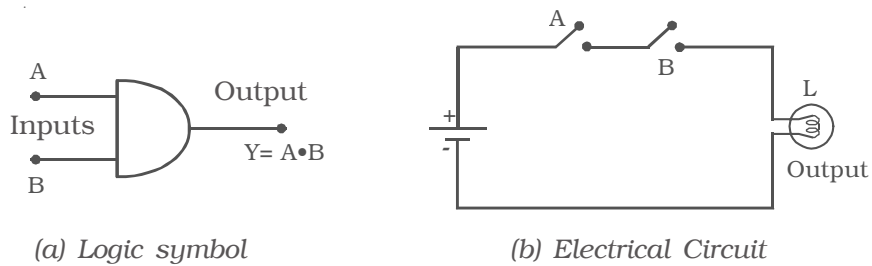


Fig 9.48 AND gate

The Boolean expression to represent AND gate is given by $Y = A \cdot B$ (\cdot should be read as AND)

AND gate may be thought of an electrical circuit as shown in Fig 9.48b, in which the switches are connected in series. Only if A and B are closed, the lamp will glow, and the output is high.

Diode AND gate

Fig 9.49 shows a simple circuit using diodes to build a two-input AND gate. The working of the circuit can be explained as follows :

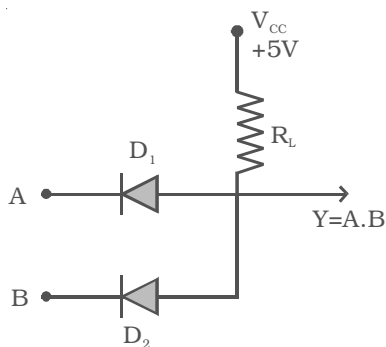


Fig 9.49 AND gate using diodes

Table 9.2 Truth table of AND gate

Inputs		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

Case (i) $A = 0$ and $B = 0$

When A and B are zero, both diodes are in forward bias condition and they conduct and hence the output will be zero, because the supply voltage V_{CC} will be dropped across R_L only. Therefore $Y = 0$.

Case (ii) $A = 0$ and $B = 1$

When $A = 0$ and B is high, diode D_1 is forward biased and diode D_2 is reverse biased. The diode D_1 will now conduct due to forward biasing. Therefore, output $Y = 0$.

Case (iii) $A = 1$ and $B = 0$

In this case, diode D_2 will be conducting and hence the output $Y = 0$.

Case (iv) $A = 1$ and $B = 1$

In this case, both the diodes are not conducting. Since D_1 and D_2 are in OFF condition, no current flows through R_L . The output is equal to the supply voltage. Therefore $Y = 1$.

Thus the output will be high only when the inputs A and B are high. The Table 9.2 summarises the function of an AND gate.

(iii) NOT gate (Inverter)

The NOT gate is a gate with only one input and one output. It is so called, because its output is complement to the input. It is also known as inverter. Fig 9.50a shows the logic symbol for NOT gate.

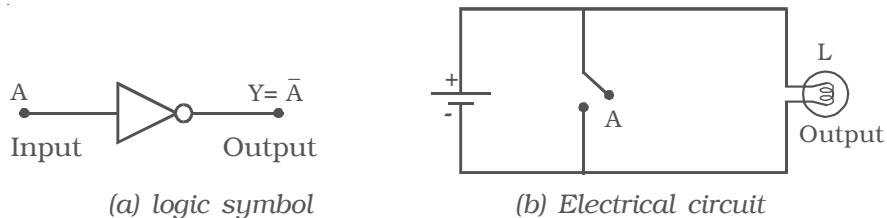


Fig 9.50 NOT gate

The Boolean expression to represent NOT operation is $Y = \bar{A}$.

The NOT gate can be thought of like an electrical circuit as shown in Fig 9.50b. When switch A is closed, input is high and the bulb will not glow (i.e) the output is low and vice versa.

Fig 9.51 is a transistor in CE mode, which is used as NOT gate. When the input A is high, the transistor is driven into saturation and

hence the output Y is low. If A is low, the transistor is in cutoff and hence the output Y is high. Hence, it is seen that whenever input is high, the output is low and vice versa. The operation of NOT gate is shown in Table 9.3.

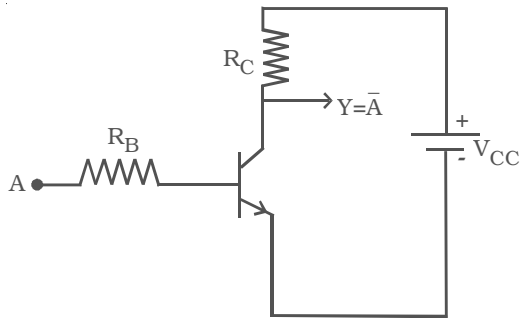


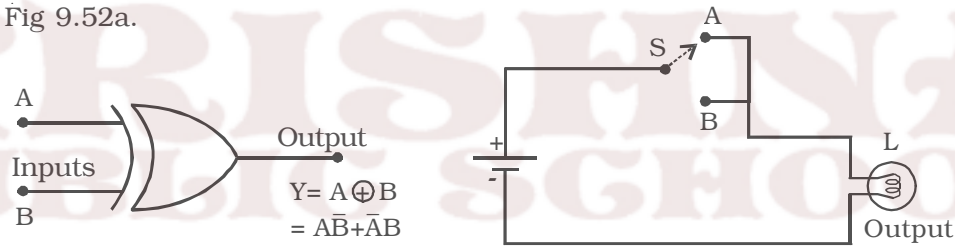
Table 9.3 Truth Table of NOT gate

Input	Output
A	$Y = \bar{A}$
0	1
1	0

Fig 9.51 NOT gate using transistor

9.18.3 Exclusive OR gate (EXOR gate)

The logic symbol for exclusive OR (EXOR) gate is shown in Fig 9.52a.



(a) Logic symbol

(b) Electrical circuit

Fig 9.52 Exclusive OR gate

The Boolean expression to represent EXOR operation is

$$Y = A \oplus B = A\bar{B} + \bar{A}B$$

EXOR gate has an output 1, only when the inputs are complement to each other. The equivalent switching circuit is shown in Fig 9.52b.

Switch positions A and B will individually make the lamp to be ON. But the combination of A and B is not possible.

The EXOR operation is represented in Table 9.4.

Table 9.4 Truth table of EXOR gate

Inputs		Output
A	B	$Y = A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

9.18.4 NAND gate

This is a NOT-AND gate. It can be obtained by connecting a NOT gate at the output of an AND gate (Fig 9.53a).

The logic symbol for NAND gate is shown in Fig 9.53b.

The Boolean expression to represent NAND Operation is $Y = \overline{AB}$

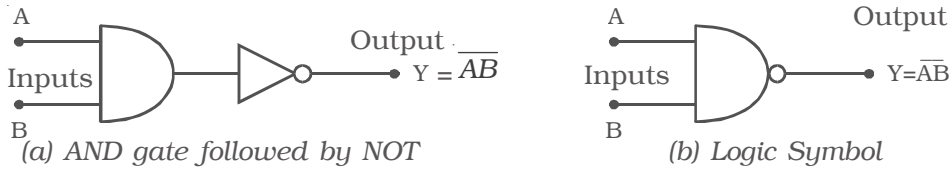


Fig 9.53 NAND gate

NAND gate function is reverse of AND gate function. A NAND gate will have an output, only if both inputs are not 1. In other words, it gives an output 1, if either A or B or both are 0. The operation of a NAND gate is represented in Table 9.5.

Table 9.5 Truth table of NAND gate

Inputs		Output
A	B	$Y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

9.18.5 NOR gate

This is a NOT-OR gate. It can be made out of an OR gate by connecting an inverter at its output (Fig 9.54a).

The logic symbol for NOR gate is given in Fig 9.54b.

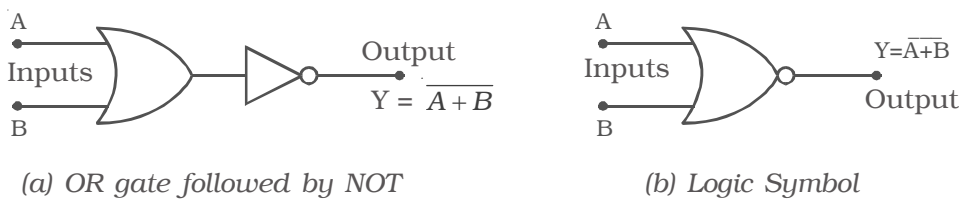


Fig 9.54 NOR gate

The Boolean expression to represent NOR gate is $Y = \overline{A+B}$

The NOR gate function is the reverse of OR gate function. A NOR gate will have an output, only when all inputs are 0. In a NOR gate, output is high, only when all inputs are low. The NOR operation is represented in Table 9.6.

Table 9.6 Truth table of NOR gate

Inputs		Output
A	B	$Y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

9.18.6 De-Morgan's theorems

The two De Morgan's theorems are very important in dealing with NOR and NAND gates. They state that a NOR gate that performs the $\overline{A+B}$ function is equivalent to the function $\overline{A} \cdot \overline{B}$ and NAND gate, that performs the $\overline{A \cdot B}$ function is equivalent to the function $\overline{A+B}$.

First theorem

"The complement of a sum is equal to the product of the complements." If A and B are the inputs, then $\overline{A+B} = \overline{A} \cdot \overline{B}$

Second theorem

"The complement of a product is equal to the sum of the complements." If A and B are the inputs, then $\overline{A \cdot B} = \overline{A+B}$.

The theorems can be proved, first by considering the two variable cases and then extending this result as shown in Table 9.7.

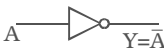
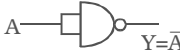

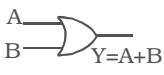
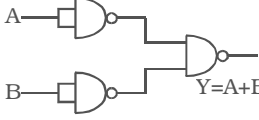
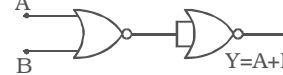
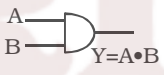
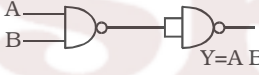
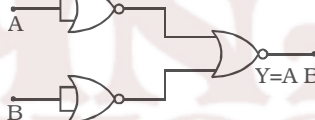
Table 9.7 Truth table to prove De-Morgan's theorems

A	B	\overline{A}	\overline{B}	$\overline{A \cdot B}$	$\overline{A+B}$	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1	1	1	1
0	1	1	0	1	1	0	0
1	0	0	1	1	1	0	0
1	1	0	0	0	0	0	0

9.18.7 NAND and NOR as Universal gates

NAND and NOR gates are called Universal gates because they can perform all the three basic logic functions. Table 9.8 gives the construction of basic logic gates NOT, OR and AND using NAND and NOR gates.

Table 9.8 Substituting NAND / NOR gates

Logic function	Symbol	Circuits using NAND gates only	Circuits using NOR gates only
NOT			
OR			
AND			

9.18.8 Boolean algebra

Boolean algebra, named after a mathematician George Boole is the algebra of logic, which is applied to the operation of computer devices. The rules of this algebra is simple, speed and accurate. This algebra is helpful in simplifying the complicated logical expression.

Laws and theorems of Boolean algebra

The fundamental laws of Boolean algebra are given below which are necessary for manipulating different Boolean expressions.

Basic laws :

Commutative laws

$$A + B = B + A$$

$$AB = BA$$

Associative Laws

$$A + (B + C) = (A + B) + C$$

$$A (BC) = (AB) C$$

Distributive law

$$A(B+C) = AB + AC$$

New operations :

$$A + 0 = A$$

$$A + 1 = 1$$

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A + A = A$$

$$A + \bar{A} = 1$$

$$A \cdot A = A$$

$$A \cdot \bar{A} = 0$$

$$\bar{\bar{A}} = A$$

Special theorems :

$$A + AB = A$$

$$(A + B)(A + C) = A + BC$$

$$A(A + B) = A$$

$$A + \bar{A}B = A + B$$

$$A(\bar{A} + B) = AB$$

$$(A + B)(\bar{A} + C) = AC + \bar{A}B$$

$$AB + \bar{A}C = (A + C)(\bar{A} + B)$$

Theorems involving a single variable can be proved by considering every possible value of the variable. For example considering theorem $A + 0 = A$,

If $A = 0$ then, $0 + 0 = 0 = A$

and if $A = 1$ then, $1 + 0 = 1 = A$

and hence the theorem is proved.

Theorems involving more than one variable can be proved by making a truth table. For example, theorem $A + AB = A$ can be proved by making the Truth table given in Table 9.9.

Table 9.9 Truth table to prove theorem $A + AB = A$

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

From the Table 9.9, it is observed that there are 4 possible combinations of the two variables A and B. For each combination the value of A is the same as that of $A + AB$, which proves the theorem.

9.18.9 Simplification of Boolean expression

By manipulating a Boolean expression according to laws and theorems of Boolean algebra, a simple expression that will require fewer gates, can be obtained.

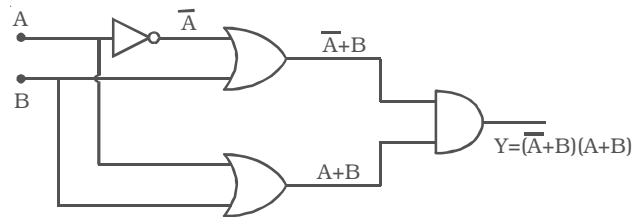


Fig 9.55 Simplification of Boolean expression

To see how Boolean algebra manipulation is used to simplify digital circuits, let us consider the following example.

In the circuit given in Fig 9.55, the outputs of the OR gates are $\bar{A} + B$ and $A + B$. These are the inputs for the AND gate and the output of the circuit is $Y = (\bar{A} + B)(A + B)$.

Now using the distributive law twice, we can write

$$\begin{aligned} (\bar{A} + B)(A + B) &= \bar{A}A + \bar{A}B + BA + BB \\ &= \bar{A}B + AB + B \quad (\because \bar{A}A = 0 \text{ and } BB = B) \\ &= B(\bar{A} + A + 1) = B \quad (\because \bar{A} + A + 1 = 1) \end{aligned}$$

Since the output of the circuit is simply B, we don't need a logic circuit.

9.19 Operational amplifier (OP - AMP)

Linear integrated circuits are being used in a number of electronic applications, such as in the fields like communication, medical electronics, instrumentation control etc. An important linear IC is an operational amplifier.

OP-AMP is a solid state device capable of sensing and amplifying dc and ac input signals. OP-AMP is an amplifier with two inputs (differential inputs) and a single output. OP-AMP consists of 20 transistors, 11 resistors and one capacitor. It usually requires a positive and negative power supply (dual power supply). This allows the output voltage to swing positive and negative with respect to ground.

The most important characteristics of OP-AMP are : (i) very high input impedance or even infinity which produces negligible current at the inputs, (ii) very high gain, (iii) very low output impedance or even zero, so as not to affect the output of the amplifier by loading.

An OP-AMP is so named, because it was originally designed to perform mathematical operations such as addition, subtraction, multiplication, division, integration, differentiation etc in analog computer. Nowadays OP-AMPs are used in analog computer operations and in timing circuits.

9.19.1 Circuit symbol and Pin-out configuration of an OP-AMP

The OP – AMP is represented by a triangular symbol as shown in Fig 9.56. It has two input terminals and one output terminal. The terminal with *negative* sign is called as the inverting input and the

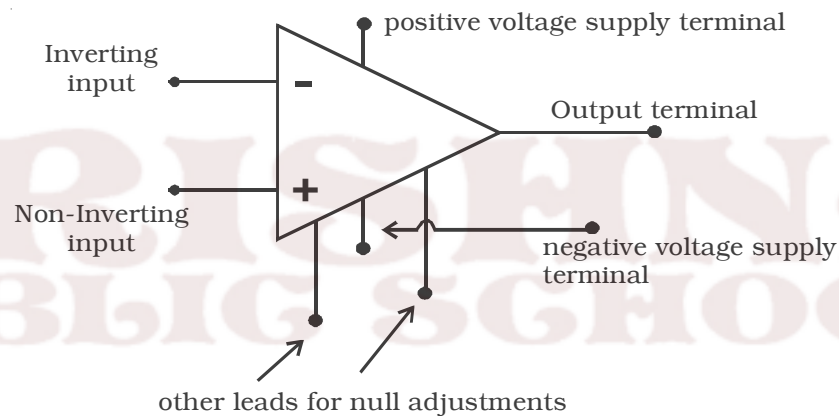


Fig 9.56 Symbol for OP-AMP

terminal with *positive* sign is called as the non-inverting input. The input terminals are at the base of the triangle. The output terminal is shown at the apex of the triangle.

The widely used very popular type Op-Amp IC 741, which is available in DIP. Referring to the top view of the dual-in-package, the pin configuration of IC 741 can be described (Fig 9.57) as follows. The top pin on the left side of the notch indicates Pin 1. The pin number 2 is inverting input terminal and 3 is non-inverting input terminal. Pin 6 is the output terminal. A d.c. voltage or a.c signal placed on the inverting input will be 180° out of phase at the output. A d.c. voltage or a.c. signal placed on the non-inverting input will be

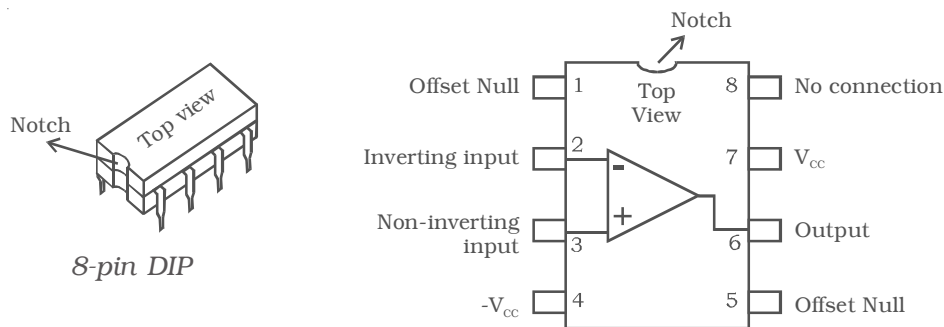


Fig 9.57 OP-AMP Pin-out configuration

inphase at the output. Pins 7 and 4 are the power supply terminals. Terminals 1 and 5 are used for null adjustment. Null adjustment pins are used to null the output voltage when equal voltages are applied to the input terminals for perfect balance. Pin 8 indicates no connection.

9.19.2 Basic OP-AMP circuits

This section concentrates on the principles involved with basic OP-AMP circuit viz, (i) inverting and (ii) non-inverting amplifiers.

(i) Inverting amplifier

The basic OP-AMP inverting amplifier is shown in Fig 9.58. The input voltage V_{in} is applied to the inverting input through the input resistor R_{in} . The non inverting input is grounded. The feedback resistor R_f is connected between the output and the inverting input.

Since the input impedance of an op-amp is considered very high, no current can flow into or out of the input terminals. Therefore I_{in} must flow through R_f and is indicated by I_f (the feedback current). Since R_{in} and R_f are in series, then $I_{in} = I_f$. The voltage between inverting and non-inverting inputs

is essentially equal to zero volt. Therefore, the inverting input terminal is also at 0 volt. For this reason the inverting input is said to be at virtual ground. The output voltage (V_{out}) is taken across R_f .

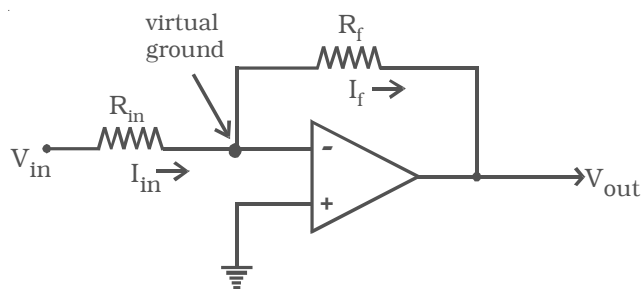


Fig 9.58 Inverting amplifier

It can be proved that

$$I_f = -\frac{V_{out}}{R_f}$$

Since $I_{in} = I_f$, then

$$\frac{V_{in}}{R_{in}} = \frac{-V_{out}}{R_f}$$

Rearranging the equation, we obtain

$$\frac{-V_{out}}{V_{in}} = \frac{R_f}{R_{in}}$$

∴ The voltage gain of an inverting amplifier can be expressed as

$$A_v = \frac{-R_f}{R_{in}}$$

The amplifier gain is the ratio of R_f to R_{in} .

Finally, the output voltage can be found by

$$V_{out} = \frac{-R_f}{R_{in}} \times V_{in}$$

The output voltage is out of phase with the input voltage.

(ii) Non-inverting amplifier

The basic OP-AMP non-inverting amplifier is shown in Fig 9.59. The input signal V_{in} is applied to the non-inverting input terminal. The resistor R_{in} is connected from the inverting input to ground. The feedback resistor R_f is connected between the output and the inverting input.

Resistors R_f and R_{in} form a resistive ratio network to produce the feedback voltage (V_A) needed at the inverting input. Feedback voltage (V_A) is developed across R_{in} . Since the potential at the inverting input

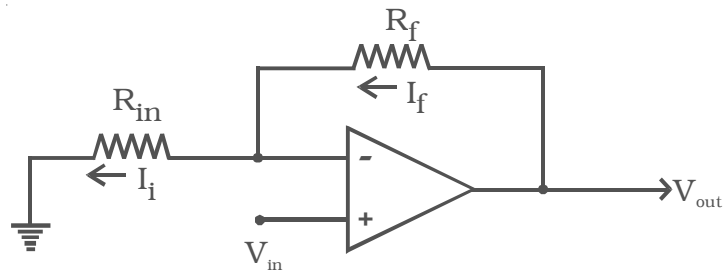


Fig 9.59 Non-inverting amplifier

tends to be the same as the non-inverting input (as pointed out with the description of virtual ground), $V_{in} = V_A$.

Since $V_A = V_{in}$, the gain of the amplifier can be expressed as

$$A_v = \frac{V_{out}}{V_A}$$

However, V_A is determined by the resistance ratio of R_{in} and R_f ; thus,

$$V_A = \frac{R_{in}}{R_f + R_{in}} V_{out}$$

$$(or) \quad \frac{V_{out}}{V_A} = \frac{R_f + R_{in}}{R_{in}}$$

$$\frac{V_{out}}{V_A} = 1 + \frac{R_f}{R_{in}}$$

$$\therefore A_v = 1 + \frac{R_f}{R_{in}}$$

Finally, the output voltage can be found by, $V_{out} = \left(1 + \frac{R_f}{R_{in}}\right) V_{in}$

It is seen that the input and output voltages are in phase.

9.19.3 Summing amplifier

The summing amplifier provides an output voltage equal to the algebraic sum of the input voltages.

Fig 9.60 shows an inverting amplifier, used to sum two input voltages. The input voltages v_1 and v_2 are applied through the resistors R_1 and R_2 to the summing junction (P) and R_f is the feedback resistor. At the point P,

$$i_1 + i_2 = i_f$$

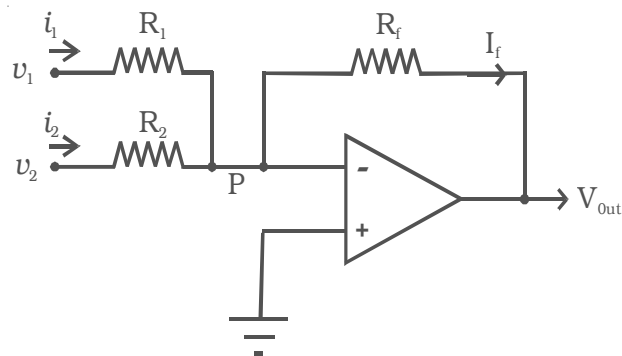


Fig 9.60 Summing amplifier

Since the voltage at the point P is ideally 0,

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} = -\frac{v_{out}}{R_f}$$

Hence the output voltage,

$$v_{out} = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right)$$

If $R_1 = R_2 = R_f = R$, then $v_{out} = -(v_1 + v_2)$

Hence the output voltage is equal to the sum of the input voltages and the circuit acts as a summing amplifier. The negative sign indicates that OP-AMP is used in the inverting mode.

9.19.4 Difference amplifier

The difference amplifier is shown in Fig 9.61. The output voltage can be obtained by using superposition principle. To find the output voltage v_{O1} due to v_1 alone, assume that v_2 is shorted to ground. Then

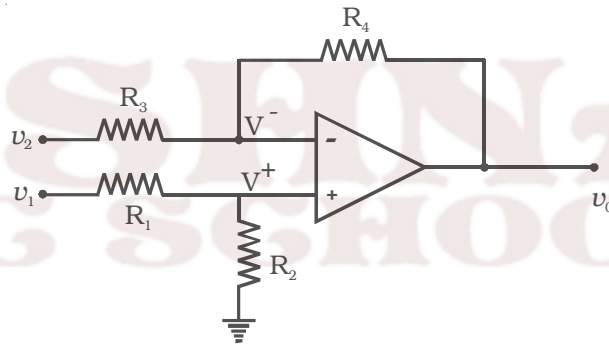


Fig 9.61 Difference amplifier

$$V^+ = \frac{R_2}{R_1 + R_2}v_1$$

$$\text{and } v_{O1} = \frac{R_3 + R_4}{R_3}V^+ = \left(\frac{R_3 + R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right)v_1$$

Now assuming that v_1 is shorted to ground, the output voltage v_{O2} due to v_2 alone is given by,

$$v_{O2} = -\frac{R_4}{R_3}v_2$$

Therefore, with both inputs present, the output is

$$v_o = v_{01} + v_{02}$$

$$= \left(\frac{R_3 + R_4}{R_3} \right) \left(\frac{R_2}{R_1 + R_2} \right) v_1 - \left(\frac{R_4}{R_3} \right) v_2$$

If $R_1 = R_2 = R_3 = R_4 = R$

then $v_o = v_1 - v_2$

If all the external resistors are equal, the voltage difference amplifier functions as a voltage subtractor.

9.20 Electronic measuring instruments

The active and passive elements in an electronic circuit should be tested for their correctness and reliability before making electrical connections with them. Electronic circuits should be tested for their performance. Electronic measuring instruments are used for this purpose. Measurement or testing is done by measuring current, voltage, resistance, capacitance, etc. The construction and working of a few measuring instruments such as cathode ray oscilloscope and multimeter are discussed in the following sections.

9.20.1 Cathode ray oscilloscope (CRO)

This device is the most useful electronic instrument. This gives visual representation of electrical quantities such as voltage and frequency in any electronic circuit. It makes use of the properties of cathode rays, which are deflected in electric and magnetic fields and produce scintillation on fluorescent screen (Fig 9.62).

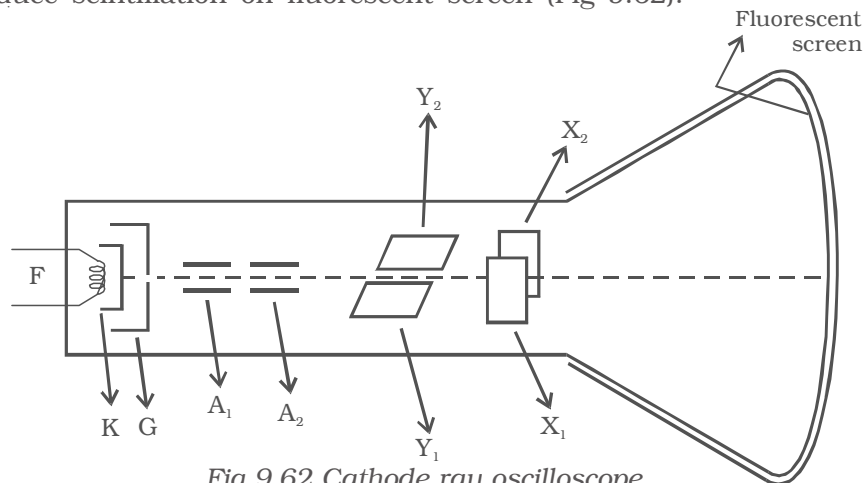


Fig 9.62 Cathode ray oscilloscope

CRO consists of three sections (i) an electron gun (ii) a deflection system (iii) a fluorescent screen. These are enclosed in a glass tube, which is completely evacuated.

(i) Electron gun

The electron gun consists of a cathode K, a control grid G and along with them two anodes A_1 and A_2 . Electrons which are emitted from the indirectly heated cathode K, passes through the hole in the cylindrical grid G, towards the anode. Due to the negative potential of G with respect to cathode, these electrons bunch together, forming a beam of electrons coming out of G. The electron beam is accelerated by the two cylindrical anodes A_1 and A_2 . Then the electron beam strikes the fluorescent screen.

(ii) Deflection system

With the help of two pairs of the plates X_1 , X_2 and Y_1 , Y_2 , the electron beam travelling towards the screen can be deflected. These plates are placed between the path of the beam beyond the anode A_2 . Y_1 , Y_2 plates are called Y-plates and X_1 , X_2 plates are called X-plates. The Y plates are kept horizontal and X plates kept vertical along the direction of the beams.

The electron beam will be deflected along the vertical direction, if a DC potential is applied to Y plates and the electron beam will be deflected along the horizontal direction, with the potential on X-plates.

(iii) Fluorescent screen

The glass tube is coated with a fluorescent material. When the beam strikes the screen a light spot is visible. The electrons striking the screen are returned to the power supply via the inner surfaces of the glass envelope, usually coated with graphite. This coating is connected to anode A_2 by a lead-in wire.

Uses

- (i) It is used to measure a.c and d.c voltage.
- (ii) It is used to study the waveforms of a.c voltages.
- (iii) It is used to find the frequency of a.c voltage.
- (iv) It is used to study the beating of heart in cardiology.

9.20.2 Multimeter

Multimeter is an electronic instrument, which is used to measure voltage, current and resistance. This is called as AVO meter (ampere, voltage, ohm). The internal circuit of the multimeter is shown in Fig 9.63. It consists of a moving coil galvanometer. By incorporating suitable circuits with the galvanometer, the measurement of voltage, current and resistance can be done. Let us consider a galvanometer of resistance $G (= 100 \Omega)$, which gives a full scale deflection for 1 mA.

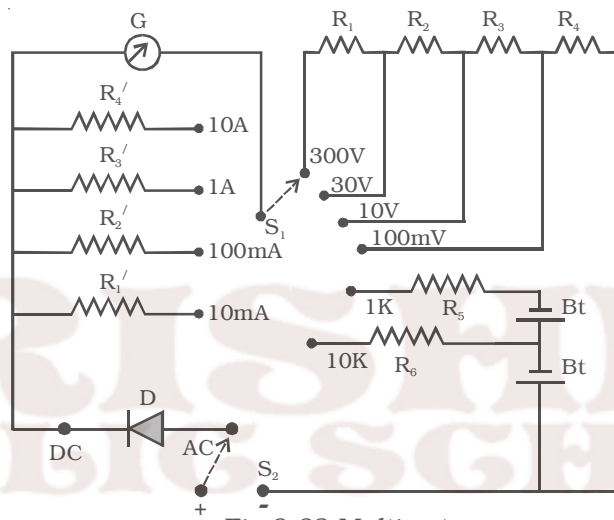


Fig 9.63 Multimeter

(a) as a voltmeter

The moving coil galvanometer is converted into a voltmeter by connecting in series a high resistance of suitable value. If a resistance of 900Ω is connected in series with the galvanometer, the range of the voltmeter becomes $10^{-3} \times 1000 = 1 \text{ V}$. The same meter can be used to measure voltage upto 10 V, if a resistance 9900Ω is connected in series. Thus, in order to measure different ranges, different high resistances are put in series with the galvanometer by a switching arrangement.

(b) as an ammeter

The galvanometer is converted into an ammeter by shunting it with suitable low resistances, one for each range. For example, the

galvanometer can be converted into an ammeter with range 0-1 A by shunting it with a resistance of $100/999 \Omega$ or 0.1Ω (approximately) and the range can be increased to 10 A by shunting it with $100/9999 \Omega$ (0.01Ω approximately) and so on.

(c) as an ohm-meter

The galvanometer is converted into an ohm-meter by connecting a battery and a suitable resistance in series (Fig 9.64). If a battery of 1 V is connected in series and the above galvanometer shows full scale deflection, then the total resistance in the circuit (including that of the galvanometer) be 1000Ω .

Measurement of resistance

On the resistance scale 0 is marked against the current of 1 mA. Now, if a unknown resistance is connected between the terminals T_1 and T_2 , the current passing through the galvanometer decreases to 0.5 mA, then the unknown resistance is 1000Ω . Hence, 1 k Ω is marked against 0.5 mA. If the galvanometer current is 0.25 mA for another unknown resistance connected between T_1 and T_2 , then that resistance is 3000Ω . Hence, 3 k Ω is marked against 0.25 mA.

Usually an adjustable resistance of small value is connected in series, so that the zero adjustment can be made, even if the emf of the cell decreases slightly.

To measure a.c. voltage and current, a rectifier unit is connected in series and by turning a switch to a.c., the instrument is used to measure a.c. quantities. If the switch is turned to d.c., the rectifier unit gets disconnected and the instrument is then used to measure d.c. current and voltage.

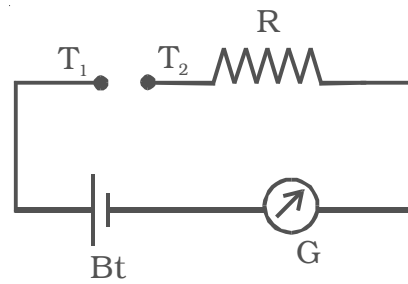
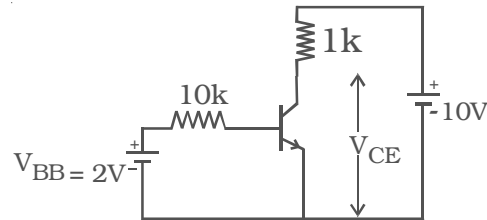


Fig 9.64 Measurement of resistance

Solved Problems

- 9.1 The current gain β of the silicon transistor used in the circuit as shown in figure is 50. (Barrier potential for silicon is 0.69 V)



Find (i) I_B (ii) I_E (iii) I_C and (iv) V_{CE}

Data :

$$V_{BB} = 2 \text{ V}, V_{CC} = 10 \text{ V}; \beta = 50$$

$$R_B = 10 \text{ k}\Omega ; R_C = 1 \text{ k}\Omega$$

The barrier potential for silicon transistor $V_{BE} = 0.69 \text{ V}$

Solution : $V_{BB} = I_B R_B + V_{BE}$

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{2 - 0.69}{10 \times 10^3} = 131 \mu\text{A}$$

$$\text{Current gain } \beta = \frac{I_C}{I_B}$$

$$I_C = I_B \beta = 131 \times 10^{-6} \times 50 = 6.5 \text{ mA}$$

$$\begin{aligned} \text{Emitter current } I_E &= I_C + I_B \\ &= 6.5 \text{ mA} + 131 \mu\text{A} \\ &= 6.5 \text{ mA} + 0.131 \text{ mA} \\ &= 6.631 \text{ mA} \end{aligned}$$

$$V_{CC} = V_{CE} + I_C R_C$$

$$\begin{aligned} V_{CE} &= V_{CC} - I_C R_C \\ &= 10 - (6.5 \times 10^{-3} \times 1 \times 10^3) \\ &= 3.5 \text{ V} \end{aligned}$$

- 9.2 A transistor is connected in CE configuration. The voltage drop across the load resistance (R_C) 3 k Ω is 6 V. Find the base current. The current gain α of the transistor is 0.97

Data : Voltage across the collector load resistance (R_C) = 6 V

$$\alpha = 0.97; R_C = 3 \text{ k}\Omega$$

Solution : The voltage across the collector resistance is, $R_C = I_C R_C = 6V$

$$\text{Hence, } I_C = \frac{6}{R_C} = \frac{6}{3 \times 10^3} = 2\text{mA}$$

$$\text{Current gain } \beta = \frac{\alpha}{1-\alpha} = \frac{0.97}{1-0.97} = 32.33$$

$$\therefore I_B = \frac{I_C}{\beta} = \frac{2 \times 10^{-3}}{32.33} = 61.86 \mu\text{A}$$

9.3 When the negative feedback is applied to an amplifier of gain 50, the gain after feedback falls to 25. Calculate the feedback ratio.

Data : $A = 50$; $A_f = 25$

Solution : Voltage gain after feedback,

$$A_f = \frac{A}{1+A\beta}$$

$$25 = \frac{50}{1+50\beta}$$

Hence, the feedback ratio $\beta = 0.02$

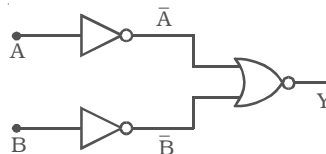
9.4 Prove the Boolean identity ; $(A+B)(A+C) = A+BC$

Solution : Proof : Applying the law of distribution on LHS of the equation, we get

$$\begin{aligned} (A+B)(A+C) &= AA + AC + BA + BC \\ &= A + AC + AB + BC = A(1+C+B) + BC = A + BC \\ & \quad [\because 1 + C + B = 1] \end{aligned}$$

\therefore LHS = RHS, the given identity is proved.

9.5 The outputs of two NOT gates are NORed, as shown in figure. What is this combination equivalent to?



Solution : From the logic circuit it follows that the output

$$y = \overline{\overline{A + B}}$$

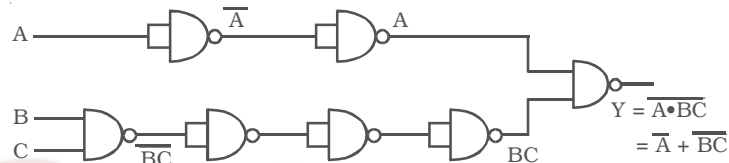
Applying DeMorgan's first theorem,

$$\text{we get, } y = \overline{\overline{A}} \cdot \overline{\overline{B}} = AB$$

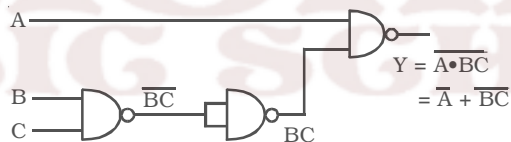
Hence given logic circuit is AND operation.

9.6 Construct a logic circuit using NAND gates only for $Y = \overline{A} + \overline{BC}$

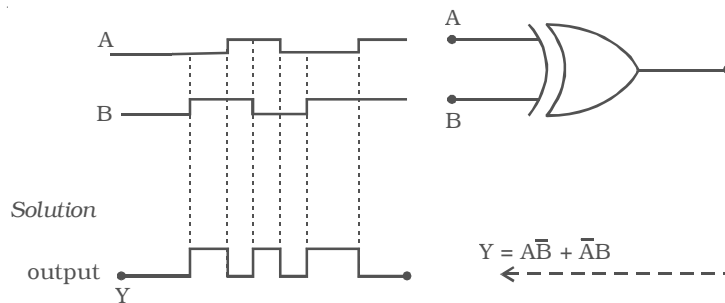
Solution : Each and every term in the given logic expression is replaced by the corresponding basic gates using NAND gates. Hence, the logic circuit for $Y = \overline{A} + \overline{BC}$ using NAND gates is given below



$\therefore \overline{\overline{A}} = A$, the above logic circuit can be simplified as,

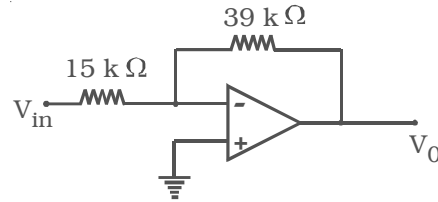


9.7 Determine the output wave form for the circuit given below, if the input waveforms are as indicated by A and B.



9.8 Find the output of the ideal operational amplifier shown in the figure for each of the following input signals

- a) $V_{in} = 120 \text{ mV dc}$
- b) $V_{in} = 0.5 \sin \omega t$
- c) $V_{in} = -2.5 \text{ V dc}$
- d) $V_{in} = 4 - \sin \omega t \text{ V}$
- e) $V_{in} = 0.8 \sin (\omega t + 75^\circ) \text{ V}$



Data : $R_f = 39 \text{ k}\Omega$; $R_1 = 15 \text{ k}\Omega$

Solution : The gain of the amplifier $A_v = -\frac{R_f}{R_{in}} = -\frac{39 \times 10^3}{15 \times 10^3} = -2.6$

$$\begin{aligned} \text{a) output of the OP.AMP } (V_o) &= -\frac{R_f}{R_{in}} \times V_{in} \\ &= -2.6 \times 120 \times 10^{-3} = -0.312 \text{ V} \end{aligned}$$

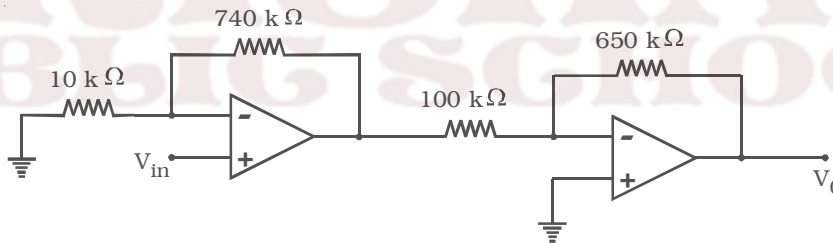
$$\text{b) } V_o = -2.6 \times (0.5 \sin \omega t) = -1.3 \sin \omega t \text{ ac}$$

$$\text{c) } V_o = -2.6 \times (-2.5 \text{ V}) = +6.5 \text{ V d.c.}$$

$$\text{d) } V_o = -2.6 (4 - \sin \omega t \text{ V}) = -10.4 + 2.6 \sin \omega t \text{ V}$$

$$\text{e) } V_o = -2.6 [(0.8 \sin (\omega t + 75^\circ) \text{ V})] = -2.08 \sin (\omega t + 75^\circ) \text{ V}$$

9.9 Find the output of the circuit given below



Data : $R_f = 740 \text{ k}\Omega$; $R_f' = 650 \text{ k}\Omega$; $R_1 = 10 \text{ k}\Omega$;
 $R_1' = 100 \text{ k}\Omega$

Solution :

Let V_1 be the output voltage of the first amplifier

$$V_1 = \left(1 + \frac{R_f}{R_1}\right) V_{in} = \left(1 + \frac{740 \times 10^3}{10 \times 10^3}\right) V_{in} = 75 V_{in}$$

$$\therefore V_o = -\left(\frac{R_f'}{R_{in}}\right) V_1 = -\left(\frac{650 \times 10^3}{100 \times 10^3}\right) 75 V_{in} = -487.5 V_{in}$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

- 9.1 The electrons in the atom of an element which determine its chemical and electrical properties are called
(a) valence electrons (b) revolving electrons
(c) excess electrons (d) active electrons
- 9.2 In an N-type semiconductor, there are
(a) immobile negative ions (b) no minority carriers
(c) immobile positive ions (d) holes as majority carriers
- 9.3 The reverse saturation current in a PN junction diode is only due to
(a) majority carriers (b) minority carriers
(c) acceptor ions (d) donor ions
- 9.4 In the forward bias characteristic curve, a diode appears as
(a) a high resistance (b) a capacitor
(c) an OFF switch (d) an ON switch
- 9.5 Avalanche breakdown is primarily dependent on the phenomenon of
(a) collision (b) ionisation
(c) doping (d) recombination
- 9.6 The colour of light emitted by a LED depends on
(a) its reverse bias (b) the amount of forward current
(c) its forward bias (d) type of semiconductor material
- 9.7 The emitter base junction of a given transistor is forward biased and its collector–base junction is reverse biased. If the base current is increased, then its
(a) V_{CE} will increase (b) I_C will decrease
(c) I_C will increase (d) V_{CC} will increase.
- 9.8 Improper biasing of a transistor circuit produces
(a) heavy loading of emitter current
(b) distortion in the output signal
(c) excessive heat at collector terminal
(d) faulty location of load line

9.9 An oscillator is

- (a) an amplifier with feedback
- (b) a convertor of ac to dc energy
- (c) nothing but an amplifier
- (d) an amplifier without feedback

9.10 In a Colpitt's oscillator circuit

- (a) capacitive feedback is used
- (b) tapped coil is used
- (c) no tuned LC circuit is used
- (d) no capacitor is used

9.11 Since the input impedance of an ideal operational amplifier is infinite,

- (a) its input current is zero
- (b) its output resistance is high
- (c) its output voltage becomes independent of load resistance
- (d) it becomes a current controlled device

9.12 The following arrangement performs the logic function of _____ gate

- (a) AND
- (c) NAND

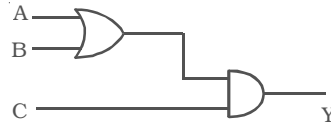
- (b) OR
- (d) EXOR



9.13 If the output (Y) of the following circuit is 1, the inputs A B C must be

- (a) 0 1 0
- (c) 1 0 1

- (b) 1 0 0
- (d) 1 1 0



9.14 According to the laws of Boolean algebra, the expression $(A + AB)$ is equal to

- (a) A
- (c) B

- (b) AB
- (d) \bar{A}

9.15 The Boolean expression \overline{ABC} can be simplified as

- (a) $AB + \bar{C}$

- (b) $\bar{A} \cdot \bar{B} \cdot \bar{C}$

- (c) $AB + BC + CA$

- (d) $\bar{A} + \bar{B} + \bar{C}$

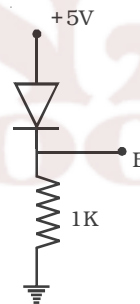
- 9.16 Describe the valence band, conduction band and forbidden energy gap with the help of energy level diagram.
- 9.17 Describe the energy band structure of insulator, semiconductor and conductor.
- 9.18 What do you understand by intrinsic and extrinsic semiconductor?
- 9.19 What is rectification?
- 9.20 Explain the working of a half wave diode rectifier.
- 9.21 Explain the working of bridge rectifier.
- 9.22 What is zener breakdown?
- 9.23 Describe the construction of Zener diode.
- 9.24 Explain with necessary circuit how zener diode can be used as a voltage regulator.
- 9.25 Describe the working of PNP and NPN transistors.
- 9.26 Deduce the relation between α and β of a transistor.
- 9.27 Explain an experiment to determine the characteristics of a transistor in CE configuration. Explain how the transistor parameters can be evaluated.
- 9.28 Why is a transistor called as current amplification device?
- 9.29 Why CE configuration is preferred over CB configuration for operating transistor as an amplifier?
- 9.30 Describe the working of a transistor amplifier.
- 9.31 Define bandwidth of an amplifier.
- 9.32 What is meant by feedback? Name the two types of feedback.
- 9.33 Derive an expression for voltage gain of an amplifier with negative feedback.
- 9.34 What are the advantages of negative feedback?
- 9.35 Give the Barkhausen criteria for oscillations.
- 9.36 Sketch the circuit of Colpitt's oscillator. Explain its working.
- 9.37 Give the function of 'OR' and 'NAND' gates.
- 9.38 What are universal gates? Why are they called so?
- 9.39 What is an EXOR gate? Give the Boolean expression for the EXOR operation.

- 9.40 State and prove DeMorgan's theorems.
- 9.41 What is an integrated circuit?
- 9.42 Identify the analog and digital signals from the following.
(i) square wave, (ii) sine wave
- 9.43 Differentiate between linear ICs and digital ICs.
- 9.44 Describe an operational amplifier. Explain its action as (i) inverting amplifier and (ii) noninverting amplifier.
- 9.45 Explain the term virtual ground of an operational amplifier.
- 9.46 Give the important parameters of an operational amplifier.
- 9.47 Explain how operational amplifier is used as a summer.
- 9.48 Describe the action of an operational amplifier as difference amplifier.
- 9.49 Explain how multimeter is used as ohm meter.

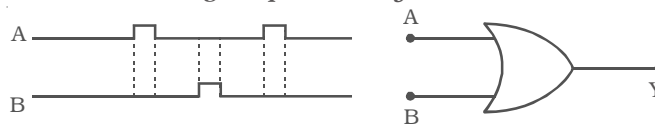
Problems

9.50 The base current of the transistor is $50 \mu\text{A}$ and collector current is 25 mA . Determine the values of β and α .

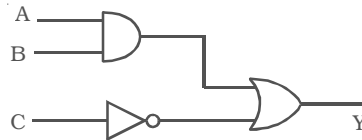
9.51 Find the voltage at the point B in the figure (Silicon diode is used).



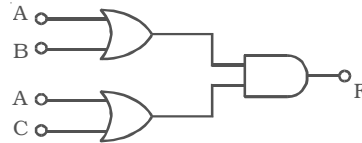
- 9.52 The gain of the amplifier is 100. If 5% of the output voltage is fed back into the input through a negative feedback network. Find out the voltage gain after feedback.
- 9.53 Determine the frequency of oscillations in a Colpitt's oscillator if $C_1 = 0.01 \mu\text{F}$, $C_2 = 0.03 \mu\text{F}$ and $L = 100 \text{ mH}$.
- 9.54 If the two waveforms shown in figure are applied to the OR gate. What is the resulting output wave form?



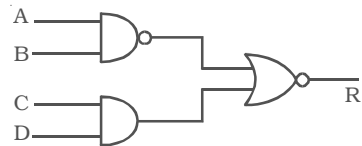
9.55 What is the Boolean expression for the logic diagram shown in figure. Evaluate its output if $A = 1$, $B = 1$ and $C = 1$.



9.56 Find the output F of the logic circuit given below:



9.57 Give the Boolean equation for the given logic diagram.

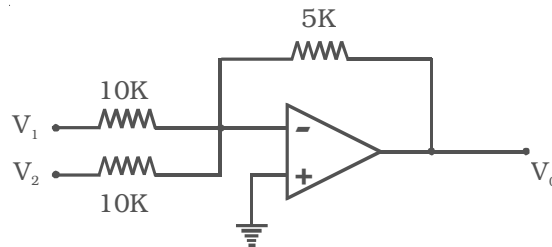


9.58 Prove the following logic expression using the laws and theorems of Boolean algebra.

$$ABC + A\bar{B}C + ABC\bar{C} = A(B+C)$$

9.59 Simplify the following logic expression using the laws and theorems of Boolean algebra. $Y = A.\bar{B} + AB + BC + CA$

9.60 Find the output of the given circuit.



Answers

- 9.1** (a) **9.2** (c) **9.3** (b) **9.4** (d) **9.5** (a)
9.6 (d) **9.7** (c) **9.8** (b) **9.9** (a) **9.10** (a)
9.11 (a) **9.12** (a) **9.13** (c) **9.14** (a) **9.15** (d)

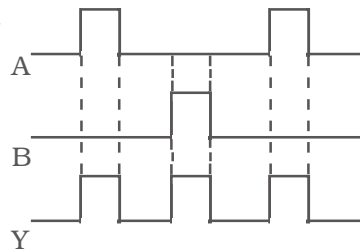
9.50 500 ; 0.998

9.51 + 4.31 V

9.52 16.66

9.53 5815 Hz

9.54



9.55 $AB + \bar{C}$; 1

9.56 $F = A + (B.C)$

9.57 $R = AB (\bar{C} + \bar{D})$

9.59 $A + BC$

9.60 $V_o = -\frac{1}{2} (V_1 + V_2)$

KRISHNA
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10. Communication Systems

The present era is the era of 'information and communication revolution'. In electronics, the term 'communication' refers to sending, receiving and processing of information electronically. In 1840's, communication started with telegraphy. Few decades later, telephony was developed followed by radio at the beginning of twentieth century. Radio communication was made possible by the invention of the electronic valves. It subsequently became even more widely used and refined through the invention and use of the transistors, integrated circuits and other semiconductor devices.

More recently, the use of satellites and fiber optics has made communications even more widespread, with an increasing emphasis on computer and other data communications. Radar, telemetry and satellite links play vital role in navigation, defence, scientific research etc. For communication purposes, only a part of the electromagnetic waves like radio waves and microwaves are being used.

10.1 Propagation of electromagnetic waves

The propagation of electromagnetic waves depend on the properties of the waves and the environment. Radio waves ordinarily travel in straight lines except where the earth and its atmosphere alter their path. The useful ranges of the electromagnetic spectrum for communication are summarised in Table 10.1.

Radio wave is propagated from the transmitting to the receiving antenna mainly in three different ways depending on the frequency of the wave. They are :

- (i) Ground (surface) wave propagation
- (ii) Space wave propagation
- (iii) Sky wave (or) ionospheric propagation

Table 10.1 Ranges of electromagnetic spectrum used for communication (NOT FOR EXAMINATION)

Name	Frequency	Wavelength
Extremely Low Frequencies (ELF)	30-300 Hz	$10^7 - 10^6$ m
Voice Frequencies (VF)	300-3000 Hz	$10^6 - 10^5$ m
Very Low Frequencies (VLF)	3-30 kHz	$10^5 - 10^4$ m
Low Frequencies (LF)	30-300 kHz	$10^4 - 10^3$ m
Medium Frequencies (MF)	300 kHz – 3 MHz	$10^3 - 10^2$ m
High Frequencies (HF)	3 – 30 MHz	$10^2 - 10$ m
Very High Frequencies (VHF)	30 – 300 MHz	10 – 1 m
Ultra High Frequencies (UHF)	300 MHz – 3 GHz	$1 - 10^{-1}$ m
Super High Frequencies (SHF)	3 – 30 GHz	$10^{-1} - 10^{-2}$ m
Extremely High Frequencies (EHF)	30 – 300 GHz	$10^{-2} - 10^{-3}$ m

10.1.1 Ground (surface) wave propagation

Ground or surface waves are the radio waves which travel along the surface of the earth as shown in Fig 10.1. Ground wave propagation takes place when the transmitting and receiving antennas are close to the ground. Ground wave propagation is of prime importance only for medium and long wave signals. All medium wave signals received during the daytime use surface wave propagation.

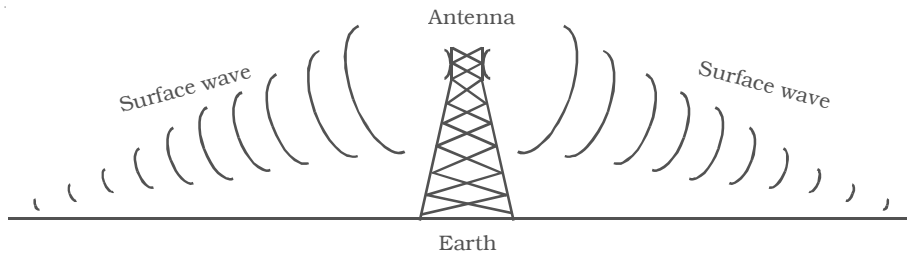


Fig 10.1 Ground or surface wave radiation from an antenna

10.1.2 Space wave propagation

Radio waves propagated through the troposphere of the Earth are known as space waves. Troposphere is the portion of the Earth's atmosphere which extends upto 15 km from the surface of the Earth. Space wave usually consists of two components as shown in Fig 10.2.

(i) A component which travels straight from the transmitter to the receiver.

(ii) A component which reaches the receiver after reflection from the surface of the Earth.

Space wave propagation is particularly suitable for the waves having frequency above 30 MHz.

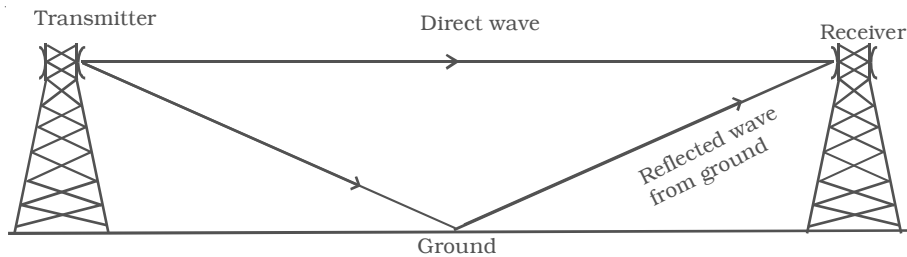


Fig 10.2 Space wave propagation

10.1.3 Sky wave (or) ionospheric propagation

The ionosphere is the upper portion of the atmosphere, which absorbs large quantities of radiant energy like ultra violet rays, cosmic rays etc., from the sun, becoming heated and ionised. This ionised region contains free electrons, positive and negative ions.

Radio waves in the short wave band, radiated from an antenna at large angles with ground, travel through the atmosphere and encounters the ionised region in the upper atmosphere. Under favourable circumstances, the radiowaves get bent downwards due to refraction from the different parts of the ionised region and again reach the earth at a far distant point. Such a radio wave is called the sky wave and such a propagation of radio wave is known as sky wave propagation or ionospheric propagation. Long distance radio communication is thus possible through the sky wave propagation.

Reflection of electromagnetic waves by ionosphere

The electromagnetic waves entering into the ionosphere, are reflected by the ionosphere. In fact, the actual mechanism involved is refraction. The refractive indices of the various layers in the ionosphere do not remain constant and it varies with respect to electron density and the frequency of the incident wave. As the ionisation density increases for a wave approaching the given layer at an angle, the refractive index of the layer is reduced. Hence, the incident wave is

gradually bent farther and farther away from the normal as shown in Fig 10.3 until some point. When the electron density is large, the angle of refraction becomes 90° and the wave, then travel towards the Earth.

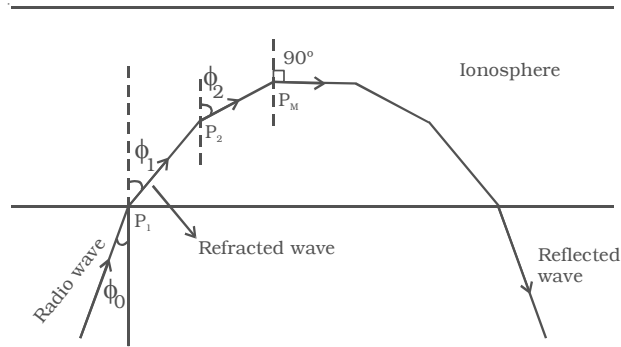


Fig 10.3 Refraction of the radio wave in ionosphere

Skip distance and skip zone

In the skywave propagation, for a fixed frequency, the shortest distance between the point of transmission and the point of reception along the surface is known as the *skip distance*.

When the angle of incidence is large for the ray R_1 as shown in Fig. 10.4, the sky wave returns to the ground at a long distance from

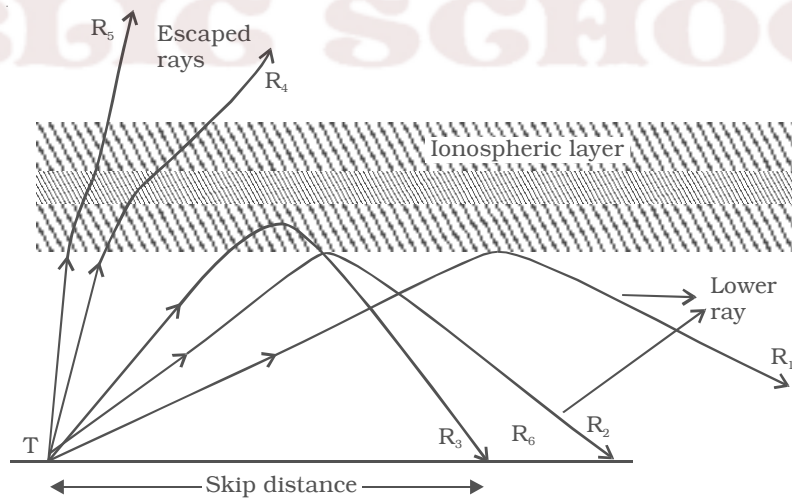


Fig 10.4 Travel of radio waves at different angles of incidence

the transmitter. As this angle is slowly reduced, naturally the wave returns closer and closer to the transmitter as shown by the rays R_2 and R_3 . If the angle of incidence is now made significantly less than that of ray R_3 , the ray will be very close to the normal to be returned to the Earth. If the angle of incidence is reduced further, the radio waves penetrate through the layer as shown by the rays R_4 and R_5 . For a particular angle of incidence, the distance between the point of transmission and the point of reception is minimum. The minimum distance between the transmitter and the ray like R_3 which strikes the Earth is called as the skip distance.

As we move away from the transmitter, the ground wave becomes lesser and lesser significant. A stage comes when there is no reception due to the ground waves. This point lies somewhere in the skip distance. The region between the point where there is no reception of ground waves and the point where the sky wave is received first is known as skip zone. In the *skip zone*, there is no reception at all.

10.2 Modulation

In radio broadcasting, it is necessary to send audio frequency signal (eg. music, speech etc.) from a broadcasting station over great distances to a receiver. The music, speech etc., are converted into audio signals using a microphone. The energy of a wave increases with frequency. So, the audio frequency (20 – 20000 Hz) is not having large amount of energy and cannot be sent over long distances. The radiation of electrical energy is practicable only at high frequencies e.g. above 20 kHz. The high frequency signals can be sent through thousands of kilometres with comparatively small power.

Therefore, if audio signal is to be transmitted properly, the audio signal must be superimposed on high frequency wave called carrier. The resultant waves are known as modulated waves and this process is called as modulation. This high frequency wave (Radio frequency wave) is transmitted in space through antenna. At the receiver end, the audio signal is extracted from the modulated wave by the process called demodulation. The audio signal is then amplified and reproduced into sound by the loud speaker.

A high frequency radio wave is used to carry the audio signal. On adding the audio signal to carrier, any one of the characteristics namely amplitude or frequency or phase of the carrier wave is changed in accordance with the intensity of the audio signal. This process is known as modulation and may be defined as the process of changing amplitude or frequency or phase of the carrier wave in accordance with the intensity of the signal. Some of the modulation process namely, (i) amplitude modulation, (ii) frequency modulation and (iii) phase modulation are discussed.

10.2.1 Amplitude modulation (AM)

When the amplitude of high frequency carrier wave is changed in accordance with the intensity of the signal, the process is called *amplitude modulation*.

In the amplitude modulation, only the amplitude of the carrier wave is changed. The frequency and the phase of the carrier wave remains constant. Fig 10.5 shows the principle of amplitude modulation.

Fig 10.5a shows the audio electrical signal of frequency f_s . Fig 10.5b shows a carrier wave of constant amplitude with frequency f_c . Fig 10.5c is the amplitude modulated wave. It is to be noted that the amplitudes of both positive and negative half cycles of carrier wave are changed in accordance with the signal. Thus the amplitude of the modulated wave possesses the frequency of the audio signal wave.

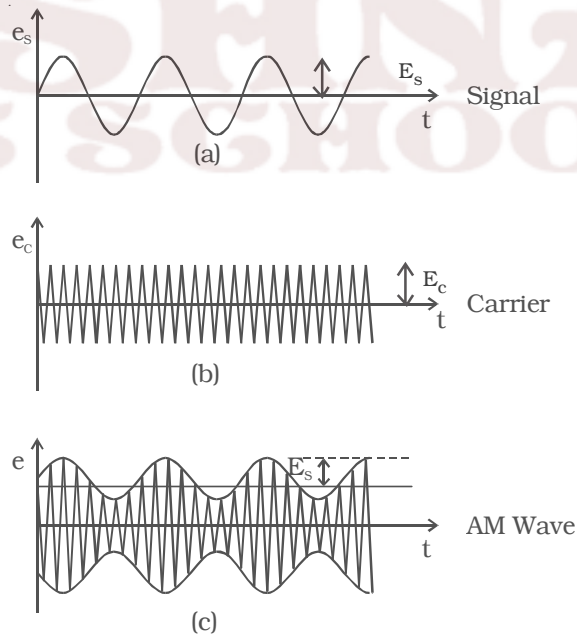


Fig 10.5 Amplitude modulation

Modulation factor

An important term in amplitude modulation is modulation factor which describes the extent to which the amplitude of the carrier wave is changed by the audio signal. It is defined as the ratio of the change of amplitude in carrier wave after modulation to the amplitude of the unmodulated carrier wave.

$$\text{i.e. modulation factor, } m = \frac{\text{Amplitude change of carrier wave after modulation}}{\text{Amplitude of carrier wave before modulation}}$$

$$m = \frac{\text{Signal amplitude}}{\text{Carrier amplitude}}$$

Modulation factor determines the strength and quality of the transmitted signal. When the modulation factor $m < 1$, the amount of carrier amplitude variation is small (Fig 10.6a). Consequently, the audio signal being transmitted will not be very strong. When the modulation factor $m > 1$, distortion is produced in the transmitted wave as shown in Fig 10.6 b. Hence, the signal wave is not exactly reproduced. For effective modulation, the degree of modulation should never exceed 100 %.

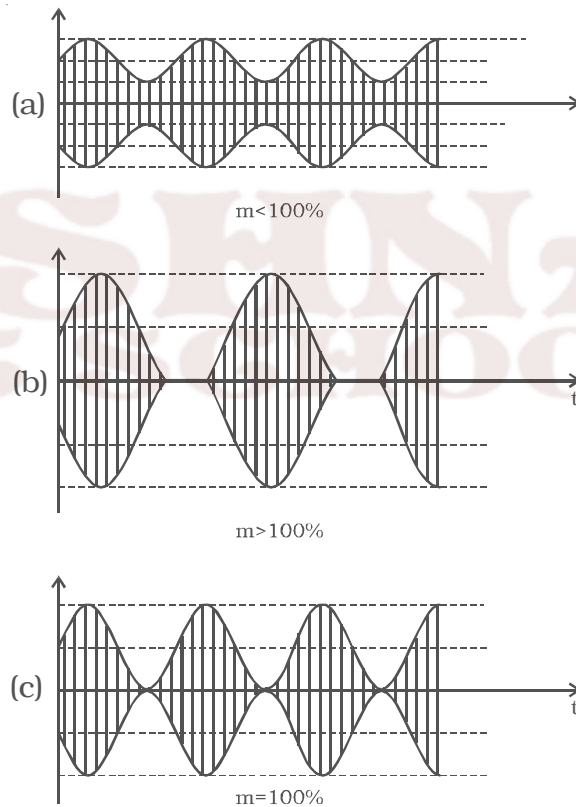


Fig 10.6 Amplitude modulated waves for different modulation factors

Analysis of amplitude modulated wave

A carrier wave may be represented as,

$$e_c = E_c \cos \omega_c t \quad \dots (1)$$

where e_c , E_c and ω_c represent the instantaneous voltage, amplitude and angular frequency of the carrier wave respectively.

In amplitude modulation, the amplitude E_c of the carrier wave is varied in accordance with the intensity of the audio signal as shown in Fig 10.5. The modulating signal may be represented as,

$$e_s = E_s \cos \omega_s t \quad \dots (2)$$

where e_s , E_s and ω_s represent instantaneous voltage, amplitude and angular frequency of the signal respectively.

Amplitude modulated wave is obtained by varying E_c of equation (1) in accordance with E_s . Thus, amplitude modulated wave is,

$$e = (E_c + E_s \cos \omega_s t) \cos \omega_c t$$

$$e = E_c \left[1 + \left(\frac{E_s}{E_c} \right) \cos \omega_s t \right] \cos \omega_c t = E_c [1 + m \cos \omega_s t] \cos \omega_c t$$

where m is the modulation factor which is equal to $\frac{E_s}{E_c}$.

$$\therefore e = E_c \cos \omega_c t + m E_c \cos \omega_c t \cdot \cos \omega_s t \quad \dots (3)$$

$$= E_c \cos \omega_c t + \frac{m E_c}{2} [2 \cos \omega_c t \cos \omega_s t]$$

$$= E_c \cos \omega_c t + \frac{m E_c}{2} [\cos (\omega_c + \omega_s) t + \cos (\omega_c - \omega_s) t]$$

$$= E_c \cos \omega_c t + \frac{m E_c}{2} \cos (\omega_c + \omega_s) t + \frac{m E_c}{2} \cos (\omega_c - \omega_s) t \quad \dots (4)$$

This expression shows that the modulated wave contains three components:

(i) $E_c \cos \omega_c t$: This component is same as the carrier wave.

(ii) $\frac{m E_c}{2} \cos (\omega_c + \omega_s) t$: This component has a frequency greater than that of the carrier and is called as the Upper Side Band (USB).

(iii) $\frac{m E_c}{2} \cos (\omega_c - \omega_s) t$: This component has a frequency lesser than that of the carrier and is called as the Lower Side Band (LSB).

Frequency spectrum

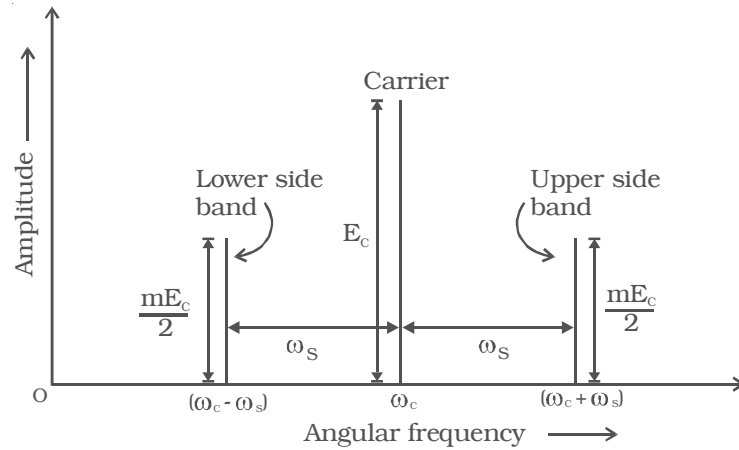


Fig 10.7 Plot of frequency spectrum of amplitude modulated voltage.

The lower side band term and upper side band term are located in the frequency spectrum on either side of the carrier at a frequency interval of ω_s as shown in Fig 10.7. The magnitude of both the upper and lower side bands is $\frac{m}{2}$ times the carrier amplitude E_c . If the modulation factor m is equal to unity, then each side band has amplitude equal to half of the carrier amplitude.

Bandwidth

In an AM wave, the bandwidth is from $(\omega_c - \omega_s)$ to $(\omega_c + \omega_s)$ i.e twice the signal frequency. In the preceding section, it is assumed that the modulating signal is composed of one frequency component only. However, in a broadcasting station, the modulating signal is the human voice or music which contains waves with a frequency range of 300 – 3000 Hz.

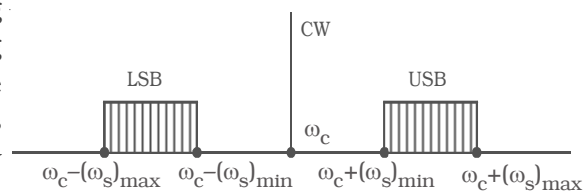


Fig 10.8 Channel width

Each of these waves has its own side bands. The upper side band (USB), in fact, contains all sum components of the signal and carrier frequency whereas lower side band (LSB) contains the difference components, as shown in Fig 10.8.

The channel width is given by the difference between extreme frequencies i.e. between maximum frequency of USB and minimum frequency of LSB.

$$\begin{aligned}\therefore \text{Channel width} &= 2 \times \text{maximum frequency of the modulating signal} \\ &= 2 \times (f_s)_{\max}\end{aligned}$$

Advantages

- (i) Easy transmission and reception
- (ii) Lesser bandwidth requirements
- (iii) Low cost

Limitations

(i) *Noisy reception* : In an AM wave, the signal appears in the amplitude variations of the carrier. Practically, all the natural and man made noises consists of electrical amplitude disturbances. As a radio receiver cannot distinguish between amplitude variation that represent noise and those that contain the desired signal, the reception is generally noisy.

(ii) *Low efficiency* : In AM, useful power is available in the side bands, since they contain signals. The sideband power for an AM wave is low. Hence the efficiency of AM is low.

(iii) *Small operating range* : Due to low efficiency of amplitude modulation, transmitters employing this method have a small operating range i.e. the messages cannot be transmitted over long distances.

10.2.2 Frequency modulation (FM)

When the frequency of carrier wave is changed in accordance with the intensity of the signal, the process is called frequency modulation.

In frequency modulation, the amplitude and phase of the carrier wave remains constant. Only, the frequency of the carrier wave is changed in accordance with the signal.

The frequency variation of the carrier wave depends upon the instantaneous amplitude of the signal as shown in Fig 10.9a. When the signal voltage is zero at A,C,E and G, the carrier frequency is unchanged. When the signal approaches its positive peaks at B and F, the carrier frequency is increased to maximum as shown by closely spaced cycles in Fig 10.9c. But during the negative peak of signal as at D, the carrier frequency is reduced to minimum as shown by widely

spaced cycles in Fig. 10.9c. The louder signal causes greater frequency change in modulated carrier as indicated by increased bunching and spreading of the waves as compared with relatively weaker signal.

The frequency of an FM transmitter without signal input is called the *resting frequency or centre frequency* (f_o) and this is the allotted frequency of the transmitter. When the signal is applied, the carrier frequency deviates up and down from its resting value f_o .

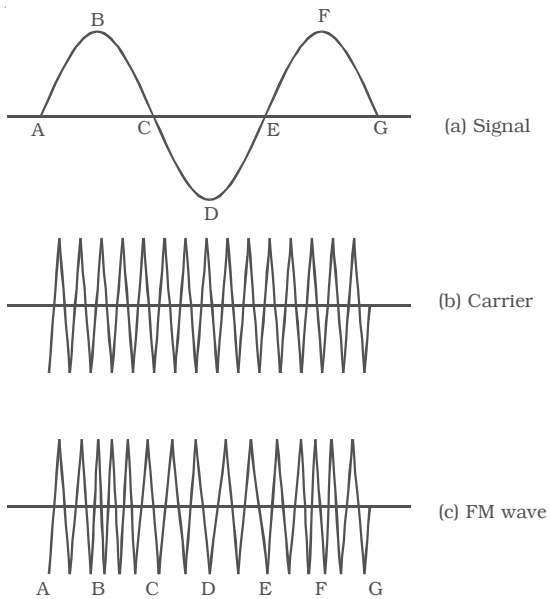


Fig 10.9 Frequency Modulation

The change or shift either above or below the resting frequency is called frequency deviation (Δf). The total variation in frequency from the lowest to the highest is called *carrier swing* (CS). Hence,

$$\begin{aligned} \text{Carrier swing} &= 2 \times \text{frequency deviation} \\ &= 2 \times \Delta f \end{aligned}$$

For the purposes of FM broadcasts, it has been internationally agreed to restrict maximum deviation to 75 kHz on each side of the centre frequency for sounds of maximum loudness.

Advantages

- (i) It gives noiseless reception. Noise is a form of amplitude variation and a FM receiver will reject such noise signals.
- (ii) The operating range is quite large.
- (iii) The efficiency of transmission is very high.

Disadvantages

- (i) A much wider channel is required by FM.
- (ii) FM transmitting and receiving equipments tends to be more complex.

10.2.3 Phase modulation (PM)

In phase modulation, the phase of the carrier wave is varied in accordance with the amplitude of the modulating signal and the rate of variation is proportional to the signal frequency. The waveform of the phase modulated wave is similar to that of FM wave. The phase modulation, generally uses a smaller bandwidth than FM. In other words, more information can be sent in a given bandwidth in phase modulation. Therefore, phase modulation facilitates highest transmission speeds on a given bandwidth. In phase modulation also, there is a frequency shift in the carrier wave frequency. The frequency shift depends on (i) amplitude of the modulating signal and (ii) the frequency of the signal. One great advantage of the phase modulation lies in the fact that the FM signal produced from PM signal is very stable. Also, the centre frequency called resting frequency is extremely stable.

10.3 Antennas

In radio, TV, Radar and mobile phone communication systems, an electromagnetic wave travels from the transmitter to the receiver through space and antennas are required at both ends for the purpose of coupling the transmitter and the receiver to the space link.

An antenna is a long conductor (wire and rod) that acts as a conversion device. It converts an electrical signal into electromagnetic energy when used as a transmitting antenna. In receiving antenna, the electromagnetic energy is converted into an electrical signal.

An antenna is characterised by an important parameter, directivity. Directivity is the ability of the antenna to concentrate the electromagnetic waves in the most desired directions (during transmission) or to have maximum reception from most preferred directions (during reception).

10.4 Radio transmission and reception

10.4.1 Radio transmission

After modulation, the radio waves are transmitted over long distances with the help of electronic circuits called transmitters. The simplest form of transmitter consists of an oscillator, generating a high frequency wave connected to an antenna.

In this section, we shall discuss the amplitude and frequency modulated transmitters.

Amplitude modulated (AM) transmitter

Fig 10.10 gives the block diagram of amplitude modulated radio transmitter. It consists of two sections (i) Audio frequency (AF) section and (ii) Radio frequency (RF) section.

AF section

The AF section of the transmitter generates the modulating wave (signal). The conversion of sound energy into electrical energy is performed by the microphone.

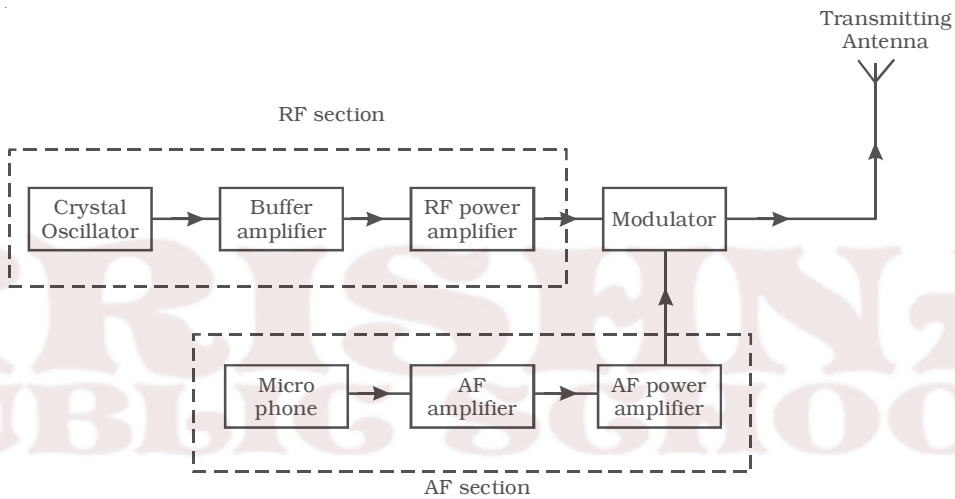


Fig 10.10 Functional block diagram of amplitude modulated radio transmitter

The electrical energy available from the microphone is very low. Hence, it is amplified through an amplifier. The output from the AF amplifier is fed to the AF power amplifier. The power amplifier provides the required audio frequency power. The output of the AF power amplifier is given to the modulator. A modulator is an electronic circuit with transistor and passive components, which performs the process of modulation.

RF section

In the RF section, the high frequency carrier wave is generated by a crystal controlled oscillator. The output of the crystal controlled

oscillator is power amplified by RF power amplifier. The buffer* isolates the RF power amplifier from the oscillator. This arrangement keeps the frequency of the crystal controlled oscillator as a constant. In the modulator the RF wave and modulating AF signal are mixed to produce the amplitude modulated wave. The output of this section is fed to the antenna for transmission.

Frequency modulated (FM) transmitter

Frequency modulated systems are operated usually at a frequency above 40 MHz. Frequency modulated broadcasting is done in television sound, mobile radio etc. The functional block diagram of a FM transmitter employing phase modulation is shown in Fig 10.11. The phase modulation is essentially a frequency modulation.

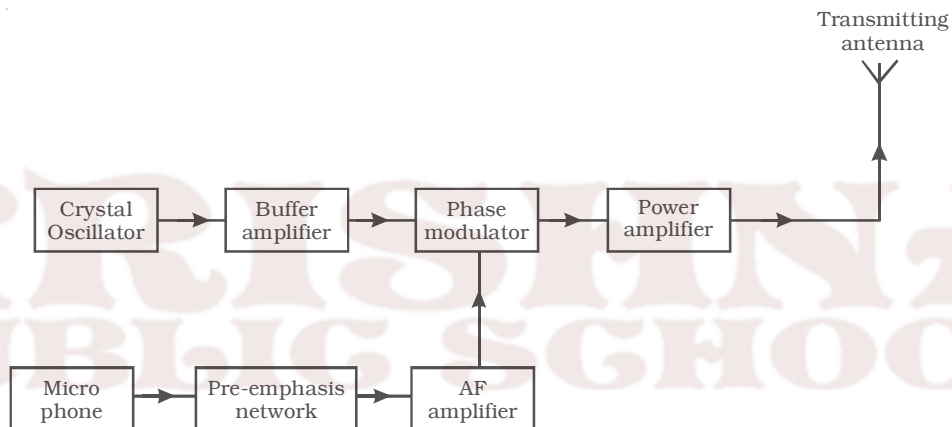


Fig 10.11 FM transmitter

It consists of a crystal oscillator, which produces the carrier wave and the output of this is fed into the phase modulator. The buffer is a low frequency amplifier which isolates the crystal oscillator from the phase modulator.

The modulating signal is produced from a microphone. Since this AF modulating signal has uneven power, it is fed into a network called pre-emphasis network, where all the frequencies in the modulating signal are made to have equal power. The output of the pre-emphasis network is then amplified and sent for phase modulation. The

* Buffer is an electronic circuit, used for the purpose of providing isolation between two other stages. This also prevents the impedance change between the two stages.

modulated output is then power amplified using a power amplifier and then fed into the transmitting antenna for transmission.

10.4.2 Radio reception

A radio receiver has the function of selecting the desired signal from all other unwanted signals, amplifying, demodulating it and finally producing it in the desired manner.

A simple (or) straight radio receiver

The functional block diagram of a simple radio receiver is shown in Fig 10.12. The receiving antenna receives the radiowaves from different broadcasting stations. The desired radiowave is selected by the radio frequency amplifier, which employs a tuned parallel circuit. The tuned RF amplifier amplifies this selected radiowave. The amplified radiowave is fed to the detector circuit which consists of a PN diode. This circuit extracts the audio signal from the radiowave. The output of the detector is the audio signal, which is amplified by one or more stages of audio amplification. The amplified audio signal is given to the loud speaker for sound reproduction.

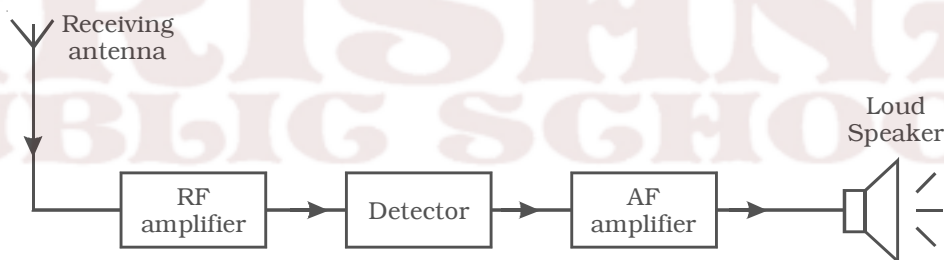


Fig 10.12 Simple radio receiver

Disadvantages

Simple radio receiver circuit has

- (i) poor sensitivity* and
- (ii) poor selectivity**

* Sensitivity : the ability to amplify the weak signals

** Selectivity : the ability to select a particular wanted signal only and rejecting the unwanted signals

Superheterodyne AM receiver

The shortcomings of straight radio receiver were overcome by the invention of superheterodyne receiver. All the modern receivers utilise the superheterodyne circuit.

The functional block diagram of AM receiving system of superheterodyne type is shown in Fig 10.13.

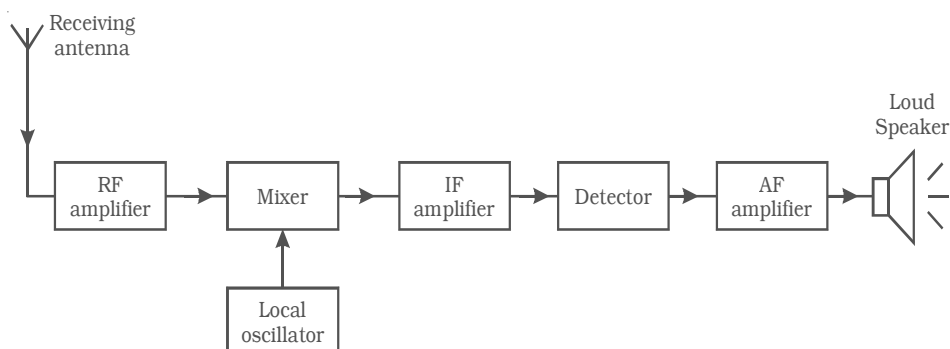


Fig 10.13 Superhet AM receiver

(i) RF amplifier

The RF amplifier uses a tuned parallel circuit. The radiowaves from various broadcasting stations are intercepted by the receiving antenna and are coupled to this stage. This stage selects the desired radiowave and enhances the strength of the wave to the desired level.

(ii) Mixer and local oscillator

The amplified output of RF amplifier is fed to the mixer stage, where it is combined with the output of a local oscillator. The two frequencies beat together and produce an intermediate frequency (IF). The intermediate frequency is the difference between oscillator frequency and radio frequency. The output of this section is always equal to the intermediate frequency 455 kHz.

For example, if 600 kHz station is tuned, then local oscillator will produce a frequency of 1055 kHz and consequently the output from the mixer will have frequency of 455 kHz. By achieving this fixed intermediate frequency, the amplifier circuit in such receivers can be made to operate with maximum stability, selectivity and sensitivity.

(iii) IF amplifier

The output of the mixer circuit is fed to the tuned IF amplifier. This amplifier is tuned to one frequency (i.e. 455 KHz) and is amplified.

(iv) Detector

The output from the IF amplifier is coupled with input of a detector. The audio signals are extracted from the IF output. Usually a diode detector circuit is used because of its low distortion and excellent audio fidelity (reproducing ability).

(v) AF amplifier

The detected AF signal is usually weak and so it is further amplified by the AF amplifier. Then, the output signal from the amplifier is fed to the loud speaker, which converts the audio signal into sound waves corresponding to the original sound at the broadcasting station.

FM Superheterodyne receiver

An FM receiver is a superheterodyne type like a typical AM receiver. The functional block diagram of an FM receiver is shown in Fig 10.14.

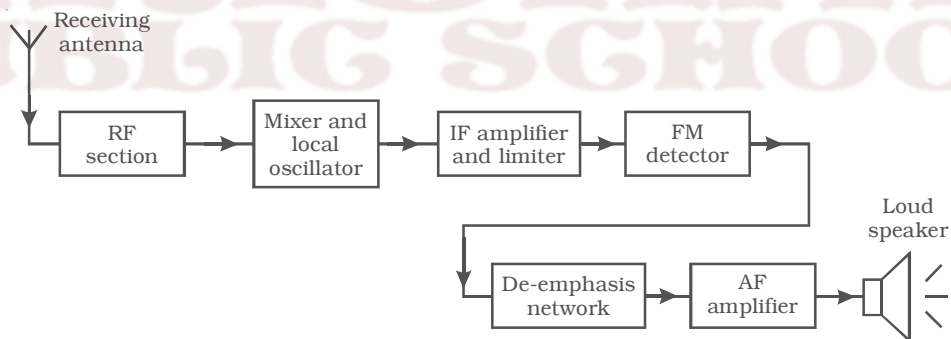


Fig 10.14 FM receiver

The RF section selects the incoming modulated signals and is amplified. It is then fed into the mixer and local oscillator. Here the frequency of the modulated signal is changed to intermediate frequency. For FM receivers, this IF is 10.7 MHz. The intermediate frequency wave is amplified using IF amplifier and then its amplitude is

maintained constant using a limiter*. The output of this section is applied to the FM detector which demodulates the modulated wave. The AF signal from the FM detector is then passed on through a de-emphasis network, where the various frequencies attain their original power distribution. Finally it is fed into the loud speaker after performing AF amplification.

10.5 Television

Television is one of the marvels that has been achieved in the field of electronics. “Television” literally means “*seeing at a distance.*” The principle of television broadcasting is essentially the same as that of sound broadcasting. In sound broadcasting, sound waves are converted into equivalent electrical signals using microphone, which are modulated and transmitted. In picture broadcast (television), the bright and dark spots of a scene are systematically scanned to produce equivalent electrical signals. These picture signals are modulated and are then transmitted. At the receiving end, the picture signals are extracted from the modulated wave. These are made to reproduce the bright and dark spots of the original scene on the fluorescent screen of a cathode ray tube called viewing tube. The sound associated with the scene is transmitted simultaneously to provide a complete picture and sound reproduction at the receiver. Although the end result required is a motion picture, television is basically a system for reproducing a still picture. Many of these are shown one after the other in rapid sequence during each second, to give the illusion of motion. Therefore, the first requirement of the television system is that it should be capable of transmitting and receiving a simple still picture. Usually sound signals are frequency modulated, while picture signals are amplitude modulated.

10.5.1 Vidicon camera tube

Vidicon camera is a television camera which converts the light energy into electrical energy. It functions on the principle of photo conductivity, where the resistance of target material decreases when exposed to light.

**A limiter is a device that prevents a signal voltage from exceeding a certain peak value. Limiters are used in FM receivers for the purpose of reducing the response to variations in signal amplitude.*

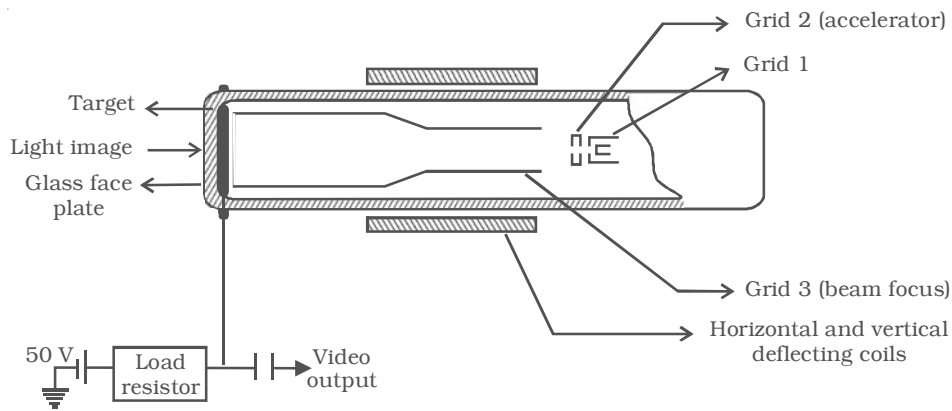


Fig 10.15 Vidicon camera tube

Construction

The Vidicon consists of a glass envelope with an optically flat face plate (Fig 10.15). A photosensitive, target plate is available on the inner side of the face plate. The target plate has two layers. To the front, facing the face plate, is a thin layer of tin oxide. This is transparent to light but electrically conductive. The other side of the target plate is coated with a semiconductor, photosensitive antimony trisulphide. The tin oxide layer is connected to a power supply of 50V.

Grid-1 is the electron gun, consisting a cathode and a control grid. The emitted electrons are accelerated by Grid-2. The accelerated electrons are focussed on the photo conductive layer by Grid-3. Vertical and Horizontal deflecting coils, placed around the tube are used to deflect the electron beam for scanning the target.

Working

The light from a scene is focussed on the target. Light passes through the face plate and tin oxide, incident on the photo conductive layer. Due to the variations in the light intensity of the scene, the resistance of the photo conductive layer varies. The emitted electrons from antimony trisulphide reach the positive tin oxide layer. So, each point on the photo conductive layer acquires positive charge. Hence, a charge image that corresponds to the incident optical image is produced. As the electron beam from the gun is incident on the charge image, drop in voltage takes place. As a result, a varying current is produced. This current produces the video-signal output of the camera.

10.5.2 Scanning and synchronising

A still picture is fundamentally an arrangement of many dark and light areas. Each small area of light or shade is called a picture element. All the elements contain the visual information in the scene. If they are transmitted and reproduced in the same degree of light or shade as original and in proper position, the picture will be reproduced.

In order to produce video signal for all the elements in the picture, it is scanned by the electron beam, one element at a time, in sequential order. The scanning is done in the same way as a written page is read to cover all the words in one line and all lines on the page. Hence, scanning is the process by which an electron beam spot is made to move across a rectangular area, so as to cover it completely. This rectangular area may be the target surface in a television camera or the screen of a picture tube in a television receiver.

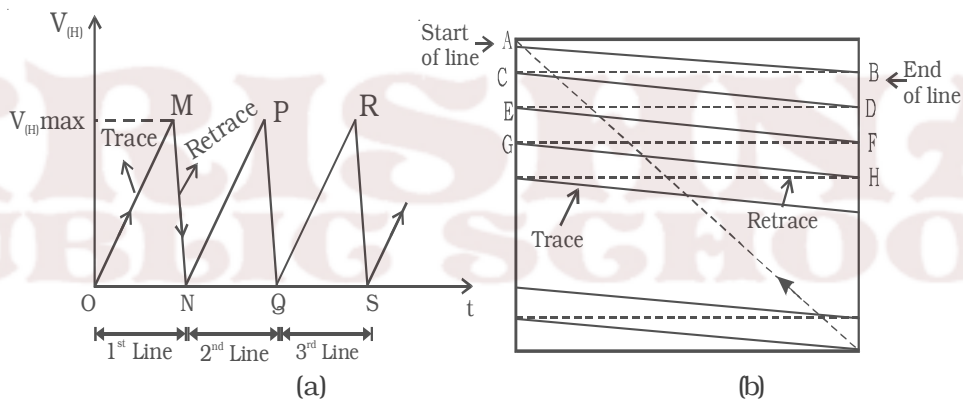


Fig 10.16 Simple scanning process

The scene is scanned rapidly both in the horizontal and vertical directions simultaneously. This provides sufficient number of complete pictures or frames per second to give the illusion of continuous motion. In most of the television systems, the frame repetition rate (scanning frequency) is 25 per second.

For scanning the picture elements, saw tooth potentials can be used. Saw tooth potentials are produced by using a unijunction transistor and a R-C network. Saw tooth potentials are applied to horizontal and vertical deflector plates in a TV camera. When the saw

tooth potential is applied to the horizontal plates called line synchronising pulse, the electron beam at A travels along a slanting line AB by the voltage variation of OM and reaches the point B (Fig 10.16a and b). From B, the scanning spot travels along a line BC by the voltage variation MN. In order that no picture should be scanned during the return journey (i.e. the beam from the right horizontal end to the beginning of the next line), a blanking pulse, which is a high negative potential, is applied to the control grid of electron gun during the duration of the return journey. This prevents the emission of electrons from electron gun. Then the electron beam starts to scan the next line and the process gets repeated till the whole picture is scanned. On reaching the right bottom corner, the scanning spot quickly moves up to the top left corner by the application of saw tooth potential to the vertical deflector plates, called frame synchronising pulse. Thus for scanning the picture, the three synchronising pulses are used. These synchronising pulses along with the output of the TV camera are modulated on an ultra high frequency carrier and transmitted. The accompanying sound is frequency modulated and transmitted via the same antenna.

Interlaced scanning

In India, the frame repetition rate has been standardised at 25 frames per second. This repetition rate is enough to cause an illusion of continuity. But, the brightness of one frame blends (mix) smoothly into the next, through this time when the screen is blanked between successive frames. This results in definite flicker of light, that is very annoying to the observer, when the screen becomes alternatively bright and dark. To eliminate this flicker, each frame is scanned twice.

In this scanning, the total number lines are divided into two groups called fields. During the presentation of the first field, only the odd numbered lines are scanned, while during the second field all the even numbered lines are scanned. Half way along the bottom of the first field, the vertical retrace returns the scanning beam to the top of the image and completes the unfinished lines. (i.e) The remaining even numbered lines are then scanned during second field. This method of scanning is known as interlaced scanning. In the 625 line TV system, for successful interlaced scanning, the 625 lines of each frame or

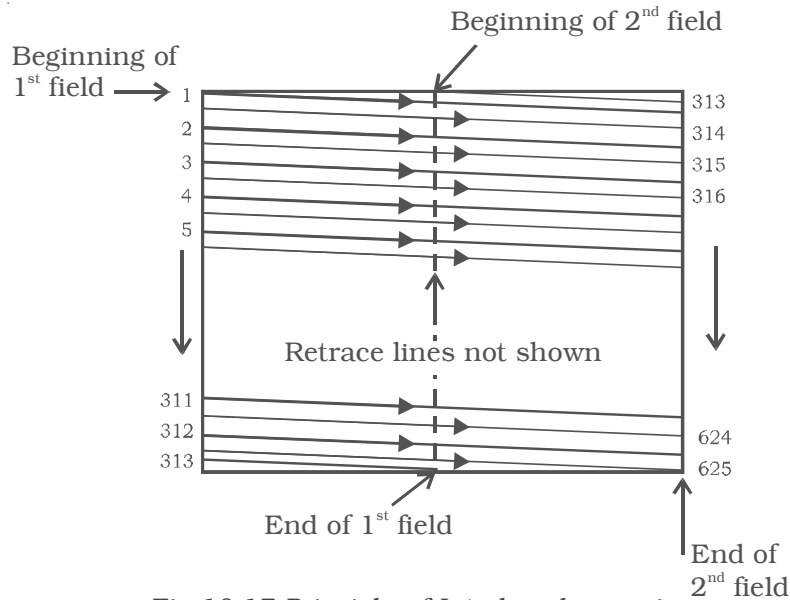


Fig 10.17 Principle of Interlaced scanning

picture are divided into sets of 312.5 lines and each set is scanned alternatively to cover the entire picture area. The principle of interlaced scanning is shown in Fig 10.17.

Hence, with the interlaced scanning the flicker effect is eliminated without increasing the speed of scanning, which in turn does not need any increase in channel bandwidth.

Horizontal and vertical scanning frequencies

The movement of electron beam spot from left to right and back, so as to start a new line in the same direction is termed as horizontal scanning. The horizontal scanning frequency is defined as the number of lines scanned per second. In a 625 line system, transmitting 25 frames per second, the horizontal frequency is $625 \times 25 = 15,625$ Hz.

Consequently, time taken to scan one line is $\frac{1}{15,625} = 64 \mu\text{s}$.

Vertical scanning is the movement of the electron beam spot in the vertical direction. One frame consists of two fields, resulting into 50 fields per second with a vertical field scan time of $\frac{1}{50} = 20$ ms.

10.6 Monochrome picture tube

The picture tube is a special form of cathode ray tube, the face plate of which serves as a screen of the television receiver. The various parts of a monochrome picture tube are shown in Fig 10.18. The cathode ray tube (CRT) is housed in a bell-shaped glass enclosure. A filament heats a cathode that emits electrons. The negatively charged electrons are attracted and accelerated by positive-bias voltages on the elements in an electron gun assembly. The electron gun also focuses the electrons into narrow beam.

A control grid that is made negative with respect to the cathode, controls the intensity of the electron beam and brightness of the spot it makes. The beam is accelerated towards the screen by a very high voltage applied to an internal metallic coating called aquadag. The face or front of the picture tube is coated internally with a phosphor, that glows and produces white light, when it is struck by the electron beam.

Around the neck of the picture tube is a structure of magnetic coils called the deflecting yoke. The horizontal and vertical current linear saw tooth waves generated by the sweep and synchronising circuits are applied to the yoke coils. This produces the magnetic field inside the tube that influence the position of the electron beam. When the electrons flow, a magnetic field is produced around the conductor through which the current flows. In a CRT, the electron beam is moved or deflected by the magnetic field produced by the deflection coils in the yoke. Thus the electron beam is swept across the face of the picture tube.

As the beam is being swept across the face of the tube to trace out the scene, the intensity of the electron beam is varied by the luminance or Y signal. The Y signal is applied to the cathode or in some cases to the control grid. The control grid is an element in the electron gun, that is negatively biased with respect to the cathode. By varying the grid voltage, the beam can be made weaker or stronger, thereby varying the intensity of the light spot produced by the beam, when it strikes the phosphor. Any shade of grey from white to black can be reproduced.

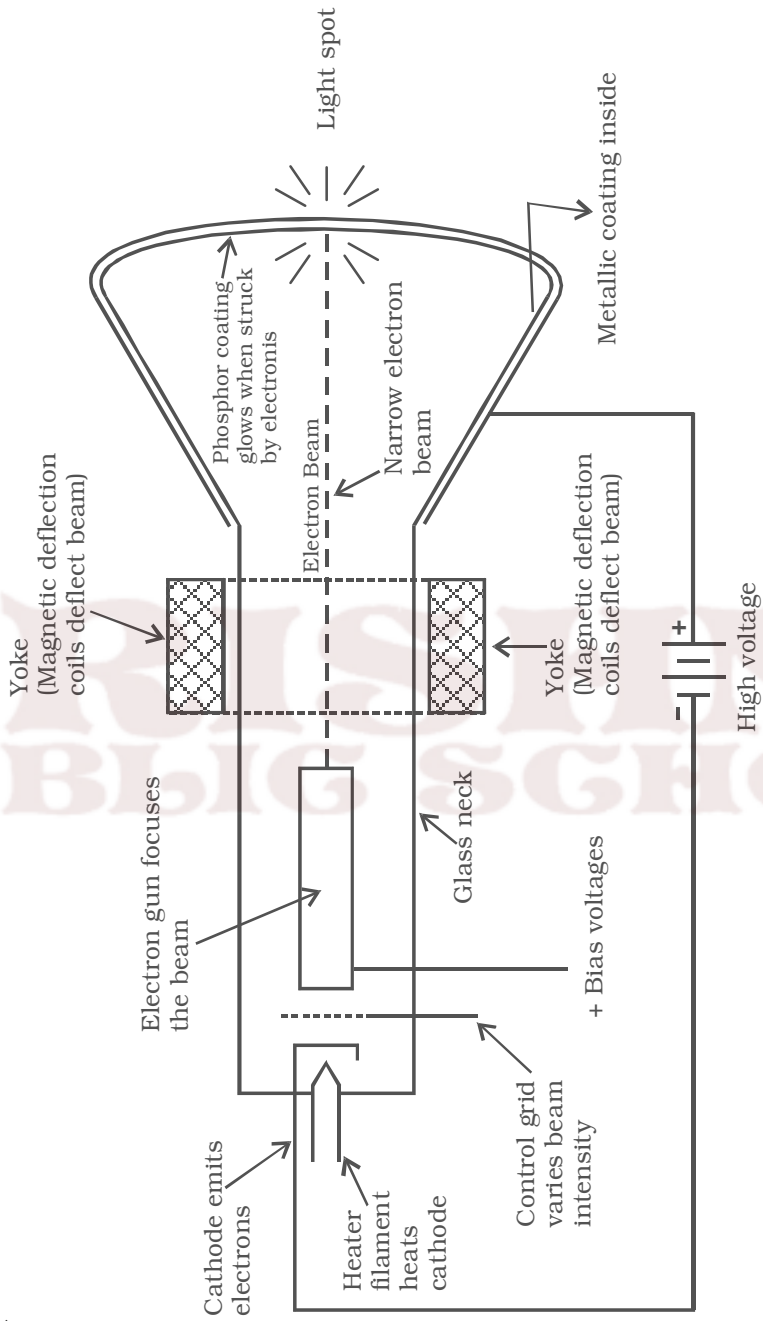


Fig 10.18 Monochrome picture tube

10.7 Monochrome TV transmission

An over simplified block diagram of a monochrome TV transmitter is shown in Fig 10.19. The functional block diagram can be broadly divided into two sections, viz. an amplitude modulated transmitter and a frequency modulated transmitter. Former is used for video modulation, whereas latter is used for audio modulation.

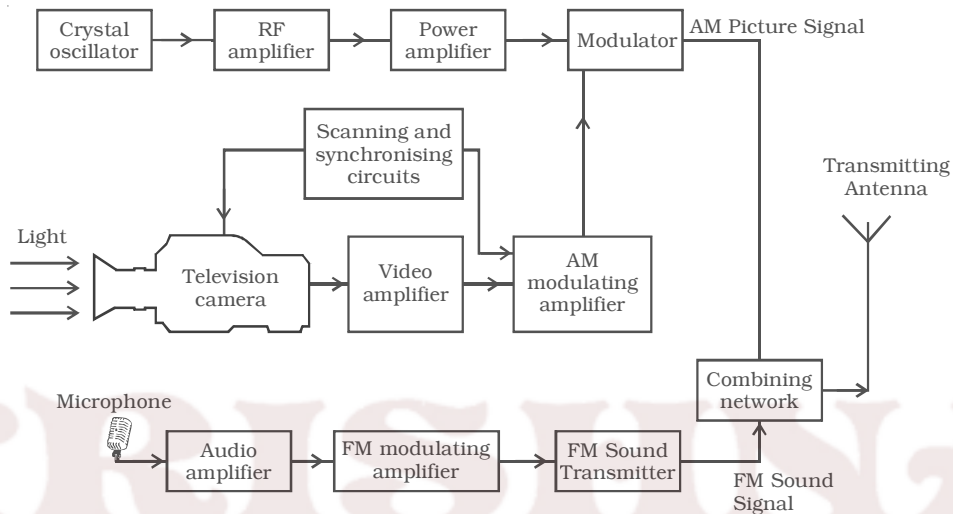


Fig 10.19 Functional block diagram of a monochrome television transmitter

The synchronising and scanning circuits produce sets of pulses for providing synchronising pulses for proper functioning of the TV system. This timing unit contains number of wave generating and wave shaping circuits. The repetition rate of its various output pulse trains is controlled by a frequency stabilised master oscillator.

The output signal of a camera tube corresponding to the image to be televised is amplified through a number of video amplifier stages.

The image signals together with the synchronising and blanking pulses are raised to a level suitable for modulating the RF carrier wave generated in the RF channel. The allotted picture carrier frequency is generated by the crystal controlled oscillator. The continuous wave output is given large amplification before feeding to the power amplifier. In the modulator, its amplitude is made to vary in accordance with the modulating signal received from the modulating amplifier.

The microphone converts the sound associated with the picture being televised into proportionate electrical signal. The audio signal from the microphone after amplification is frequency modulated, employing the assigned carrier frequency. The output of the sound FM transmitter is finally combined with the AM picture transmitter output, through a combining network and fed to a common antenna for radiation of energy in the form of electromagnetic waves.

10.8 Monochrome TV receiver

The simplified block diagram of a black and white TV receiver is shown in Fig 10.20. The receiving antenna intercepts radiated RF signals and the tuner selects the desired channel frequency band. The antenna provides RF picture and sound signals for the RF amplifier stage. The RF amplifier stage is then coupled into the mixer stage. The mixture is connected to the local oscillator. The RF audio and video signals are heterodyned into intermediate frequency by the mixer and local oscillator. The RF amplifier, mixer and local oscillator stages are combinely called as the RF tuner. The output signal from the tuner circuit is amplified by using a common IF amplifier. Then the video and audio components are separated by a detector. The sound signals are detected from FM waves, amplified and then fed into the loud speaker, which reproduce the sound.

The video components are first passed into a detector which separates the picture signal from the synchronising pulses. The line synchronising pulses and the frame synchronising pulses are fed into the horizontal and vertical deflector plates of the picture tube. The blanking pulses are given to the control grid of the electron gun of the picture tube. The picture signals are applied to the filament of the electron gun of the picture tube. According to the variations of potential in the picture, electrons are emitted from the electron gun. Thus, the intensity of the fluorescent screen of the picture tube is in accordance with the variation of potential in the picture and the picture is reproduced.

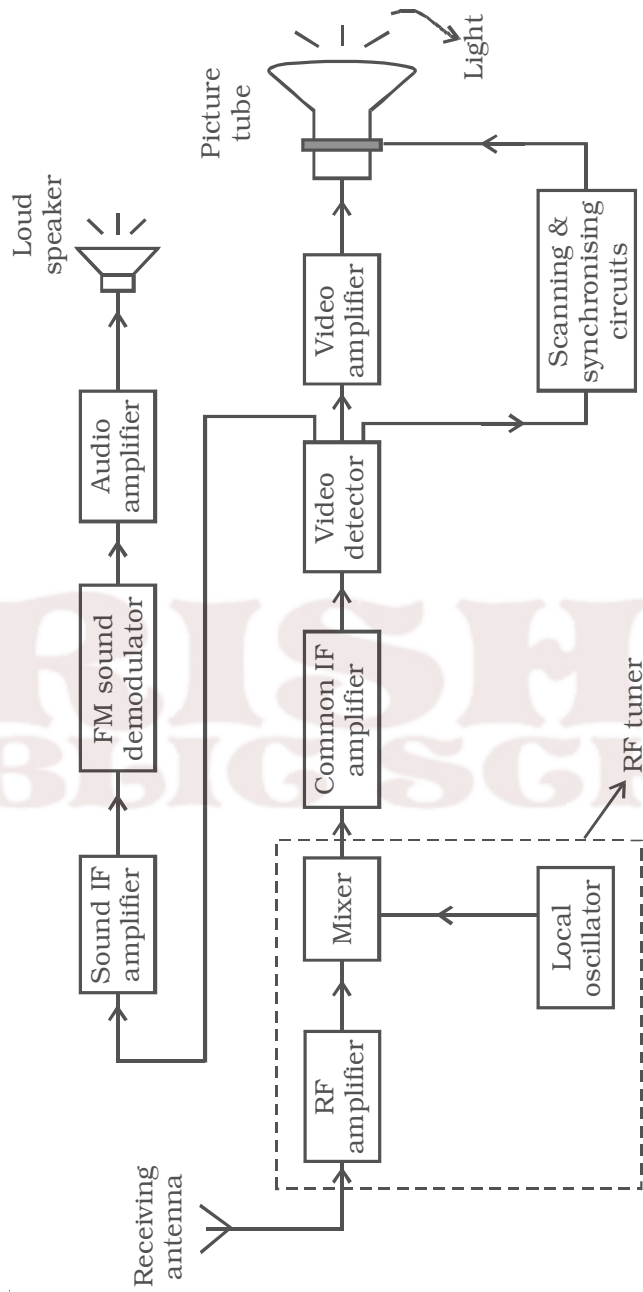


Fig 10.20 Elementary block diagram of a monochrome TV receiver

10.9 RADAR

The term RADAR is an acronym for RAdio Detection And Ranging. It is a system which uses radio waves to detect and to fix the position of targets at a distance.

10.9.1 Principle of radar

Radar works on the principle of 'radio echoes'. The transmitter in a radar, radiates the high power electrical pulses into space. When these pulses are incident on any distant target such as a mountain, ship or aircraft, they get scattered in all directions. The transmitter antenna receives a part of the scattered energy. This transmitter antenna also acts as receiving antenna for the receiving pulse. The pulse travels with the speed of light $3 \times 10^8 \text{ ms}^{-1}$. In other words, these pulses cover a distance of 300 metres for every micro second. Hence by measuring the time taken by the pulse to reach the target and back to the transmitter, the range or distance of the target can be easily determined. To locate the direction of the target, directional antennas are used.

10.9.2 Transmission and reception of radar

The block diagram of a simple radar system is shown in Fig 10.21. This block diagram indicates that the radar system consists of both the transmitting and the receiving system.

The transmitting system consists of a transmitter and a pulser. The receiving system consists of a receiver and an indicator. In most of the cases, a single antenna is used for both transmission and reception and this is achieved with the use of TR switch (Transmitter Receiver Switch). This switching arrangement is called as 'duplexer'. This connects the antenna to the transmitter during transmission and to the receiver during reception. Moreover, this switch isolates the sensitive receiver from the damaging effects of the high power transmitter.

The transmitter is essentially a high power magnetron oscillator which generates high power pulses. This transmitter is turned on and off with a periodic pulse from the pulser. Thus the transmitter generates periodic pulses of very short duration. These short pulses are fed to the antenna which radiates them into the space. The antenna is highly directional.

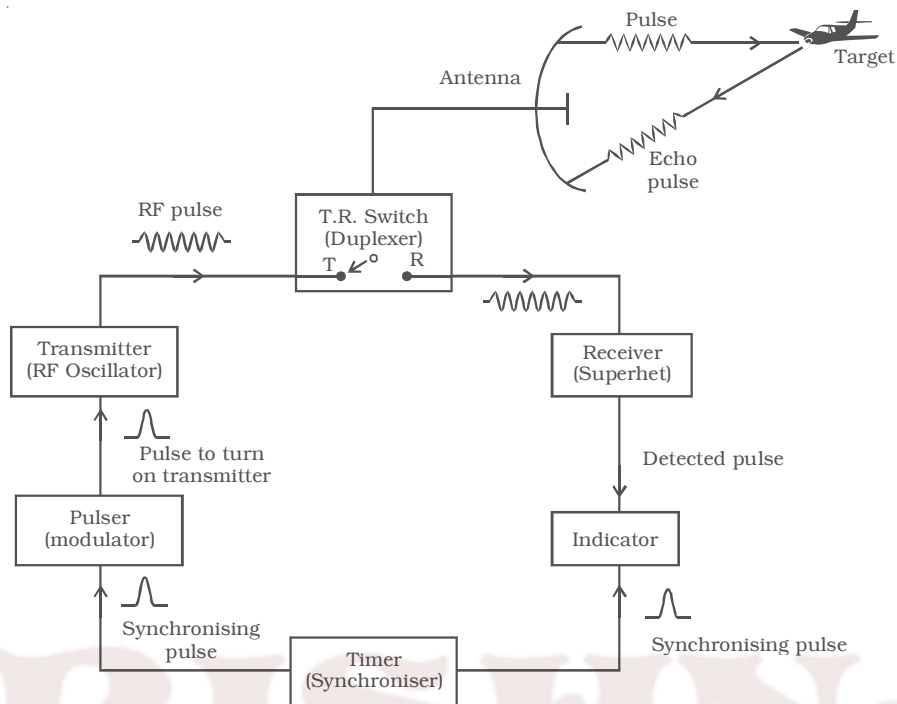


Fig 10.21 Block diagram of a radar system

If the transmitted pulse hits any target, a weak echo signal returns to the same antenna. But, now the TR switch puts the antenna in contact with the receiver. This echo signal is amplified and demodulated by the superhet receiver. The sensitivity of the receiver is very high. The detected output is sent to the indicator. The indicator is a cathode ray tube. The CRT displays the original transmitted pulse as well as the detected echo pulse along a horizontal base line. The synchronising pulse generated by the 'timer' is supplied to both transmitting and receiving systems. So, the indicator records the transmitted pulse as well as the returning pulse simultaneously. The returning echo pulse appears slightly displaced from the transmitted pulse and this displacement is a measure of the range of the target.

Some applications of radar

(i) Air and sea navigation is made entirely safe, with radar installations. High flying planes and ship at sea, can get detailed reports of mountains, ice bergs, rivers, lakes, shore lines etc., which they can avoid.

(ii) Radar systems are used for the safe landing of air crafts. On approaching the air field, the pilot is guided by signals from a radar set, so that it flies along the line of the runway and lands safely, whatever be the visibility.

(iii) Rain drops may reflect suitable radar signals and thus enable meteorologists to measure the distance of the clouds, with great accuracy for forecasting.

(iv) The pulses can be used for discovering the position of buried metals, oils and ores.

10.10 Analog communication and digital communication

Based on the type of signals transmitted, communication electronics can be classified as,

- (i) Analog communication and
- (ii) Digital communication

10.10.1 Analog communication

In analog communication, analog signals are used. An analog signal is a continuously varying voltage or current. Traditionally, in telephone and radio systems, the messages consisted of information conveyed by voice. The voice signal is an analog signal and so could take on any value within the overall range allowed. For example, if the telephone system were set up to handle voice signals, which ranged from 0 to 1 volt, the values transmitted at any instant could be 0.345 V, 0.179 V, and so on. This is called analog signal communication, because the signal can be any value within the range.

Shortcomings of analog communication

The greatest technical problem with an analog communication system is noise. Noise that comes from any part of the system – the modulator, the transmitter, the communication link, the receiver or the demodulator – can corrupt the signal.

Analog system do not offer opportunities for identifying corrupted signals.

10.10.2 Digital Communication

A digital communication system offers many advantages to the user, that cannot be achieved with an analog system. Digital communications system may make use of analog links and concepts.

A digital system is a more general case of a binary system. In binary system, only two signal values can exist. They are often called 0 and 1, but these names represent specific voltages.

The term data is commonly used in digital communication systems. Data is any form of information, that has been put into digital form, so that it can be handled by a digital system. The data itself is measured as bit. (bit is a contraction of the term 'binary digit')

The binary signals are easy to generate and process with digital circuits. These digital circuits are available in the IC form and can generate and process digital data at high speeds.

Advantages

- (i) The transmission quality is high and almost independent of the distance between the terminals.
- (ii) The capacity of the transmission system can be increased.
- (iii) The newer types of transmission media such as light beams in optical fibers and wave guides operating in the microwave frequency extensively use digital communication.

Disadvantages

- (i) A digital system requires larger bandwidth.
- (ii) It is very difficult to gradually change over from analog to digital transmission.

10.10.3 Modem

The name modem is the abbreviation of the term Modulator and Demodulator. As the name implies, both functions are included in a modem. A modem is used to convert digital signals into analog signals capable of being transmitted over telephone lines. At the receiving end of the system, modem is used to demodulate the analog signals and reconstruct the equivalent digital output. Modems are placed at both ends of the communication circuit as shown in Fig 10.22.

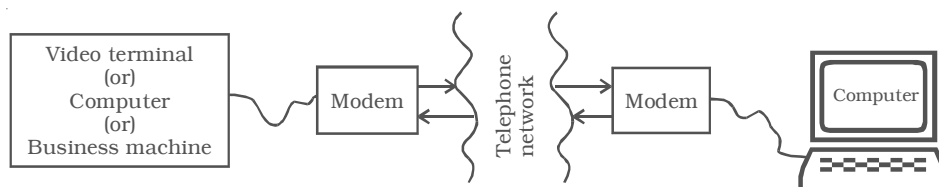


Fig 10.22 Communication circuit using modem

10.10.4 Fax (or) Facsimile

Fax (or) Facsimile, is an electronic system for transmitting graphical information by wire or radio. It is used to send printed material by scanning and converting it into electronic signals. These signals modulate a carrier to be transmitted over the telephone lines. Since modulation is involved, fax transmission can also take place by radio.

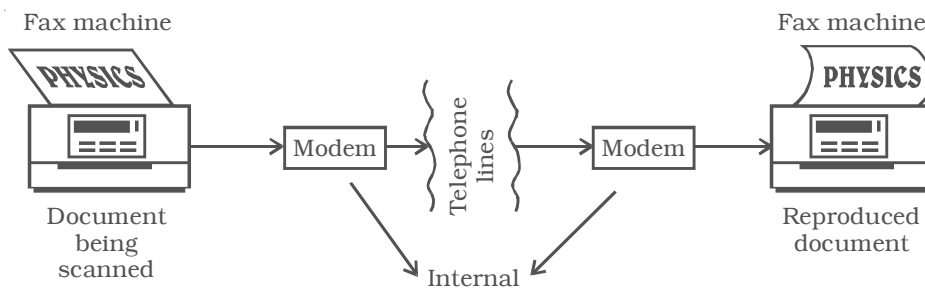


Fig 10.23 Components of a fax system

The components of a fax system are illustrated in Fig 10.23. Although facsimile is used to transmit pictures, it is not TV because it does not transmit sound messages or live scenes and motion. However, it does use scanning techniques that are in same way generally similar to those used in TV. A scanning process is used to break up a printed document into many horizontal scan lines which can be transmitted and reproduced serially.

10.10.5 Wire and Cable

The medium is the actual path for the electromagnetic energy of the link or channel of the communication system. Through the medium, the energy representing the data of the sender can reach the receiver. This path can take many forms : an electrical conductor such as wire, vacuum or optical fiber.

Copper is an inexpensive metal that is easily made into wire, Fortunately, it is also an excellent conductor of electricity. When separate wires are collected into bundles and are given special protective jacketing, the term 'cable' is often used.

The main types of wire and cable used in data communications are (a) Twisted pair (b) Multiconductor flat cable and (c) Coaxial cable and are represented in Fig 10.24.

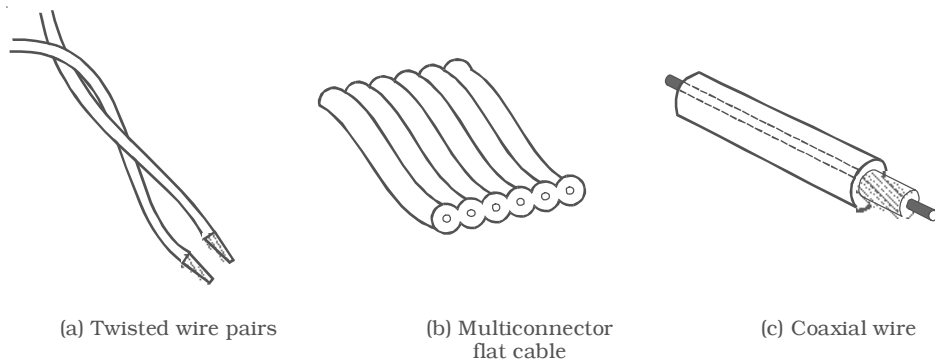


Fig 10.24 Different types of wire and cable used for communications

Twisted pair cable is the simplest and lowest cost cable. It consists of two insulated wire twisted around each other in a continuous spiral as shown in Fig 10.36a. The wire is twisted to minimise the external noise. Twisted pair is used between telephones and the central office. It is difficult to use, when many signals must be brought from one place to another.

The multiconductor flat cable consists of many parallel wires in a common plastic jacket as shown in Fig. 10.36b. A cable of this type can have any number from 10 to about 50 wires. All the wires are grouped mechanically and they can be used with a single connector at each end. Flat cable is more expensive than twisted pair.

For some applications, coaxial cable (coax) must be used. It consists of a solid-centre conductor surrounded by a plastic insulator such as teflon. Over the insulator, is a second conductor, a tubular braid or shield made of fine wires. An outer plastic insulation protects and insulates the braid. It is fairly expensive to buy and can be difficult to install, because of its mechanical stiffness and thickness.

10.10.6 Fiber optical communication

Coherent light can be generated with laser or light emitting diodes and may be detected by photo-diodes. Optical fibers are used for transmission of light.

An optical fiber is a thin transparent rod, usually made of glass or plastic, through which light can propagate. The light signals travel through the rod from the transmitter to the receiver and can be easily detected at the receiving end of the optical fiber. The principle of total

internal reflection is used for the transmission of light signals through the optical fiber.

Advantages

- (i) Transmission loss is low.
- (ii) Fiber is lighter and less bulky than equivalent copper cable.
- (iii) More information can be carried by each fiber than by equivalent copper cables.
- (iv) There is no interference in the transmission of light from electrical disturbances or electrical noise.

Applications

The various applications of fiber in communication area are, voice telephones, video phones, message services, data network etc.

10.10.7 Satellite Communication

Space technology has witnessed a phenomenal growth, since the launch of man-made satellite Sputnik in 1957. One of the most significant applications of space technology has been in the field of communications. The people over world watch international events like Olympic games via satellite. A number of countries are using satellites for military communications, which include services to ships, air crafts and land mobile terminals. Several direct TV broadcasting satellite systems are also being used.

Satellite communication is basically a microwave link repeater. A satellite receives energy from an earth station, amplifies it and returns it to each at a frequency about 2 GHz away from the uplink frequency (earth to satellite) . This prevents interference between the uplink and the downlink (satellite to earth). Satellite so used is a geostationary satellite which appears to be stationary at a given spot above the equator. Actually, it moves with the same angular velocity as the earth i.e. it completes one revolution per 24 hours and hence appears to be stationed over one spot on the globe. Satellite orbiting the earth will be geostationary when it is about 36,000 km away from the earth.

Fig 10.25 gives the general structure of a satellite communications system. A satellite in space links many earth stations. The user is connected to the earth station through terrestrial network.

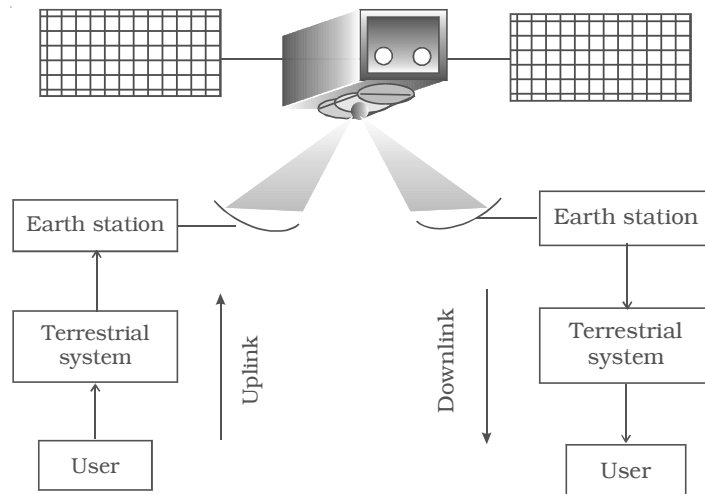


Fig 10.25 Configuration of a satellite communication

This network may assume various configurations including a telephone switch or a dedicated link to the earth station. Signal generated by the user is processed and transmitted from the earth station to the satellite. The satellite receives the modulated RF carrier at the pre-determined uplink frequencies from all the earth stations in the network, amplifies these frequencies and then re-transmits them back to earth at downlink frequencies. The downlink frequencies are kept different from the uplink frequencies in order to avoid interference. The modulated carrier received at receiving earth station is processed to get back the original baseband signal. This signal is then sent to the user through a terrestrial network.

As per WARC (World Administrative Radio Conference) 1979 allocation, commercial communication satellites use 500 MHz bandwidth near 6 GHz for uplink transmission and use 500 MHz bandwidth near 4 GHz for downlink transmission. In actual practice, uplink of 5.725 – 7.075 GHz is used while downlink of 3.4 – 4.8 GHz is used.

Merits

- (i) Mobile communication can be easily established by satellite communication.
- (ii) Satellite communication is economical compared with terrestrial communication particularly where long distances are involved.

(iii) Compared to the optical fiber communication, satellite communication has the advantages that, quality of transmitted signal and location of sending and receiving stations are independent of distance.

(iv) For thin traffic remote areas like north east regions in India, Ladakh etc., satellite communication is most economical.

(v) For search, rescue and navigation, satellite communication is far superior and economical compared to other systems.

Demerits

(i) Between talks there is a time gap which becomes quite annoying. This time delay also reduces the efficiency of satellite in data transmission.

(ii) An imperfect impedance match may cause echo, received back after a delay. Echo suppressor has to be used.

(iii) Repair of satellite is almost impossible, once it has been launched.

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Solved problems

10.1 A 10 MHz sinusoidal carrier wave of amplitude 10 mV is modulated by a 5 kHz sinusoidal audio signal wave of amplitude 6 mV. Find the frequency components of the resultant modulated wave and their amplitude.

Data: Frequency of the carrier $= f_c = 10 \text{ MHz}$
Frequency of the signal $= f_s = 5 \text{ kHz} = 0.005 \text{ MHz}$
Amplitude of the carrier signal $= E_c = 10 \text{ mV}$
Amplitude of the audio signal $= E_s = 6 \text{ mV}$
Frequency components of modulated wave = ?
Amplitude of the components in the modulated wave = ?

Solution : The modulated carrier wave contains the following frequencies :

(i) Original carrier wave of frequency $= f_c = 10 \text{ MHz}$

(ii) Upper side band frequency, $f_c + f_s = 10 + 0.005$
 $= 10.005 \text{ MHz}$

(iii) Lower side band frequency $f_c - f_s = 10 - 0.005$
 $= 9.995 \text{ MHz}$

The modulation factor is,

$$m = \frac{E_s}{E_c} = \frac{6}{10} = 0.6$$

$$\therefore \text{Amplitude of USB} = \text{Amplitude of LSB} = \frac{mE_c}{2} = \frac{0.6 \times 10}{2} = 3 \text{ mV}$$

10.2 An FM signal has a resting frequency of 105 MHz and highest frequency of 105.03 MHz when modulated by a signal. Determine (i) frequency deviation and (ii) carrier swing.

Data : Resting frequency (f) = 105 MHz

Frequency of the signal (f_s) = 5 kHz

Highest frequency of the modulated wave, (f_m) = 105.03 MHz

Frequency deviation = Δf = ? Carrier swing (CS) = ?

Solution : Frequency deviation (Δf) = $f_m - f$

$$\Delta f = 105.03 - 105 = 0.03 \text{ MHz}$$

$$\text{Carrier swing} = 2 \times \Delta f = 2 \times 0.03 = 0.06 \text{ MHz} = 60 \text{ kHz}$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

- 10.1 High frequency waves follow
- (a) the ground wave propagation
 - (b) the line of sight direction
 - (c) ionospheric propagation
 - (d) the curvature of the earth
- 10.2 The main purpose of modulation is to
- (a) combine two waves of different frequencies
 - (b) acquire wave shaping of the carrier wave
 - (c) transmit low frequency information over long distances efficiently
 - (d) produce side bands
- 10.3 In amplitude modulation
- (a) the amplitude of the carrier wave varies in accordance with the amplitude of the modulating signal.
 - (b) the amplitude of the carrier wave remains constant
 - (c) the amplitude of the carrier varies in accordance with the frequency of the modulating signal
 - (d) modulating frequency lies in the audio range
- 10.4 In amplitude modulation, the band width is
- (a) equal to the signal frequency
 - (b) twice the signal frequency
 - (c) thrice the signal frequency
 - (d) four times the signal frequency
- 10.5 In phase modulation
- (a) only the phase of the carrier wave varies
 - (b) only the frequency of the carrier wave varies.
 - (c) both the phase and the frequency of the carrier wave varies.
 - (d) there is no change in the frequency and phase of the carrier wave

- 10.6 The RF channel in a radio transmitter produces
- (a) audio signals
 - (b) high frequency carrier waves
 - (c) both audio signal and high frequency carrier waves
 - (d) low frequency carrier waves.
- 10.7 The purpose of dividing each frame into two fields so as to transmit 50 views of the picture per second is
- (a) to avoid flicker in the picture
 - (b) the fact that handling of higher frequencies is easier
 - (c) that 50 Hz is the power line frequency in India
 - (d) to avoid unwanted noises in the signals
- 10.8 Printed documents to be transmitted by fax are converted into electrical signals by the process of
- (a) reflection
 - (b) scanning
 - (c) modulation
 - (d) light variation
- 10.9 What are the different types of radio wave propagation?
- 10.10 Explain the ground wave propagation.
- 10.11 Explain the wave propagation in ionosphere.
- 10.12 What is meant by skip distance?
- 10.13 What is the necessity of modulation?
- 10.14 Explain amplitude modulation.
- 10.15 Define modulation factor.
- 10.16 Define bandwidth.
- 10.17 What are the limitations of amplitude modulation?
- 10.18 Explain frequency modulation.
- 10.19 What is phase modulation?
- 10.20 Define directivity.
- 10.21 Draw the block diagram of AM radio transmitter.
- 10.22 Explain the function of FM transmitter with neat block diagram.
- 10.23 What is meant by scanning?

- 10.24 What is interlaced scanning?
- 10.25 Explain the function of a vidicon camera tube.
- 10.26 Explain the functions of various units in the monochrome television transmission.
- 10.27 Explain the functional block diagram of a monochrome TV receiver.
- 10.28 Explain the principle of radar.
- 10.29 What are the applications of radar?
- 10.30 Explain the principle of modem.
- 10.31 What are the different types of wire and cable used for telecommunication system ?
- 10.32 What are the advantages of fiber optic communication system?
- 10.33 In a broadcasting studio, a 1000 kHz carrier is modulated by an audio signal of frequency range, 100–5000 Hz. Find (i) maximum and minimum frequencies of USB (ii) maximum and minimum frequencies of LSB and (iii) width of the channel.

Answers

10.1 (c) **10.2** (c) **10.3** (a) **10.4** (b)

10.5 (c) **10.6** (b) **10.7** (a) **10.8** (b)

10.33 1005 kHz and 1000.1 kHz,
999.9 kHz and 995 kHz, 10 kHz.